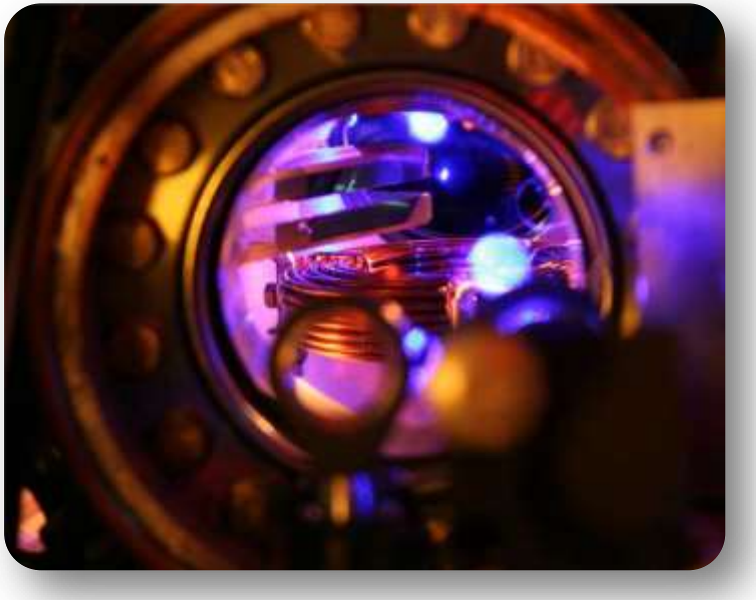
(Transportable) Optical Lattice Clocks

Christian Lisdat



Outline:

- Clock instability

  Sensitivity function, Dick effect

- Optical lattice

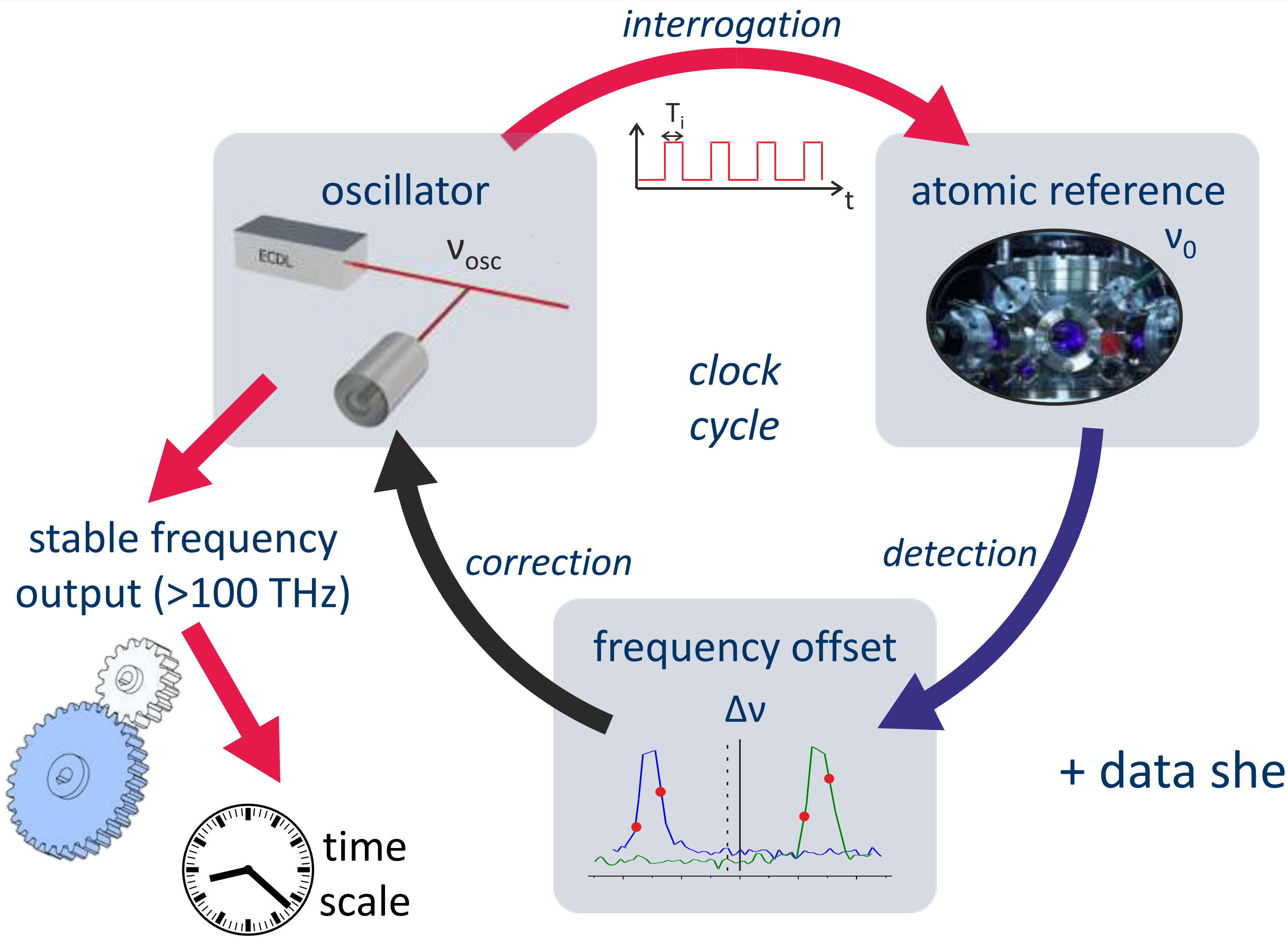
  Doppler, trapping, Lamb-Dicke

- An essential tool: the clock laser

  critical aspects, thermal noise, beyond 10^{-16} instability

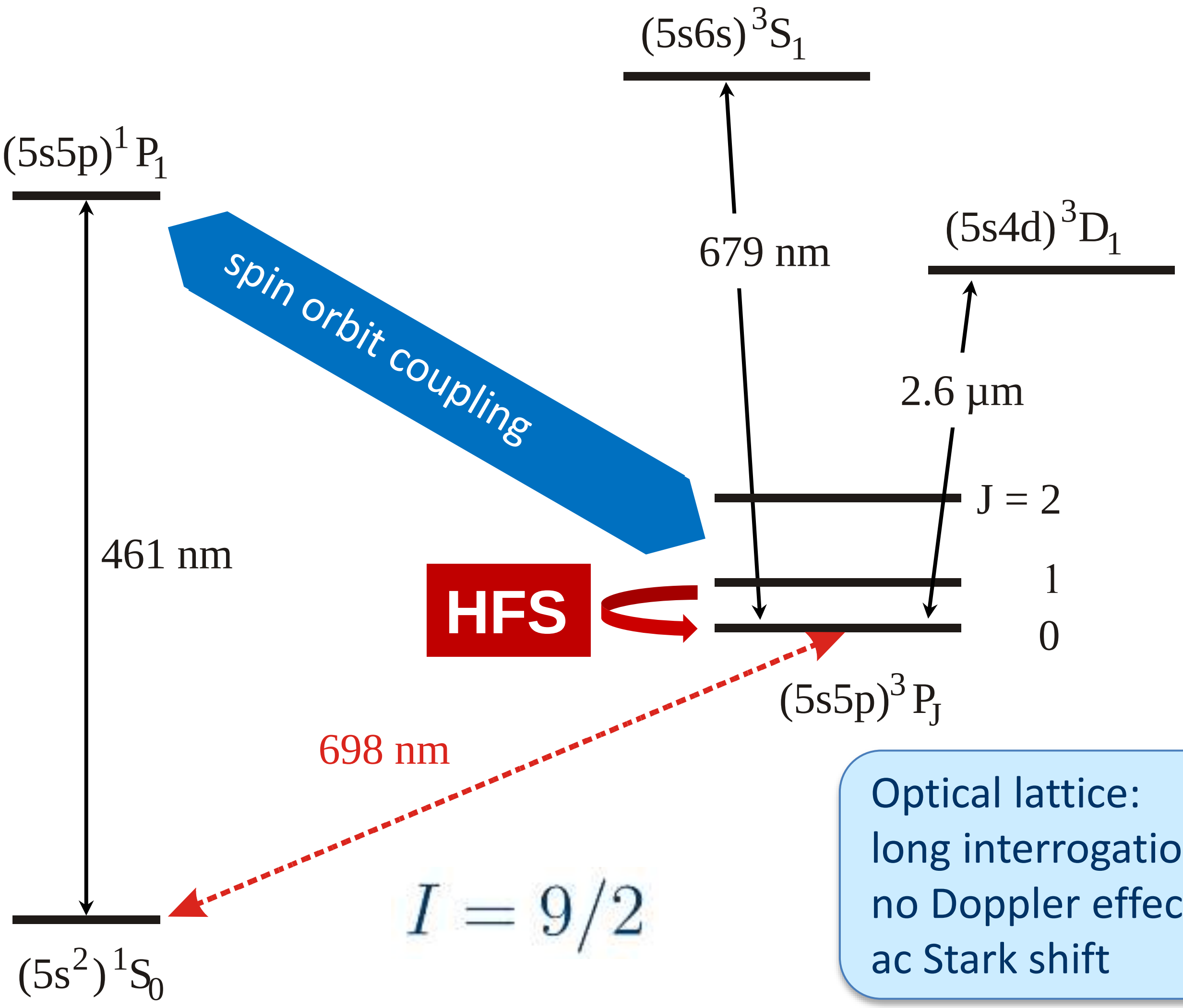
- 2nd part: Applications – clock comparisons, geodesy

A clock is...

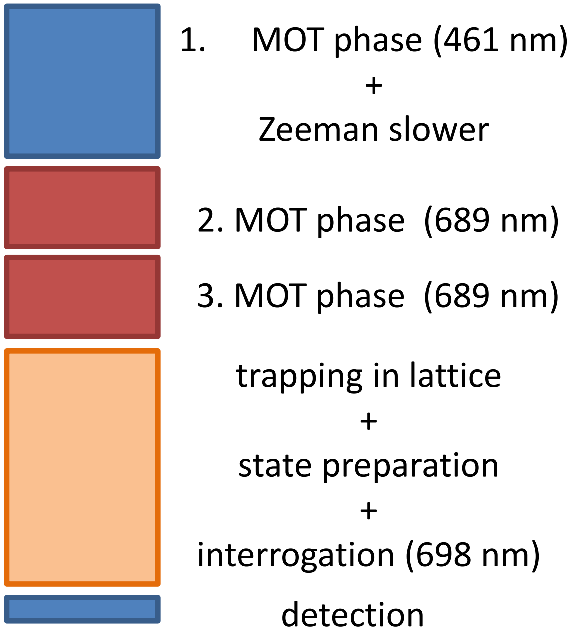


+ data sheet

Laser cooling of strontium-87



$I = 9/2$



1. MOT phase (461 nm)
+
Zeeman slower

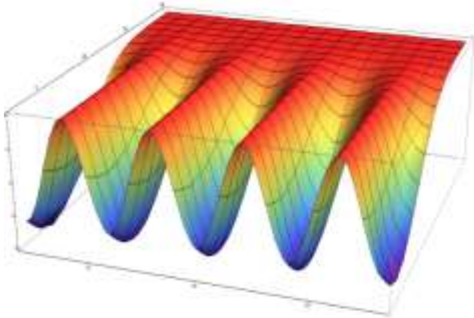
2. MOT phase (689 nm)

3. MOT phase (689 nm)

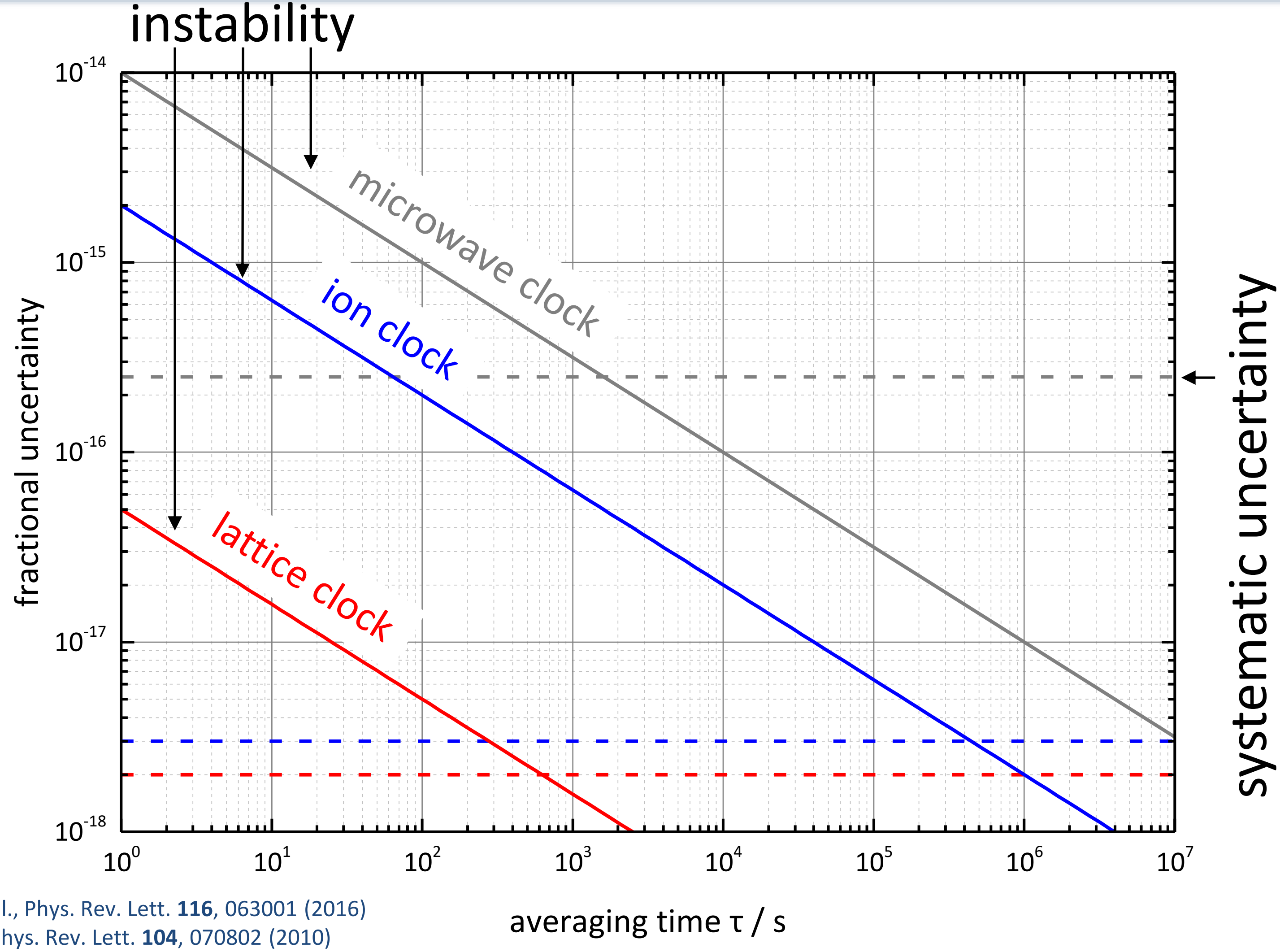
trapping in lattice
+
state preparation
+
interrogation (698 nm)

detection

Optical lattice:
long interrogation,
no Doppler effect,
ac Stark shift



Stability: resolve a frequency



N. Huntemann et al., Phys. Rev. Lett. **116**, 063001 (2016)
C. W. Chou et al., Phys. Rev. Lett. **104**, 070802 (2010)
T. Nicholson et al., Nature Com. **6**, 6896 (2015)
M. Schioppo et al., Nature Photonics **11**, 48 (2017)

Sensitivity function:

$$\sigma_{p_\mathrm{e}} = \frac{dp_\mathrm{e}}{d\nu}\sigma_\nu$$

Which change of p_e will I get for a phase/frequency jump during the interrogation?

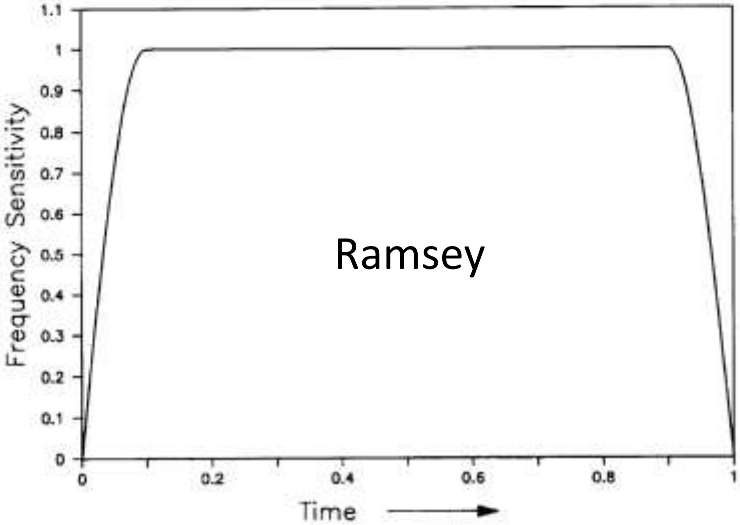
$$\delta P = \frac{1}{2}\int_0^{T_c} 2\pi g(t)\delta\nu(t)dt \qquad g(t) = 2\lim_{\Delta\phi\to 0}\delta P(t,\Delta\phi)/\Delta\phi$$

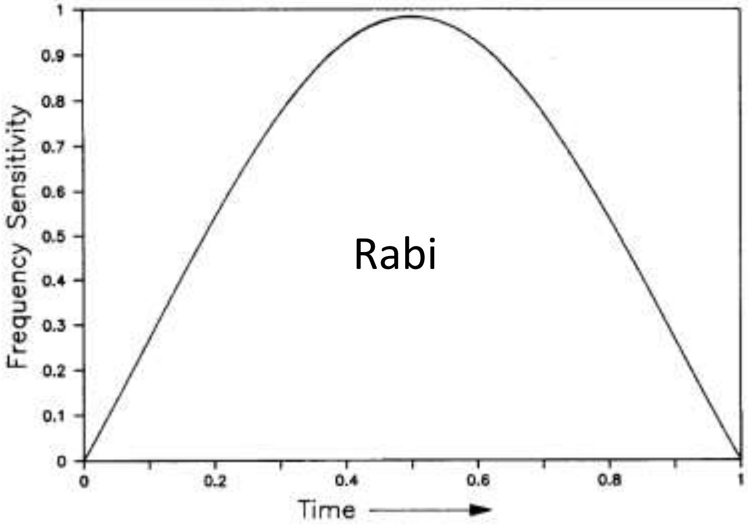
G. J. Dick, *Proceedings of 19th Annu. Precise Time and Time Interval Meeting*, Redendo Beach, 1987, 133 – 147 (1988)
G. Santarelli, C. Audoin, A. Makdissi, P. Laurent, G. J. Dick, A. Clairon *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **45**, 887 – 894 (1998)
A. Quessada, R. P. Kovacich, I. Courtillot, A. Clairon, G. Santarelli, P. Lemonde, *J. Opt. B: Quantum Semiclass. Opt.* **5**, S150 – S154 (2003)

Sensitivity function:

$$\delta P = \frac{1}{2}\int_0^{T_c} 2\pi g(t)\delta\nu(t)dt \quad g(t) = \begin{cases} \sin(\Omega t) & 0 < t < Tp \\ 1 & T_p < t < T_p + T \\ \sin(\Omega(t-T)) & T_p + T < t < 2T_p + T \\ 0 & \text{otherwise} \end{cases}$$

approximately!





G. J. Dick, *Proceedings of 19th Annu. Precise Time and Time Interval Meeting*, Redendo Beach, 1987, 133 – 147 (1988)
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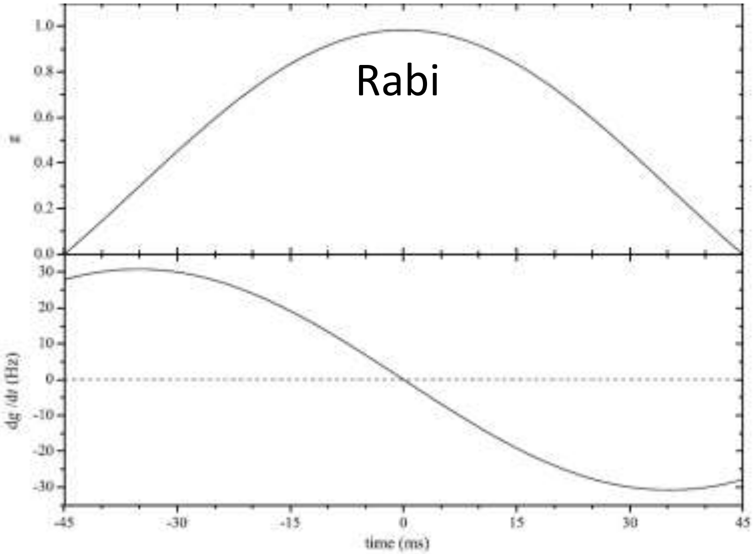
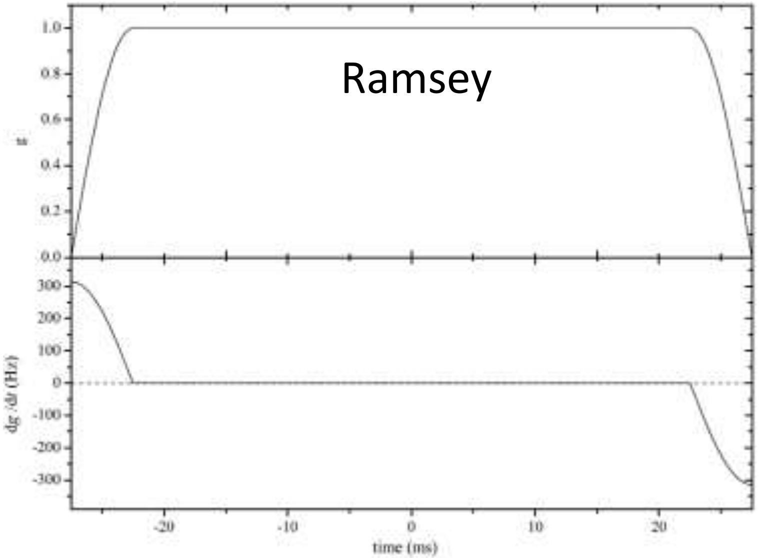
A. Quessada, R. P. Kovacich, I. Courtillot, A. Clairon, G. Santarelli, P. Lemonde, *J. Opt. B: Quantum Semiclass. Opt.* **5**, S150 – S154 (2003)
A. Al-Masoudi *et al.*, Phys. Rev. A **92**, 063814 (2015)

Sensitivity function:

$$\begin{aligned}\delta P &= \frac{1}{2}\int_0^{T_c} 2\pi g(t)\delta\nu(t)dt \\ &= -\frac{1}{2}\int_0^{T_c} \frac{\mathrm{d}}{\mathrm{d}t}g(t)\delta\phi(t)dt\end{aligned}$$





G. J. Dick, *Proceedings of 19th Annu. Precise Time and Time Interval Meeting*, Redendo Beach, 1987, 133 – 147 (1988)
G. Santarelli, C. Audoin, A. Makdissi, P. Laurent, G. J. Dick, A. Clairon *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **45**, 887 – 894 (1998)
A. Quessada, R. P. Kovacich, I. Courtillot, A. Clairon, G. Santarelli, P. Lemonde, *J. Opt. B: Quantum Semiclass. Opt.* **5**, S150 – S154 (2003)
A. Al-Masoudi *et al.*, Phys. Rev. A **92**, 063814 (2015)

Dick effect

Sensitivity function and noisy laser:

$$\sigma_y^2(\tau) = \frac{1}{\tau}\sum_{m=1}^{\infty}\left(\frac{g_m^{c2}}{g_0^2}+\frac{g_m^{s2}}{g_0^2}\right)S_y^f(m/T_c)$$

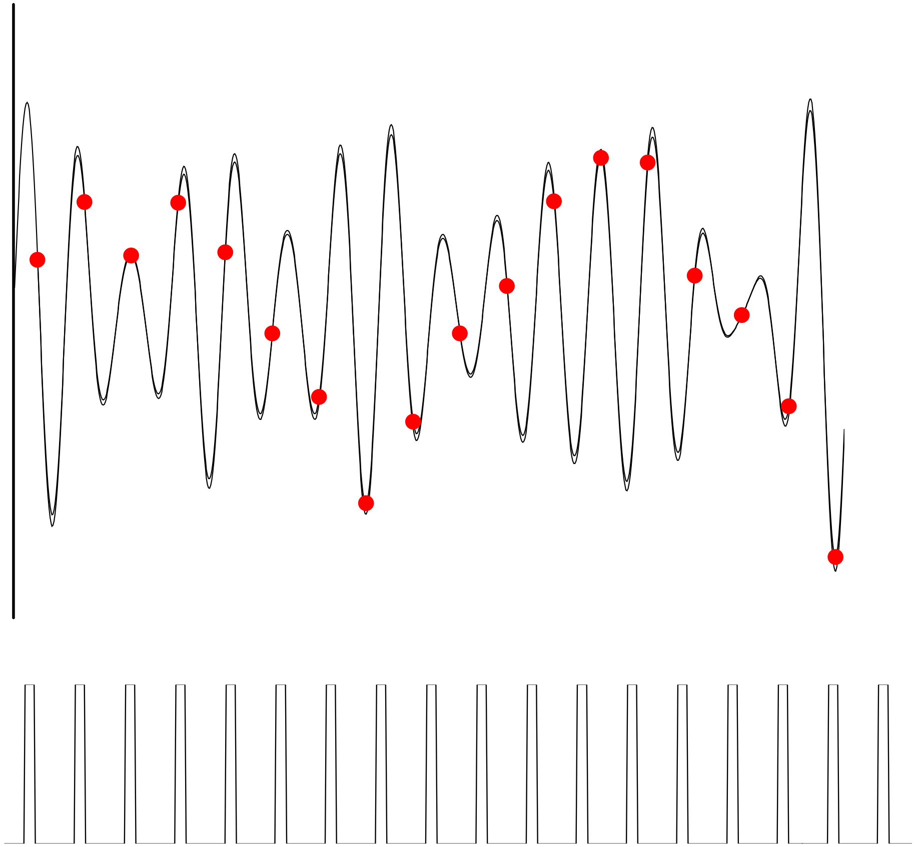
S_y^f: one-sided power spectral density

$$\begin{aligned}\begin{pmatrix} g_m^s \\ g_m^c \end{pmatrix} &= \frac{1}{T_c}\int_0^{T_c} g(t)\begin{pmatrix}\sin(2\pi m t/T_c)\\ \cos(2\pi m t/T_c)\end{pmatrix}dt\\ g_0 &= \frac{1}{T_c}\int_0^{T_c} g(t)dt\end{aligned}$$

G. Santarelli, C. Audoin, A. Makdissi, P. Laurent, G. J. Dick, A. Clairon *IEEE Trans. Ultrason. Ferroelectr. Freq. Control* **45**, 887 – 894 (1998)

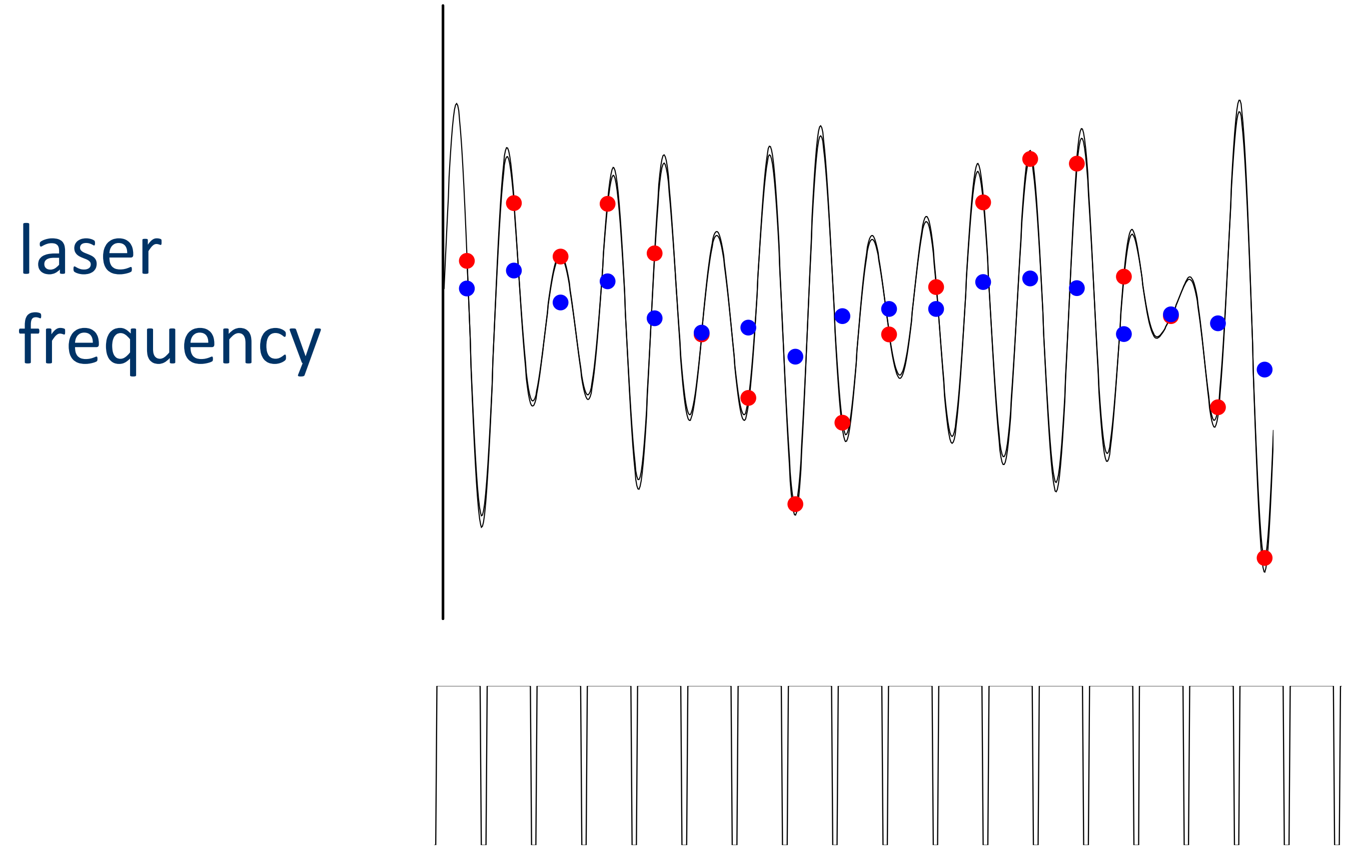
Dick effect

laser frequency



interrogation with long dead time

Dick effect



interrogation with long dead time

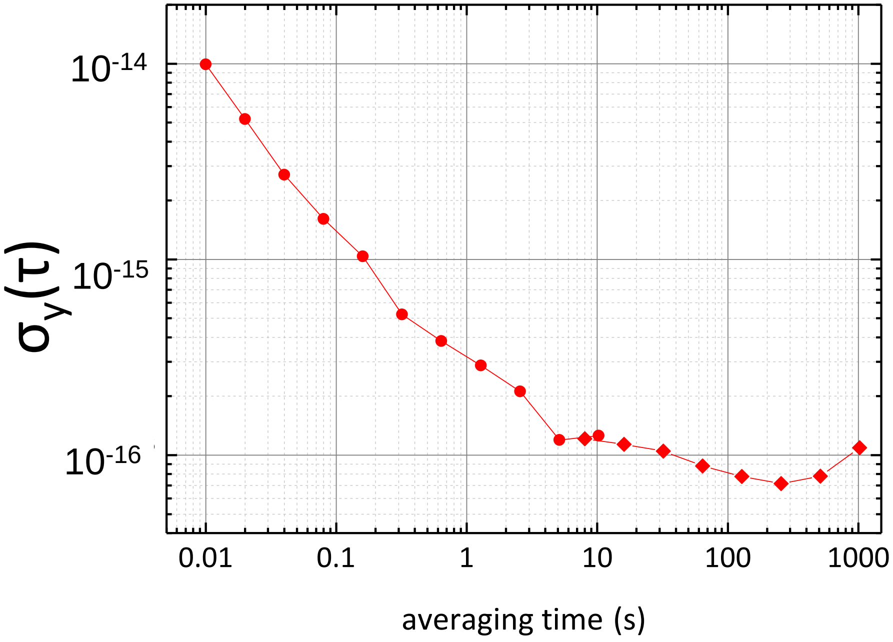
Dick effect

$$\sigma_y(\tau) = \sqrt{\frac{S_y}{\tau}\left(\frac{1}{d} - 1\right)}$$

frequency standard‘s instability for white frequency noise of the laser, *d*: duty cycle

$$S_y = 2 \cdot \sigma_y^2(1\ \mathrm{Hz})$$

Very bad for lattice clocks: loading and state preparation in every cycle



Dick effect

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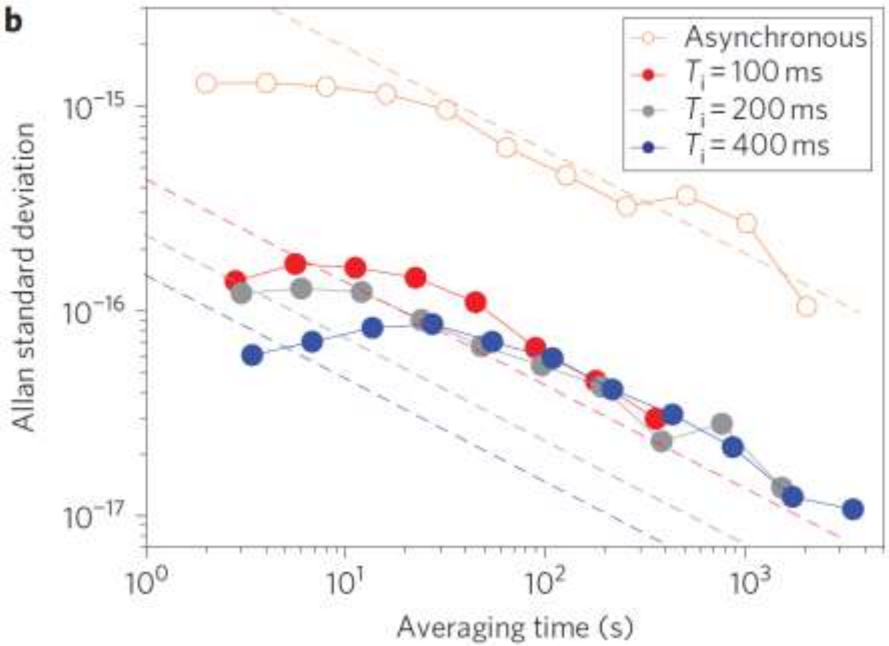
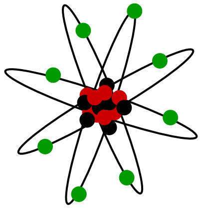


Correlated interrogation of two clocks

M. Takamoto et a.., Nature Phot. **5**, 288 (2011)

Wants and Don‘t wants

travelling light wave ν_L



atom; v = 100 m/s

Doppler shift:

$$\frac{v}{c}\nu_\mathrm{L} \approx 3 \times 10^{-7}\nu_\mathrm{L}$$

experimental tricks:

velocity ⊥ light beam

cold atoms 1 cm/s

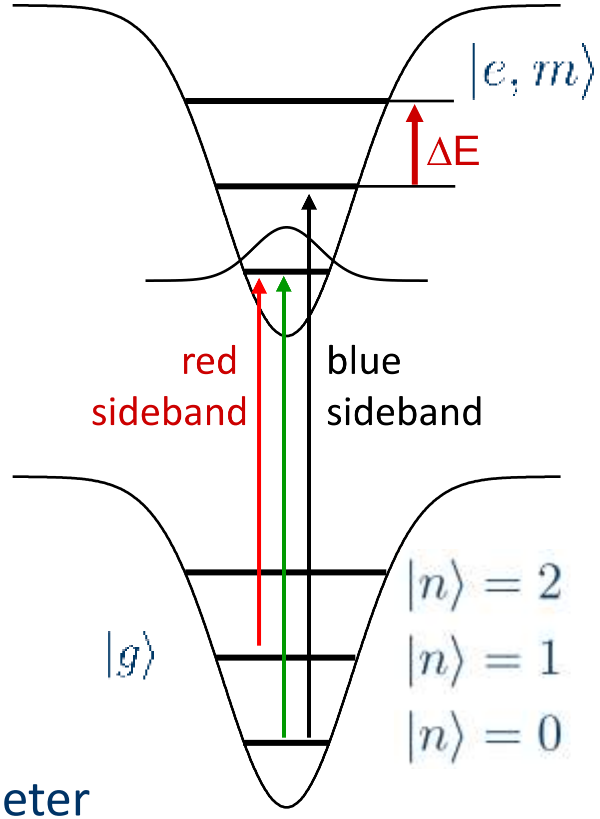
fountains

standing waves in cavities

reduction by 10^{9};
for 10^{-18} still factor 100 missing !

Wants and Don‘t wants: Doppler

Harmonic Trap $\Delta E = \hbar\omega$



$$|\Psi(x)|^2 = \sqrt{\tfrac{m\omega}{\hbar}} \exp -\tfrac{x^2 m\omega}{\hbar}$$

$$\begin{aligned} & \langle g,n|\, \vec{d}\cdot\vec{E}_0 e^{ikx}\, |e,m\rangle \\ = \quad & \langle g|\, \vec{d}\cdot\vec{E}_0\, |e\rangle \, \langle n|\, e^{ikx}\, |m\rangle \\ = \quad & \tfrac{\hbar}{2}\Omega \, \langle n|\, e^{ikx}\, |m\rangle \end{aligned}$$

strong confinement $kx \ll 1$

$$\approx \quad \tfrac{\hbar}{2}\Omega \, \langle n|\, 1 + ikx\, |m\rangle$$

ladder operators $a, a^\dagger$ $\eta = k\sqrt{\frac{\hbar}{2m\omega}}$ Lamb Dicke parameter

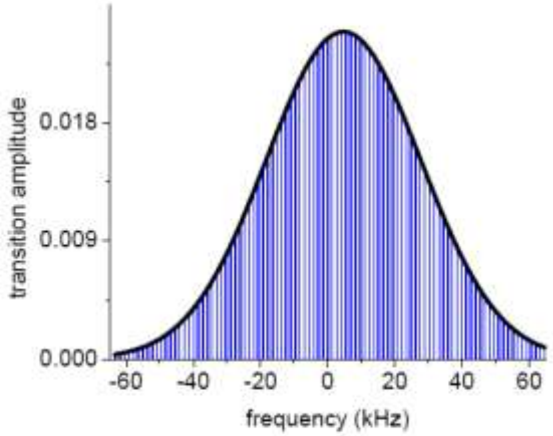
$$= \quad \tfrac{\hbar}{2}\Omega \left(\underbrace{\delta_{n,m}}_{\text{carrier}} + \underbrace{i\eta\sqrt{n}\,\delta_{m=n-1}}_{\text{red sideband}} + \underbrace{i\eta\sqrt{n+1}\,\delta_{m=n+1}}_{\text{blue sideband}} \right)$$

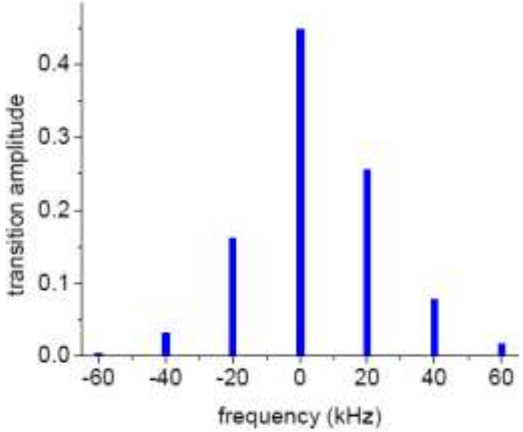
Wants and Don‘t wants: Doppler

$$\Omega_{mn} = \begin{cases} e^{-\frac{\eta^2}{2}}\sqrt{\frac{n!}{m!}}\,\eta^{m-n}L_n^{m-n}(\eta^2) & \text{if } m > n \\ e^{-\frac{\eta^2}{2}}L_n^0(\eta^2) & \text{if } m = n \\ e^{-\frac{\eta^2}{2}}\sqrt{\frac{m!}{n!}}\,\eta^{n-m}L_n^{n-m}(\eta^2) & \text{if } m < n \end{cases}$$

$$L_n^\alpha(x) = \sum_{p=0}^{n}(-1)^p \begin{pmatrix} n+\alpha \\ n-p \end{pmatrix} \frac{x^p}{p!}$$

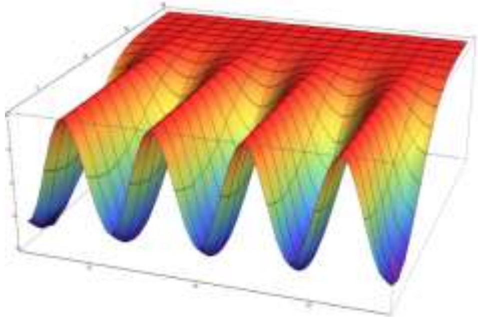
Laguerre functions



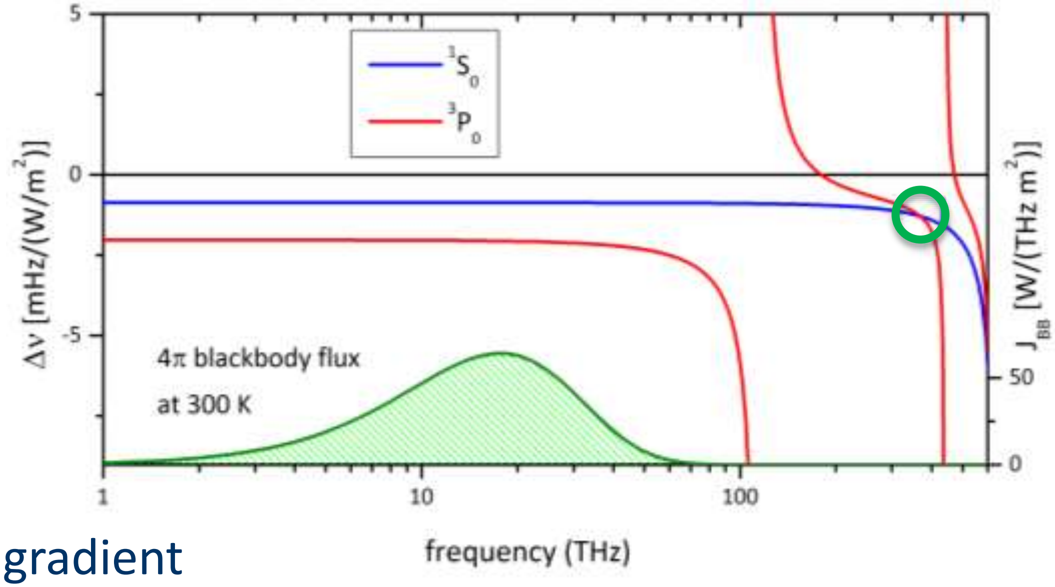


A. Ludlow, PhD thesis 2008

Lattice stuff:



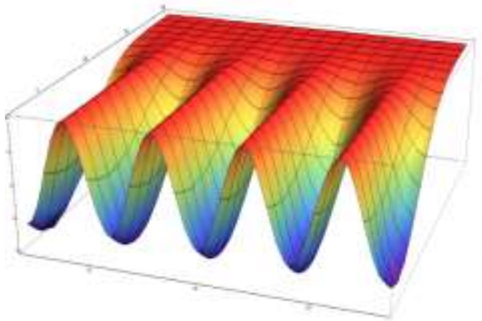
trapping in intensity gradient

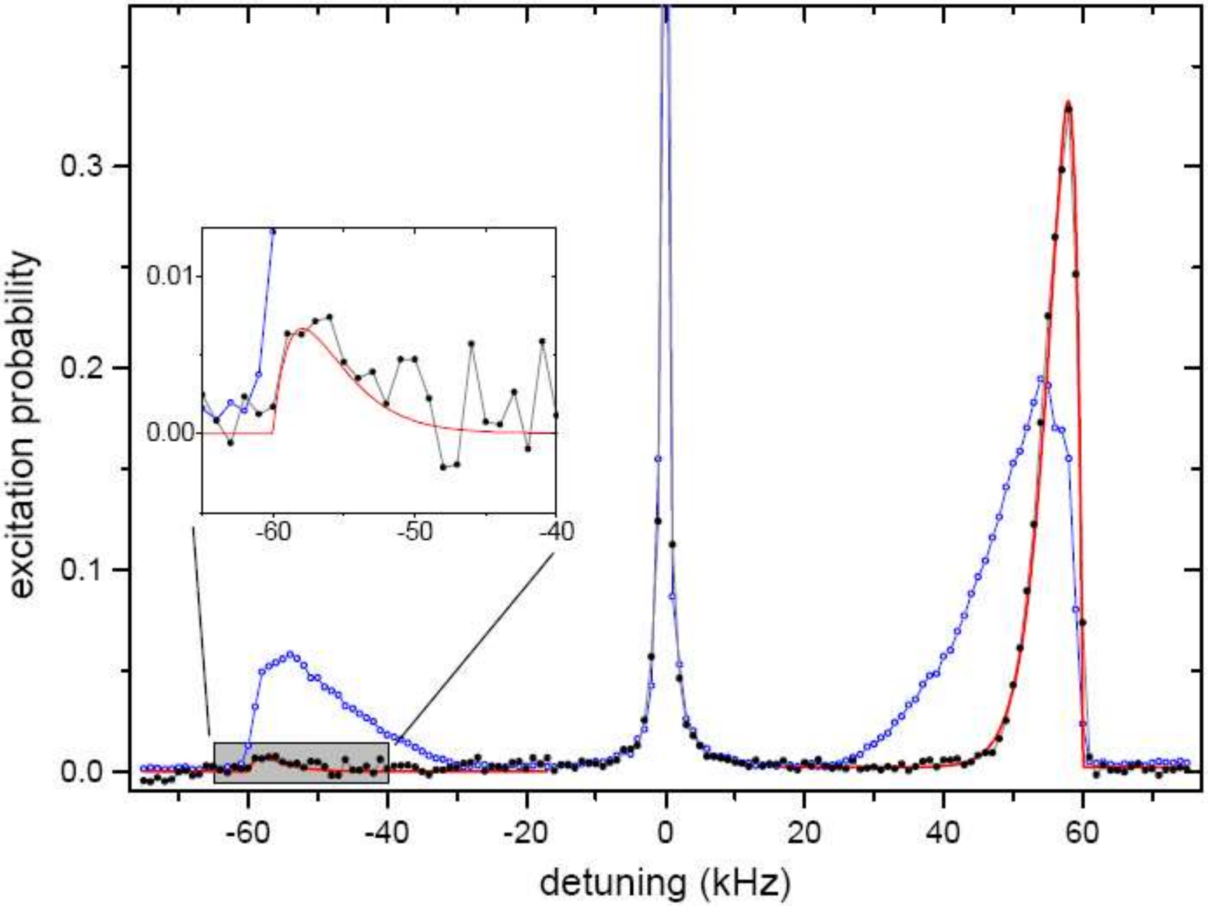


required relative shift
to beat gravity: several 10^{-10}

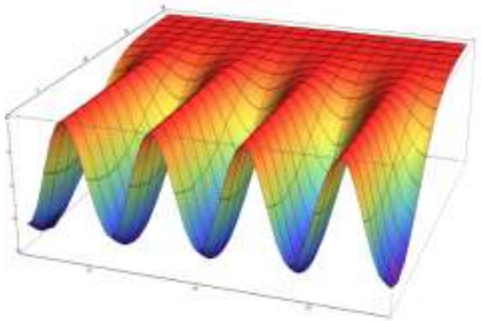
for 10^{-18}: control of the effect at 10^{-8} needed

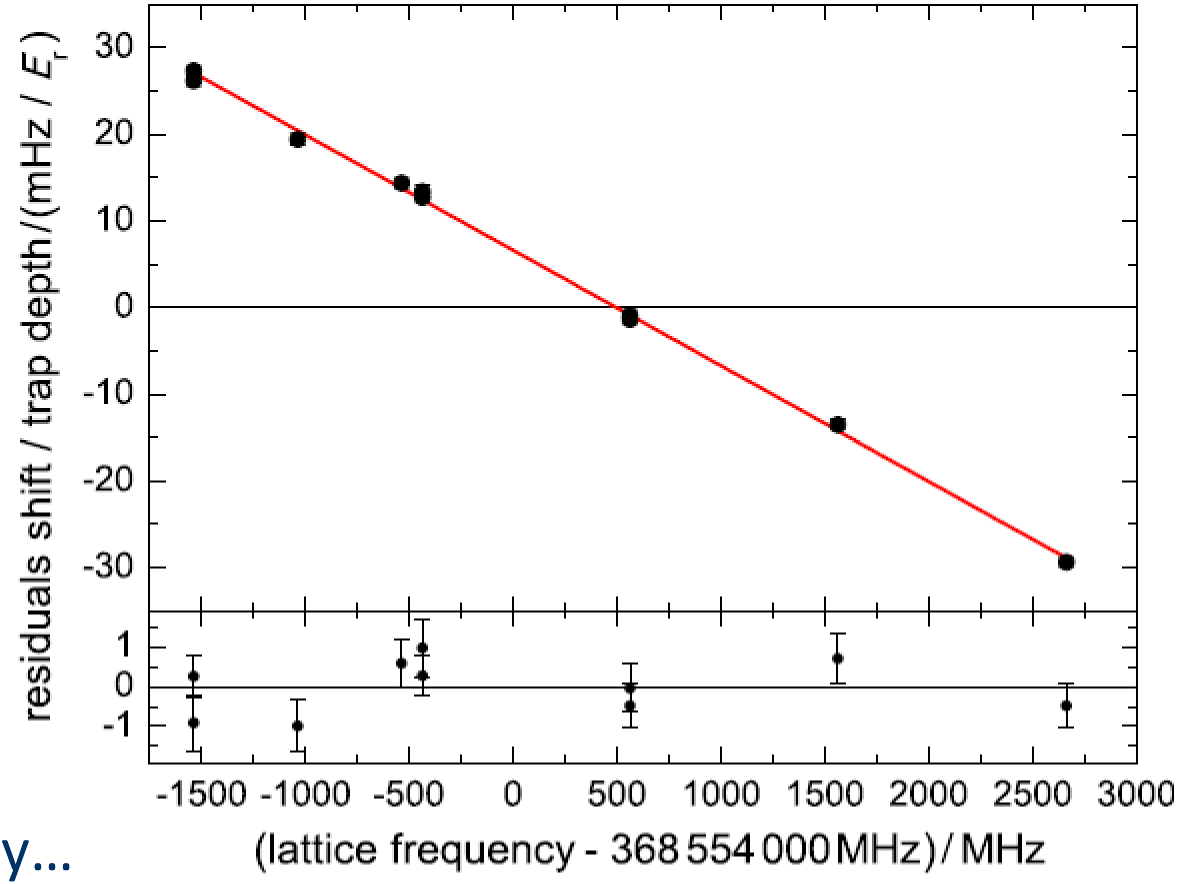
Lattice stuff:





Lattice stuff:





linear in intensity, easy…

S. Falke, H. Schnatz, J. S. R. Vellore Winfred, T. Middelmann, S. Vogt, S. Weyers, B. Lipphardt, G. Grosche, F. Riehle, U. Sterr, C. Lisdat
Metrologia **48**, 399 – 407 (2011)

Lattice stuff:

Easy? Hm…

Tunneling?

→Doppler shift

Dipole approximation

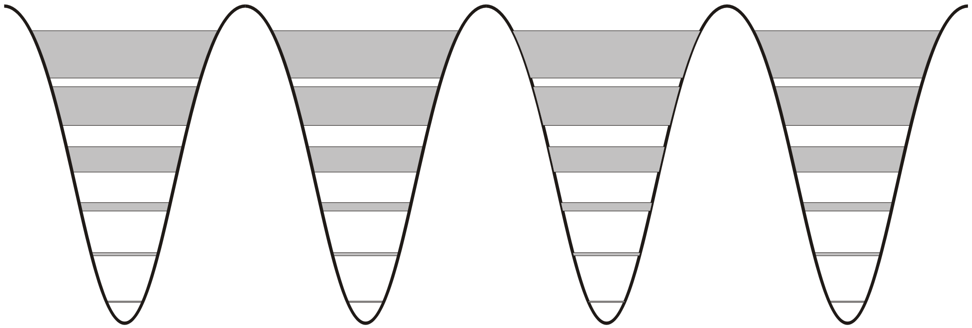
→ two-photon transitions; higher multipole transitions

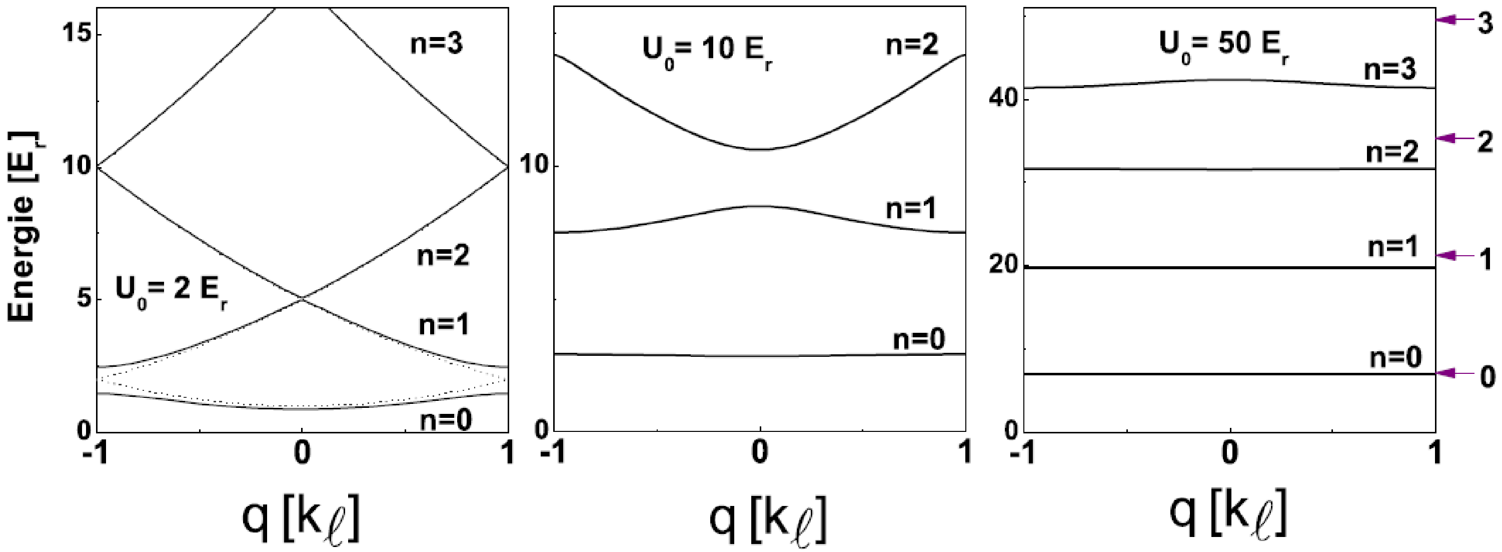
→ not linear in intensity!

Light shift will depend on polarization (if not $J = F = 0$)

→ control of polarization

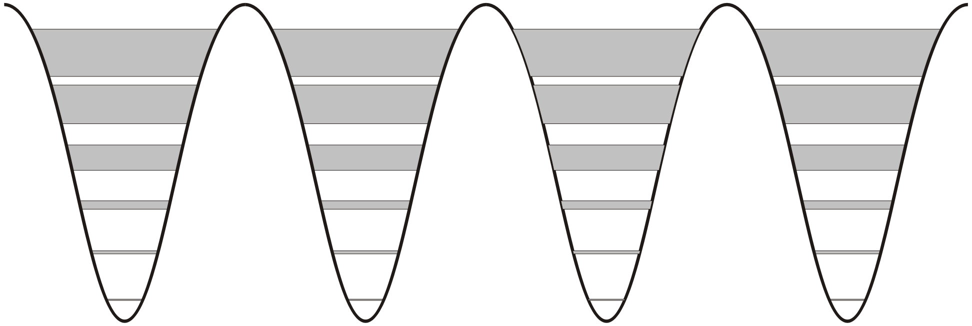
Lattice stuff:

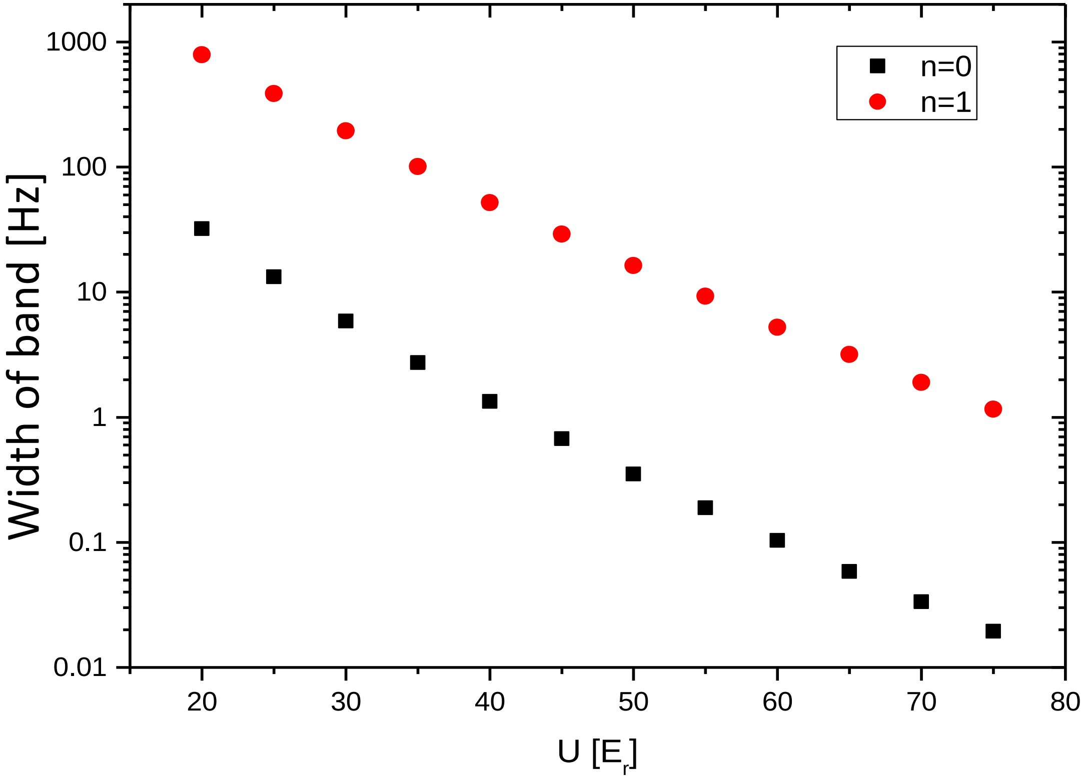




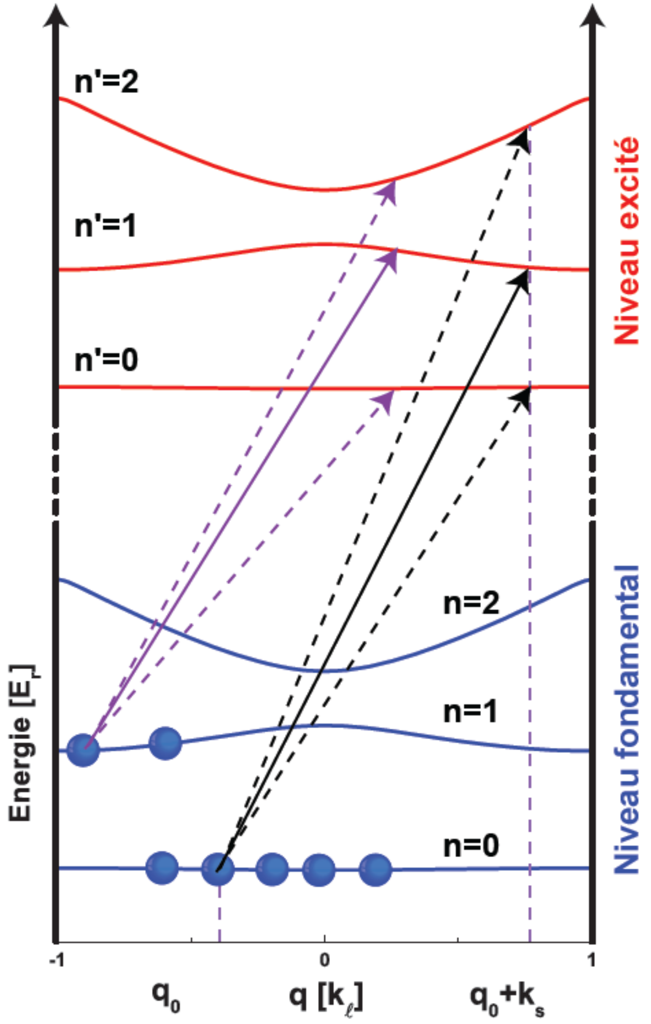
R. Le Targat PhD thesis (2007)
P. Lemonde & P. Wolf, Phys. Rev. A **72**, 033409 (2005)

Lattice stuff:





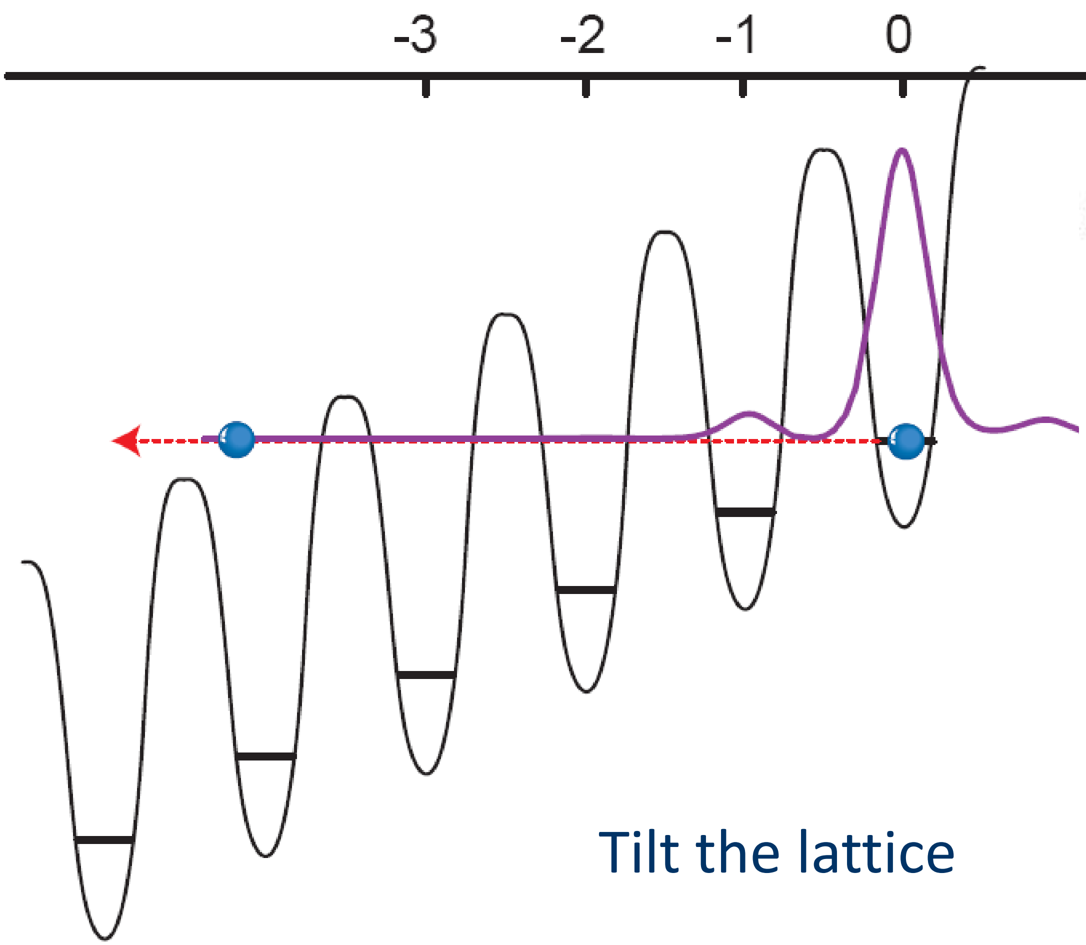
Lattice stuff:



Possible shift in order of the bandwidth!

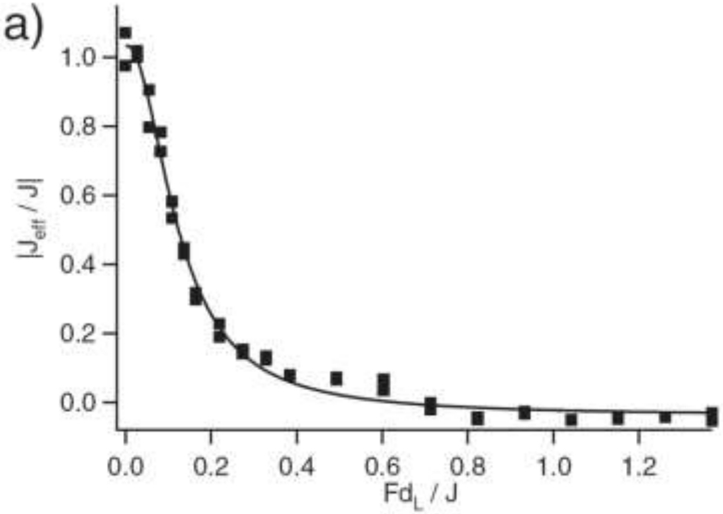
R. Le Targat PhD thesis (2007)
P. Lemonde & P. Wolf, Phys. Rev. A **72**, 033409 (2005)

Lattice stuff:



Tilt the lattice

R. Le Targat PhD thesis (2007)
P. Lemonde & P. Wolf, Phys. Rev. A **72**, 033409 (2005)



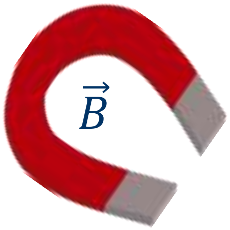
C. Sias et al. *Phys. Rev. Lett.* **100**, 040404 (2008)

Tilt of lattice: 0,16° (2 mm / 73 cm)

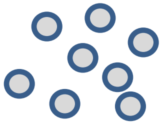
- ⇒ 2.4 Hz offset
- ⇒ suppression of tunneling

Uncertainty budget

Zeeman effect



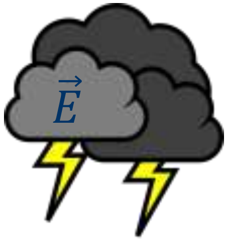
collisions

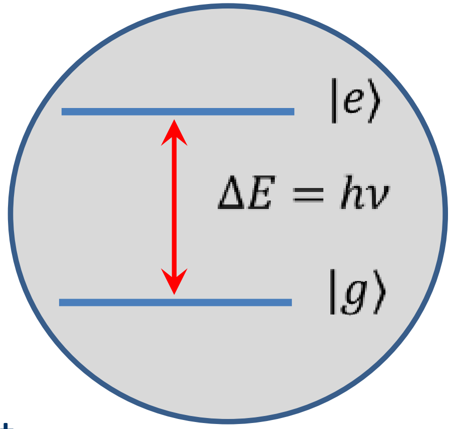


theoretical (unperturbed) frequency

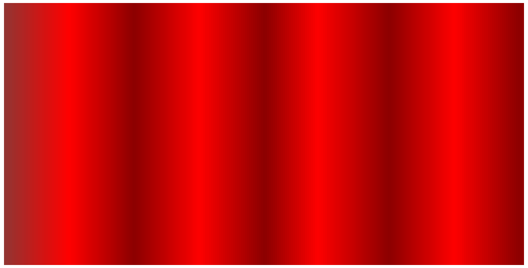
$$H\,|\Psi\rangle = E\,|\Psi\rangle$$

dc Stark effect



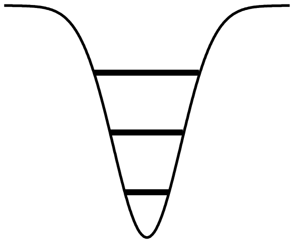


interrogation laser (Doppler effect)

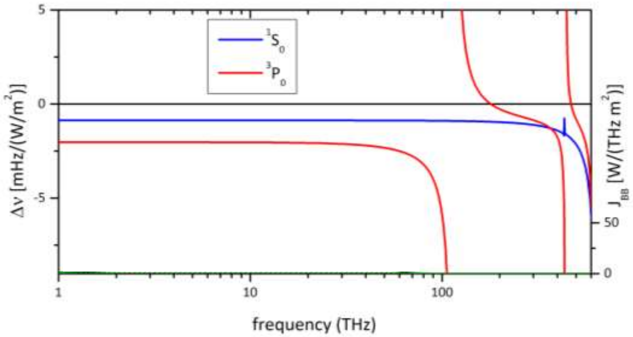


blackbody radiation shift
(ac Stark shift)

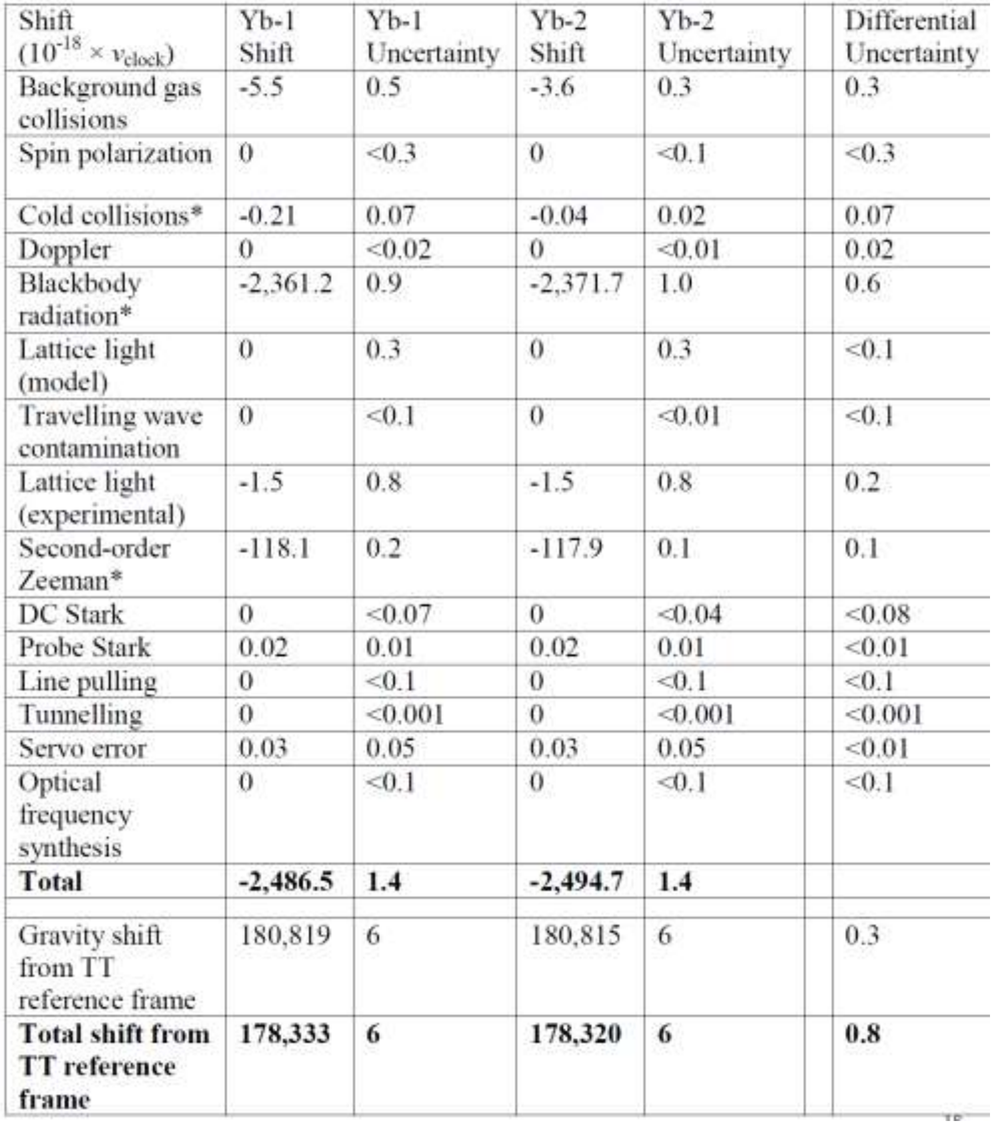
influence trap potential



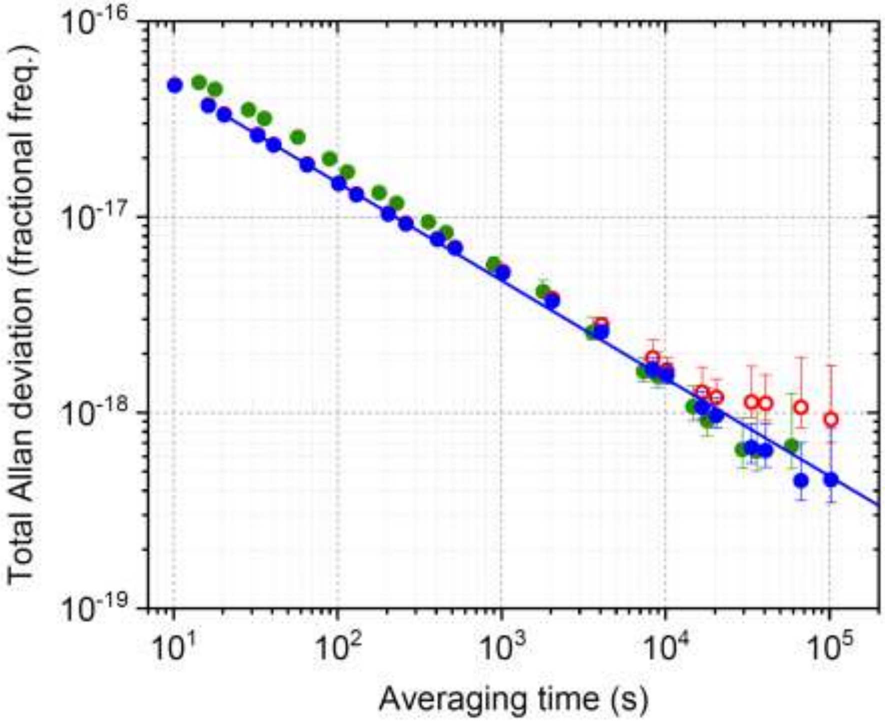
magic wavelength



Uncertainty budget



Shift ($10^{-18} \times \nu_{clock}$)	Yb-1 Shift	Yb-1 Uncertainty	Yb-2 Shift	Yb-2 Uncertainty	Differential Uncertainty
Background gas collisions	-5.5	0.5	-3.6	0.3	0.3
Spin polarization	0	<0.3	0	<0.1	<0.3
Cold collisions*	-0.21	0.07	-0.04	0.02	0.07
Doppler	0	<0.02	0	<0.01	0.02
Blackbody radiation*	-2,361.2	0.9	-2,371.7	1.0	0.6
Lattice light (model)	0	0.3	0	0.3	<0.1
Travelling wave contamination	0	<0.1	0	<0.01	<0.1
Lattice light (experimental)	-1.5	0.8	-1.5	0.8	0.2
Second-order Zeeman*	-118.1	0.2	-117.9	0.1	0.1
DC Stark	0	<0.07	0	<0.04	<0.08
Probe Stark	0.02	0.01	0.02	0.01	<0.01
Line pulling	0	<0.1	0	<0.1	<0.1
Tunnelling	0	<0.001	0	<0.001	<0.001
Servo error	0.03	0.05	0.03	0.05	<0.01
Optical frequency synthesis	0	<0.1	0	<0.1	<0.1
Total	**-2,486.5**	**1.4**	**-2,494.7**	**1.4**	
Gravity shift from TT reference frame	180,819	6	180,815	6	0.3
Total shift from TT reference frame	**178,333**	**6**	**178,320**	**6**	**0.8**



Record data from NIST Yb lattice clock

$u = 1.4 \times 10^{-18}$

McGrew *et al.*, arXiv 1807.11282

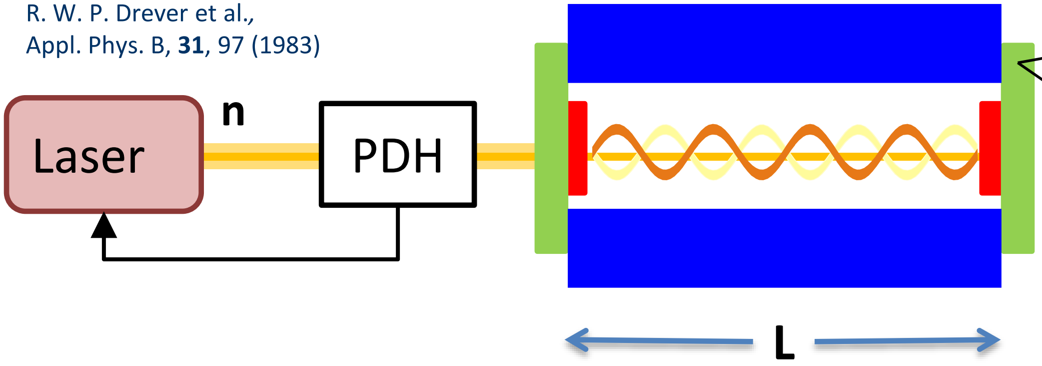
Clock lasers

Frequency stability is determined by the optical length stability:

$$\frac{\Delta\nu}{\nu} = -\frac{\Delta L}{L}$$

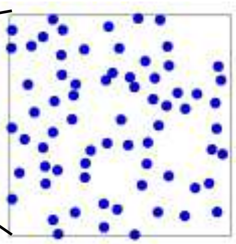
Fabry-Pérot cavity

R. W. P. Drever et al., Appl. Phys. B, **31**, 97 (1983)



fundamental noise source

Brownian thermal noise



thermal motion

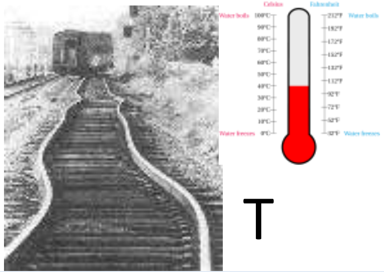
technical noise sources

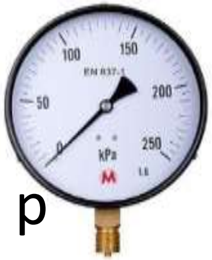
temperature fluctuations

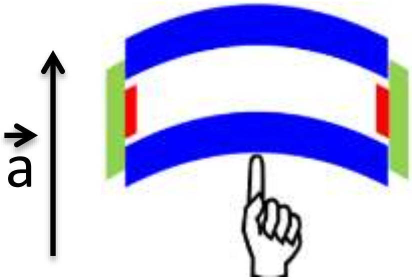
pressure variations

seismic and acoustic vibrations

others ....



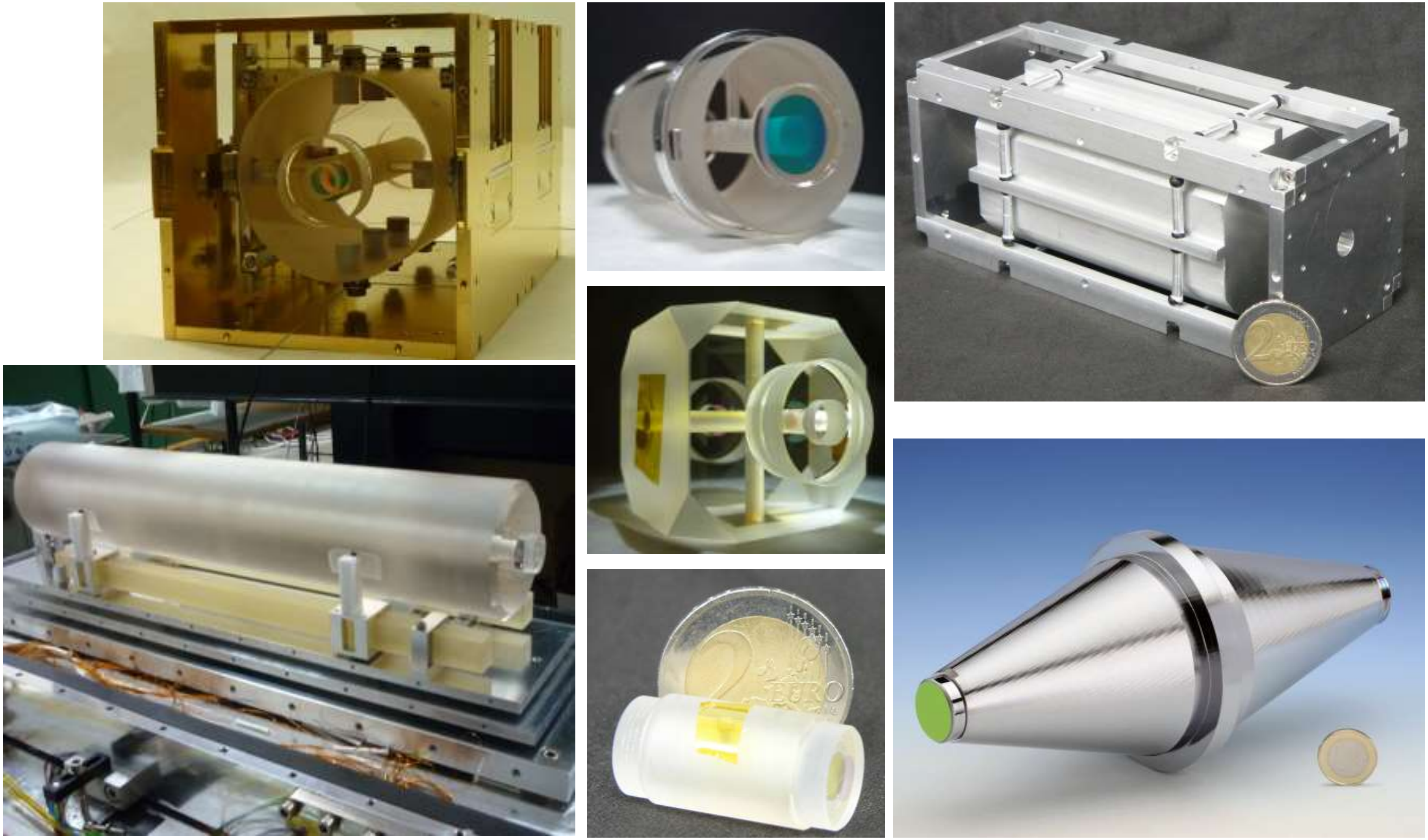




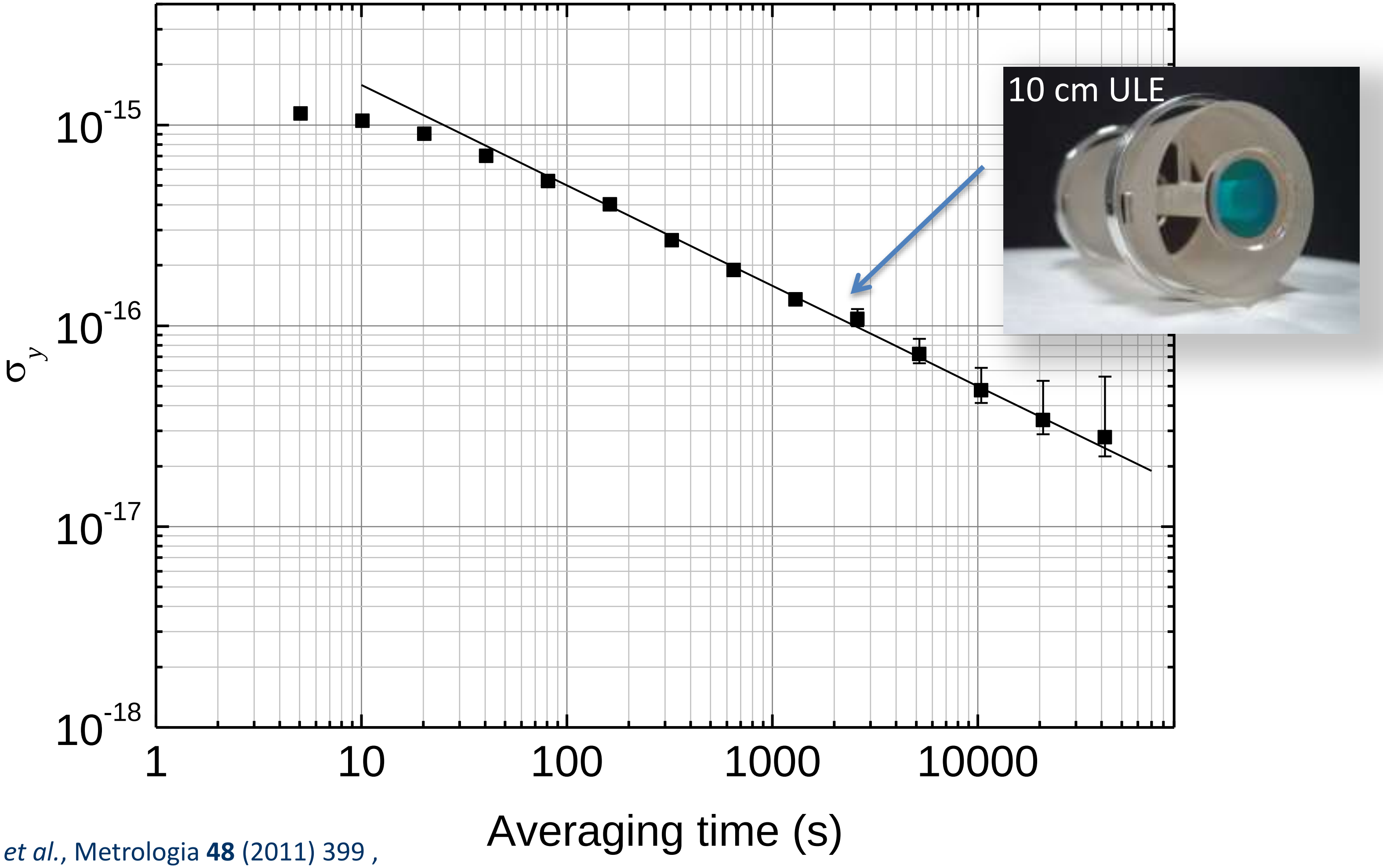
intensity fluctuations

residual amplitude modulation (RAM)

Clock lasers



Clock lasers improving clocks



St. Falke *et al.*, Metrologia **48** (2011) 399 ,
Ch. Hagemann *et al.*, IEEE Trans. Instr. Meas. **62** (2013) 1556
A. Al-Masoudi *et al.*, PRA **92**, 063814 (2015)

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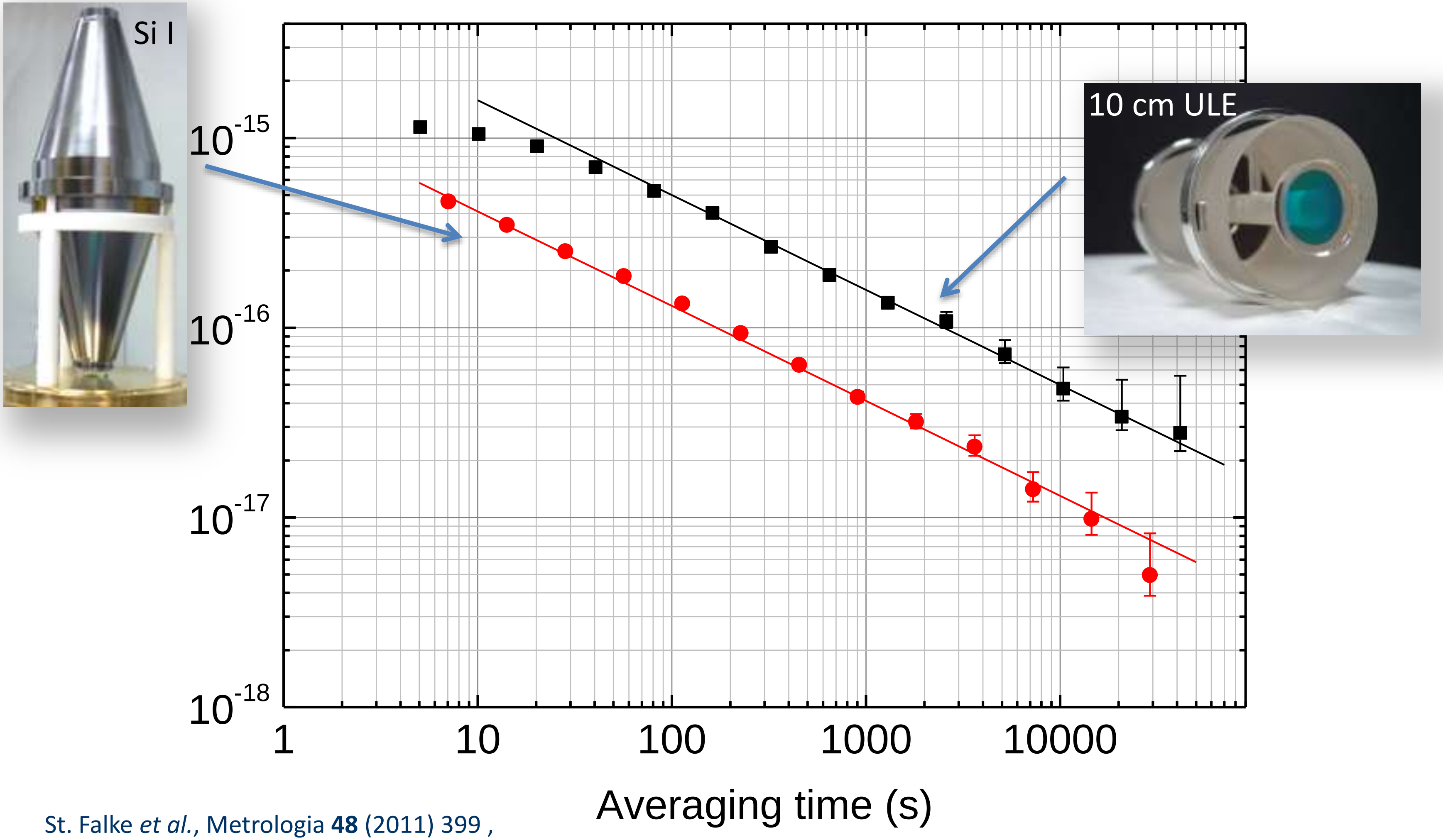

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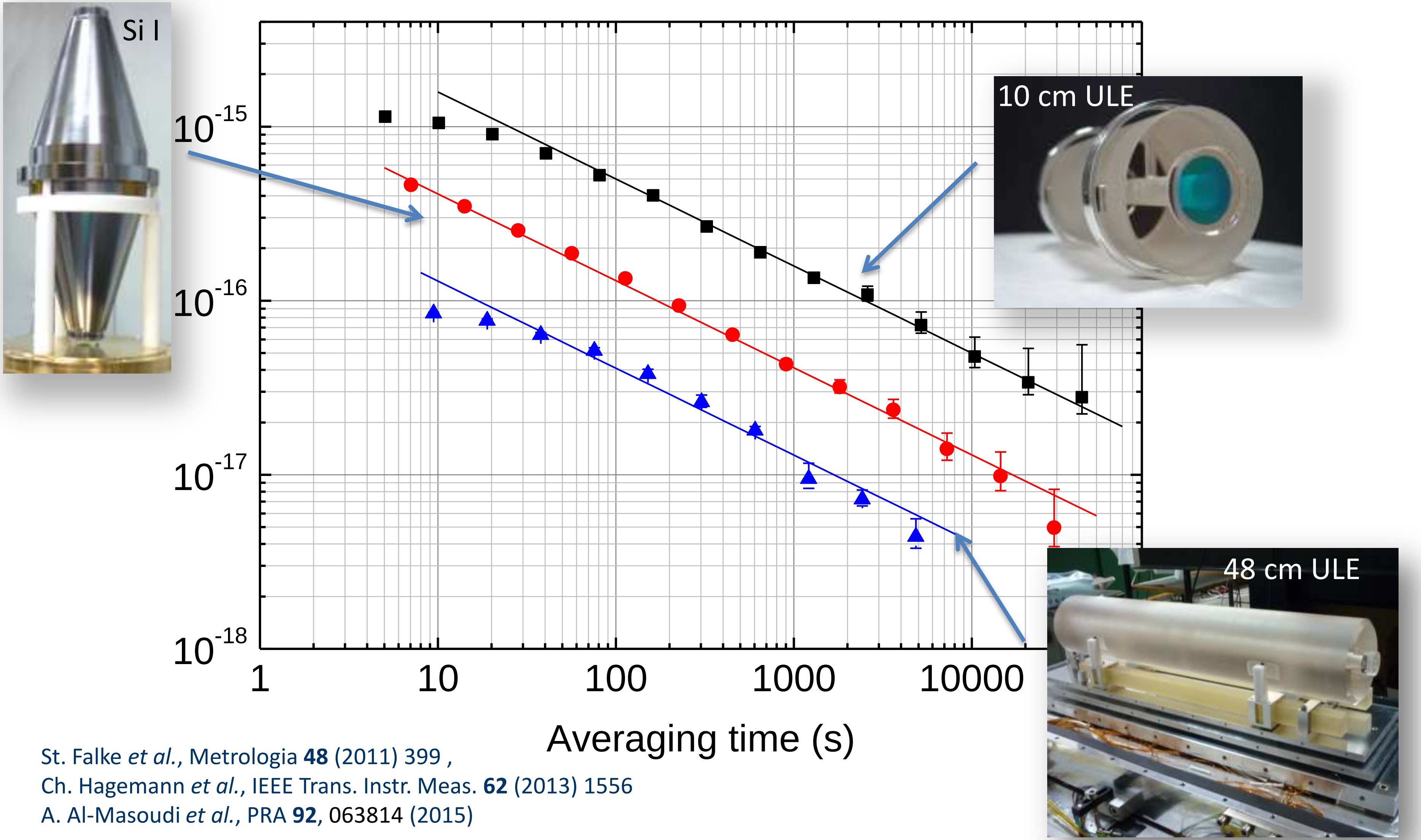
Clock lasers improving clocks



St. Falke *et al.*, Metrologia **48** (2011) 399 ,
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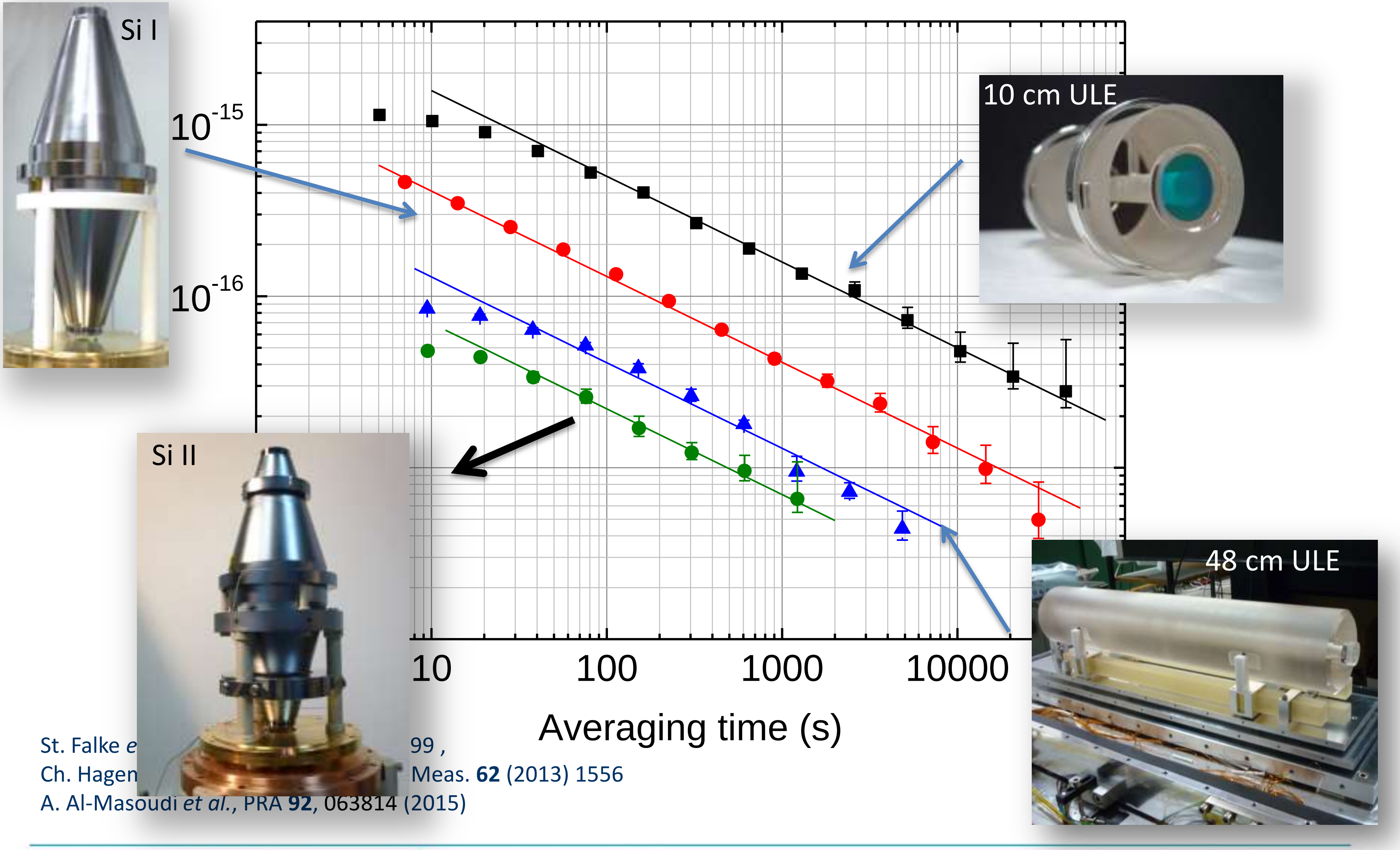


St. Falke *e* 99 ,
Ch. Hagen Meas. **62** (2013) 1556
A. Al-Masoudi *et al.*, PRA **92**, 063814 (2015)

Clock lasers improving clocks

Spacer and mirror substrates in <111> orientation

FEM optimized cavity shape for minimal vibration sensitivity

mirrors optically contacted in the same crystallographic orientation as the spacer

High finesse ion beam sputtered SiO_2/Ta_2O_5 coatings for 1542 nm.

expected thermal noise limit at T =123.5 K:

mod $\sigma_y \approx 4\times10^{-17}$

$$\frac{\Delta\nu}{\nu} = -\frac{\Delta L}{L}$$

absolute length fluctuations $\approx 8.5\times10^{-18}$ m

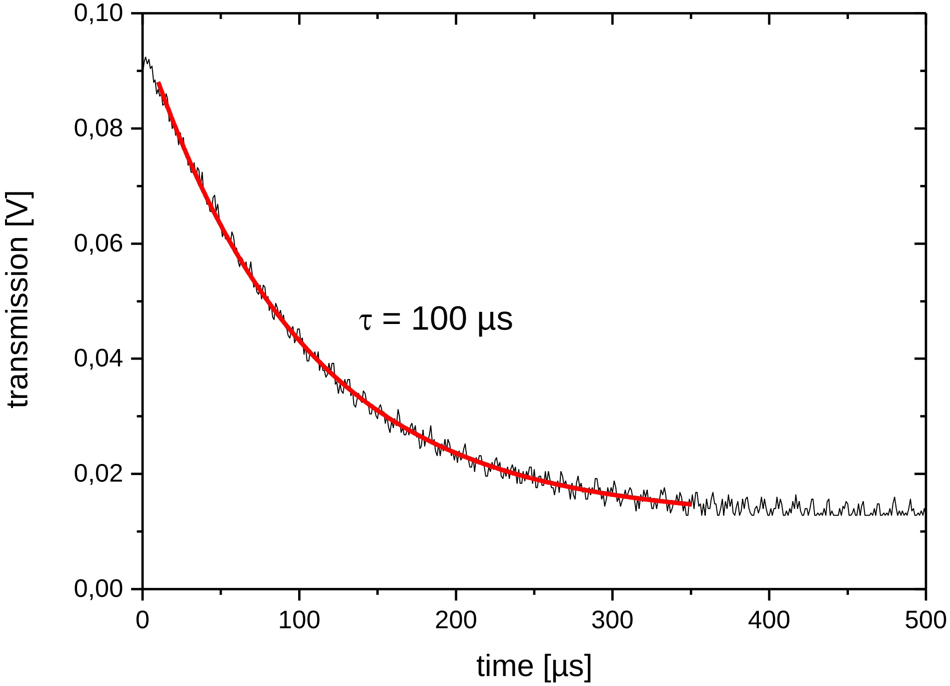
dominated by mirror coatings !

proton diameter ≈ 0.85 fm = 850×10^{-18} m

Clock lasers improving clocks

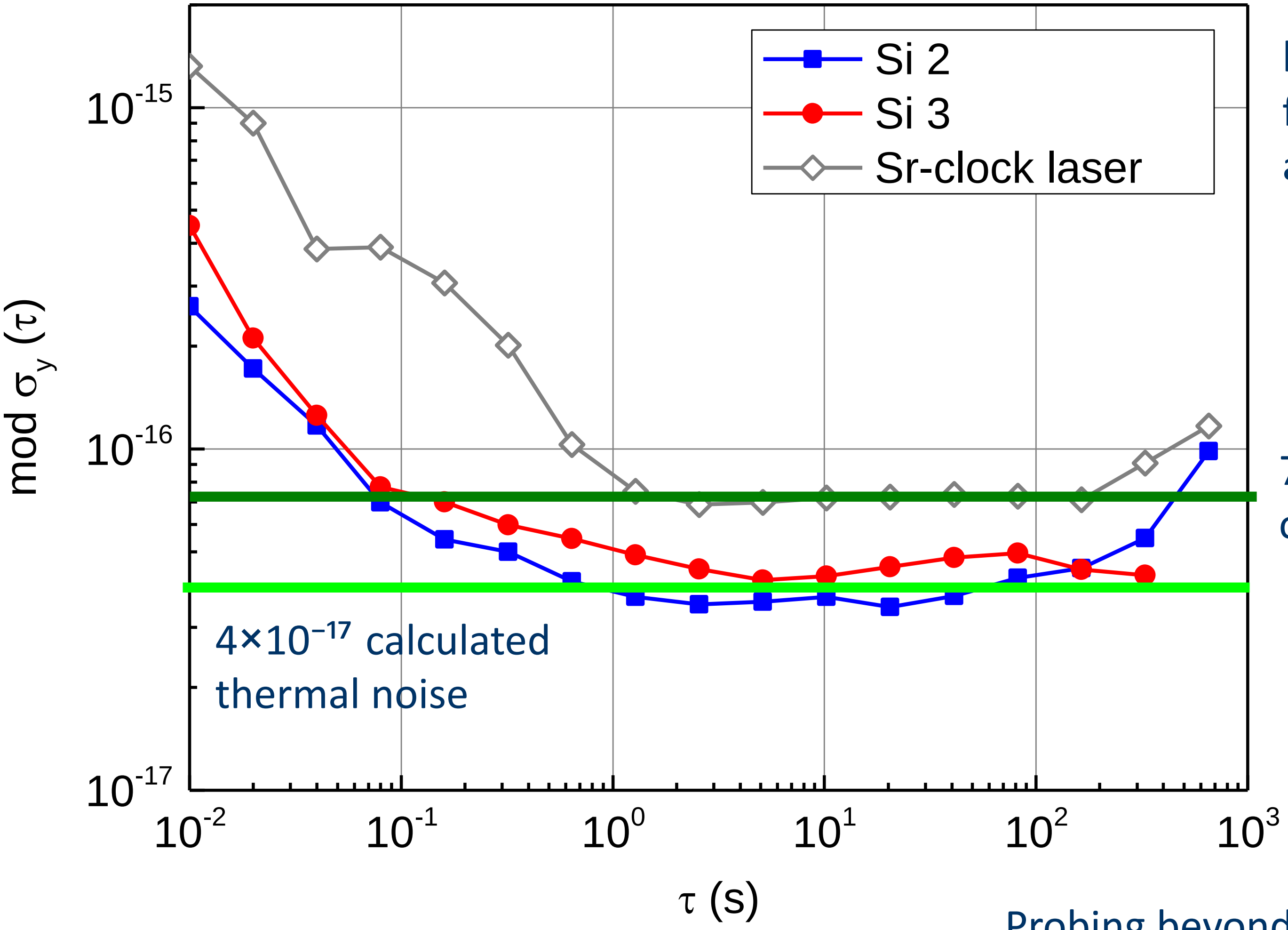
Silicon II + Silicon III

Ringdown measurement : TEM_{00} mode



F = 480 000

Clock lasers improving clocks



4×10^{-17} calculated thermal noise

linear drift removed from Si 3 – Si 2 and Sr – Si 2 beats

7×10^{-17} thermal noise of ULE cavity

Probing beyond the coherence time of the laser: Poster Roman Schwarz

Summary:

- A clock is … an oscillator with data sheet
- Noise and clock sensitivity to it

  Projection noise, sensitivity function, Dick effect
- Trapping, Lamb-Dicke regime
- Clock lasers

What can you do?

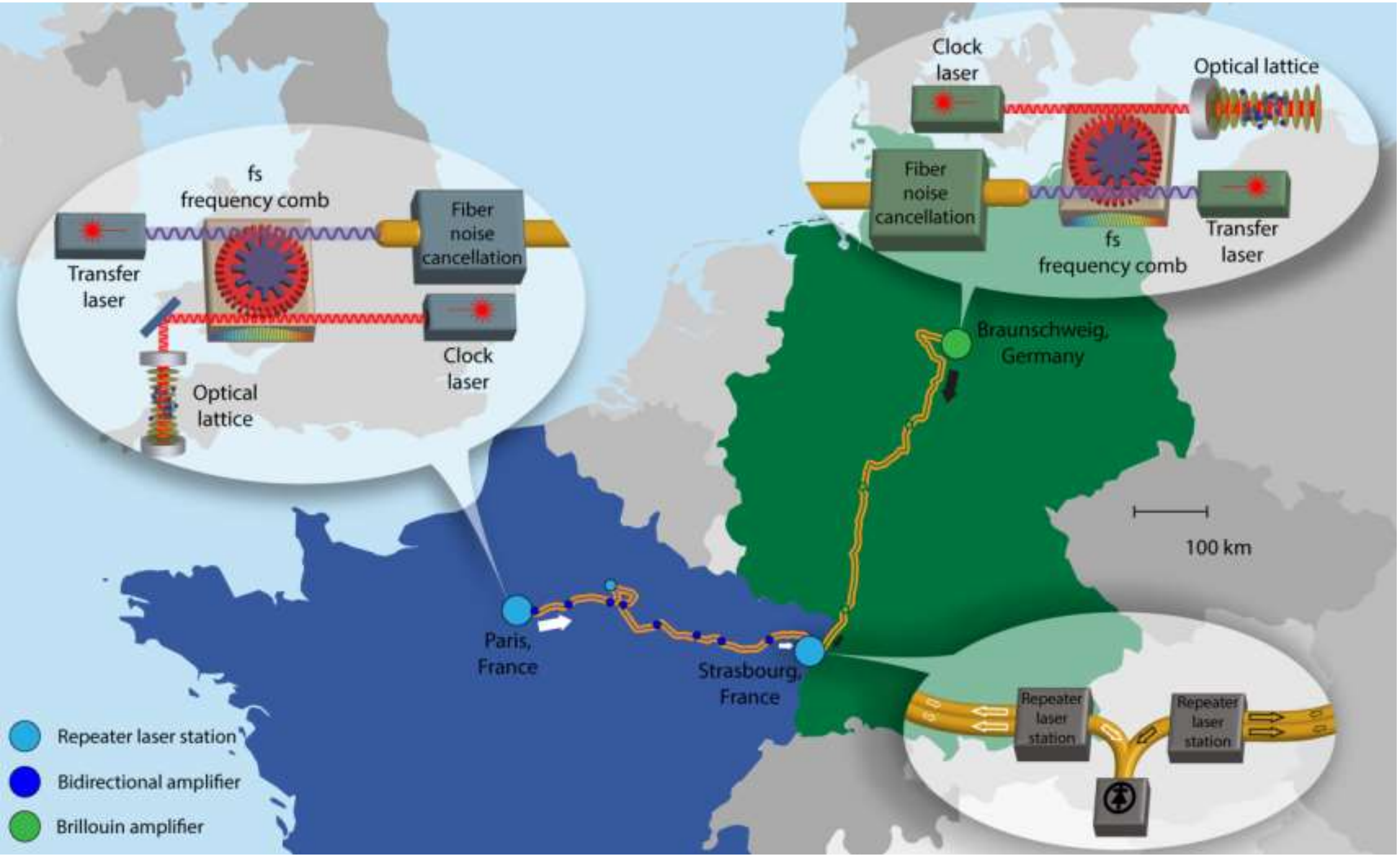
Yb$^+$ single ion clock

source: Google Maps

building A **building B**

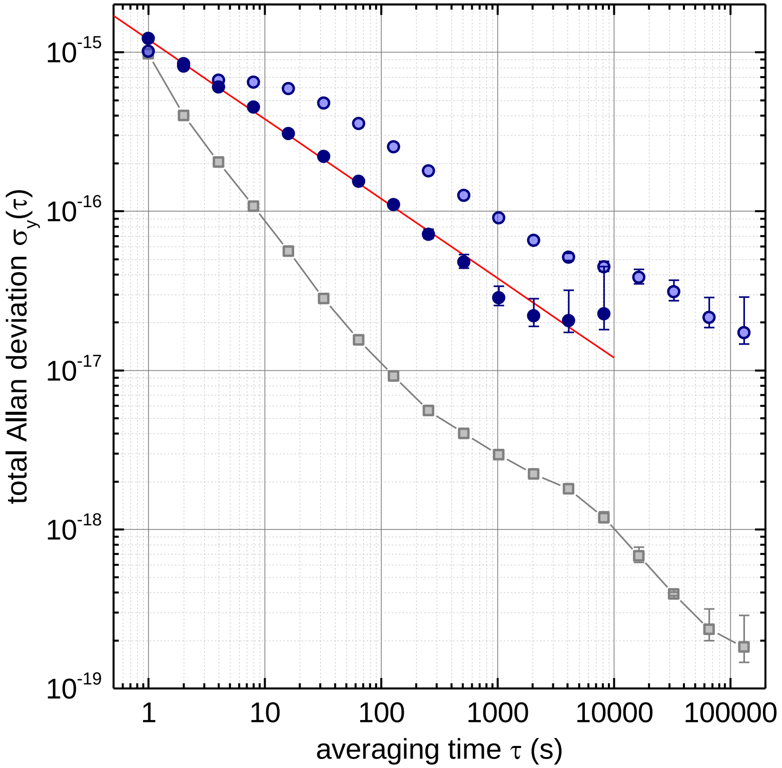
Sr lattice clock

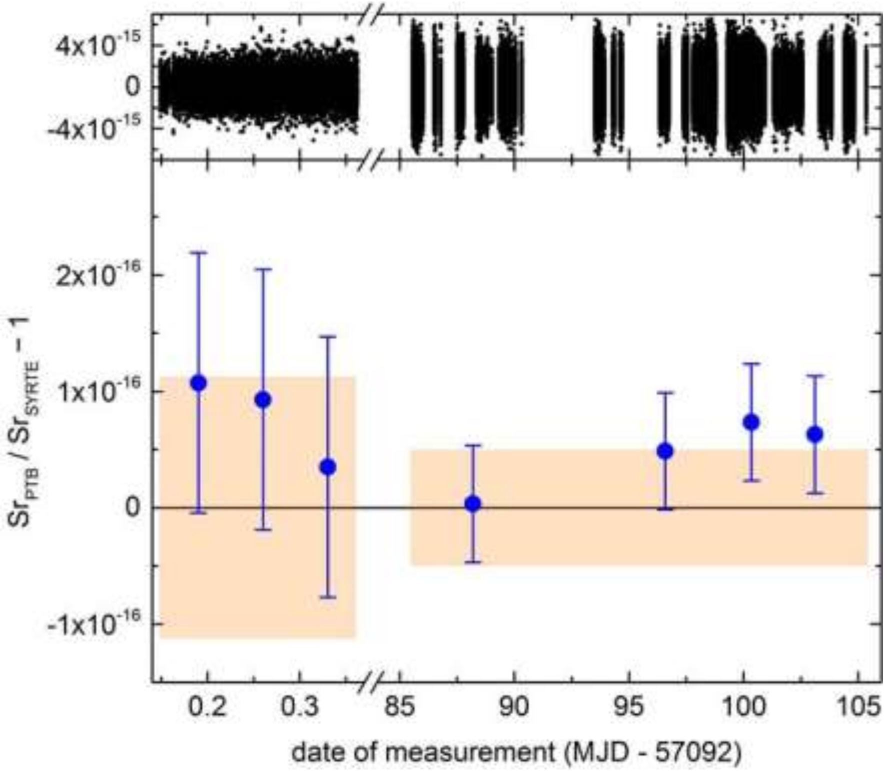
Clock comparisons: international



Ch. Lisdat *et al.*, Nature Comm. **7**, 12443 (2016)

Comparisons – Paris & Braunschweig





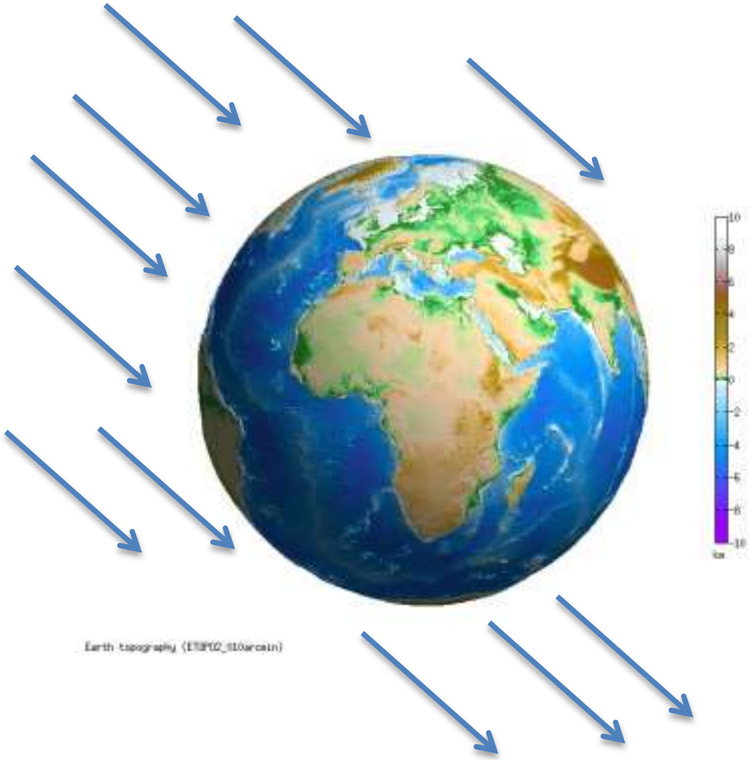
Gravity potential correction

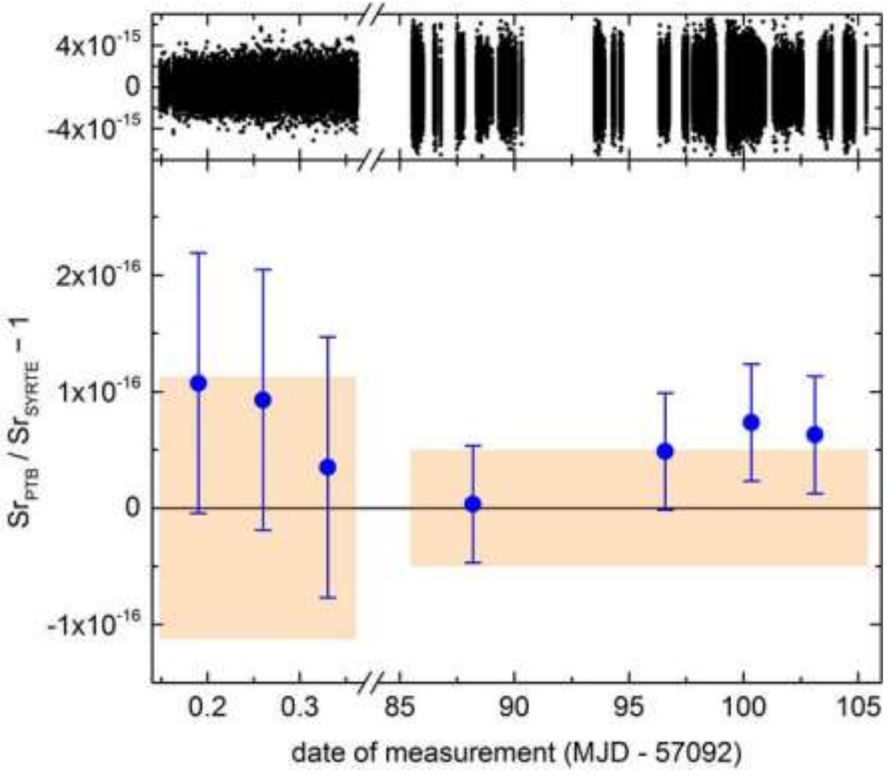
$-247.2(4)\times10^{-17}$

Ch. Lisdat *et al.*, Nature Comm. **7**, 12443 (2016)

Comparisons – Paris & Braunschweig

Local Lorentz invariance: search for daily modulation due to motion wrt. background

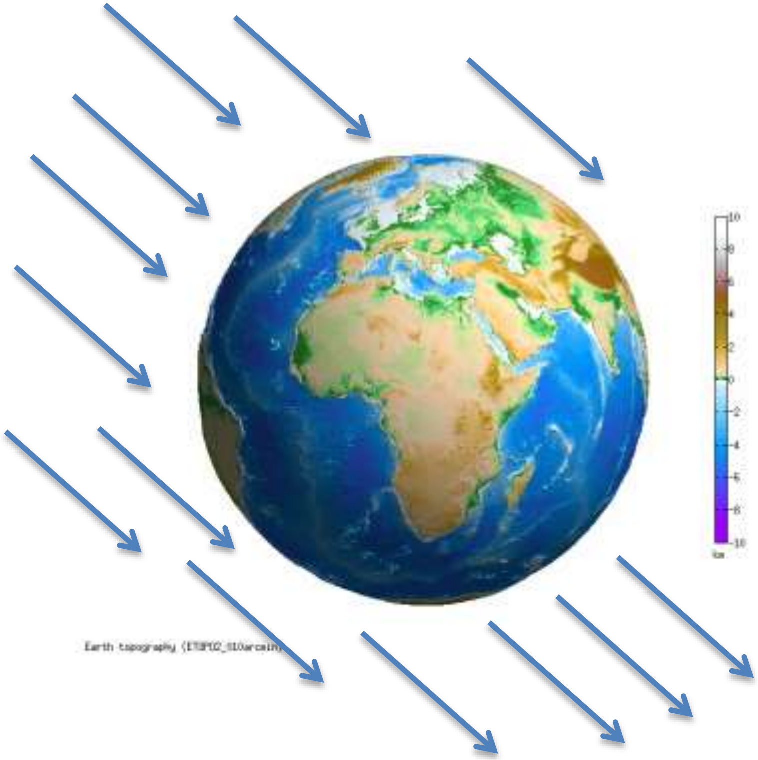




animation: A. Bezdek and J. Sebera, Computers & Geosciences **56**, 127 (2013), data set: ETOPO2 / EGM2008

Comparisons – Paris & Braunschweig

Local Lorentz invariance: search for daily modulation due to motion wrt. background



was done with Rb clocks (GPS)
P. Wolf & G. Petit, Phys. Rev. A **56**, 4405 (1997)

$|\alpha| \leq 10^{-6}$

LLI test also with fast ion beams
B. Botermann *et al.*, Phys. Rev. Lett. **113**, 120405 (2014)

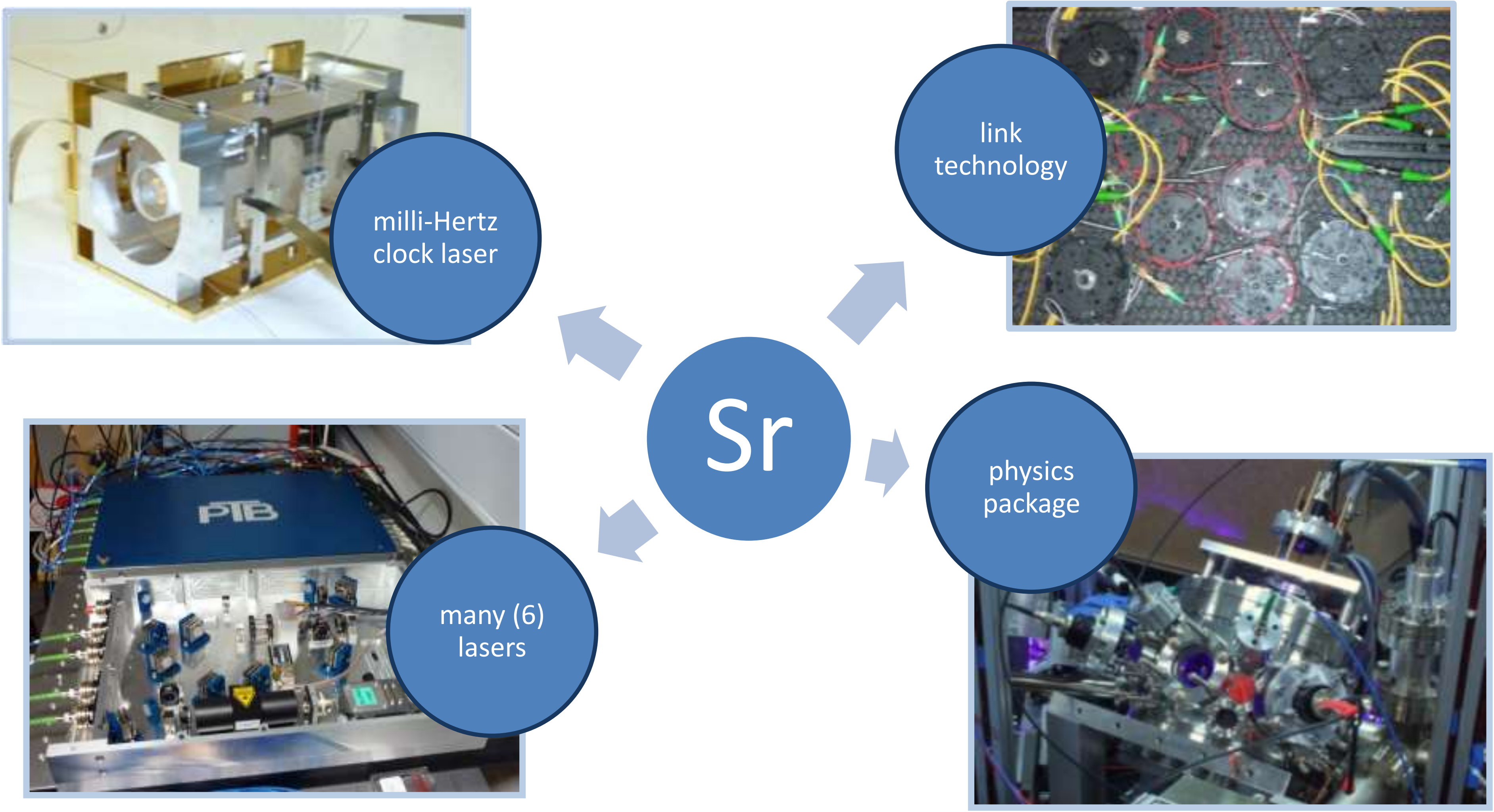
$|\alpha| \leq 2 \times 10^{-8}$

Sr clocks London, Paris, Braunschweig
P. Delva *et al.*, Phys. Rev. Lett. **118**, 221102 (2017)

$|\alpha| \leq 1.2 \times 10^{-8}$

animation: A. Bezdek and J. Sebera, Computers & Geosciences **56**, 127 (2013), data set: ETOPO2 / EGM2008

Transportable optical clocks



Sr

milli-Hertz clock laser

link technology

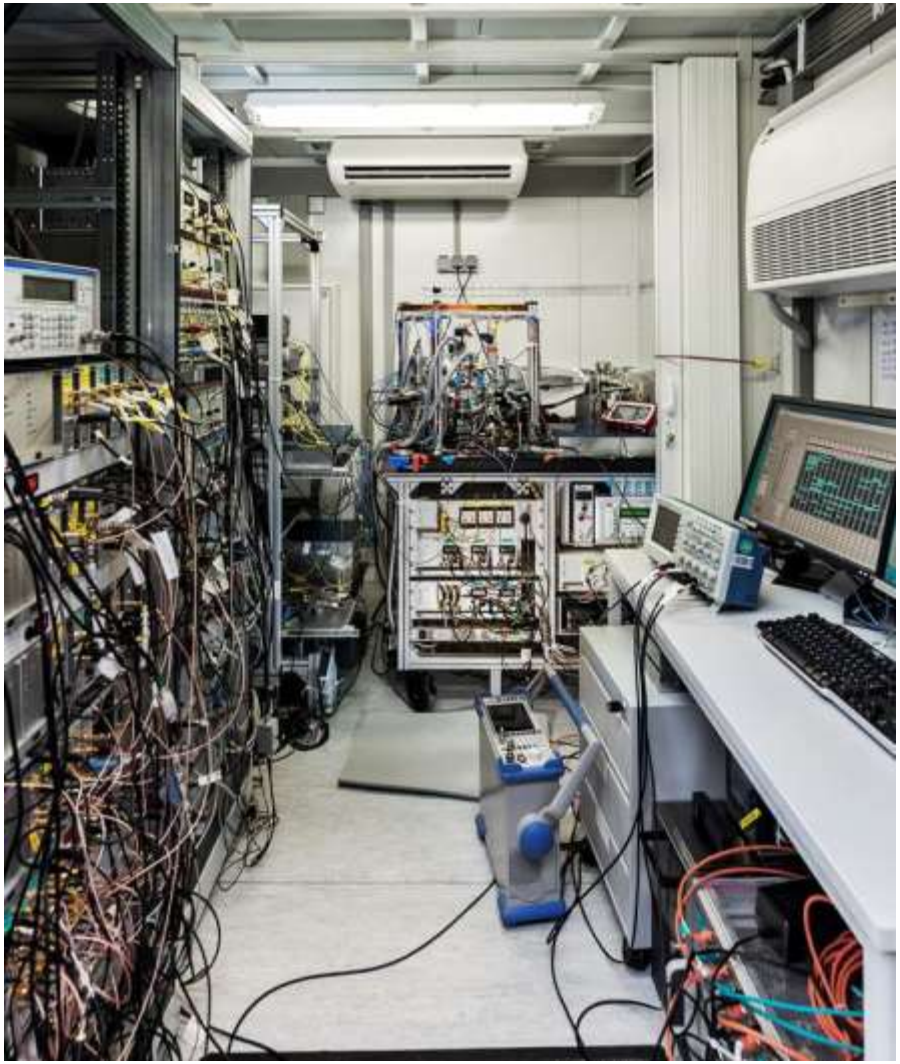
physics package

many (6) lasers

Transportable optical clocks

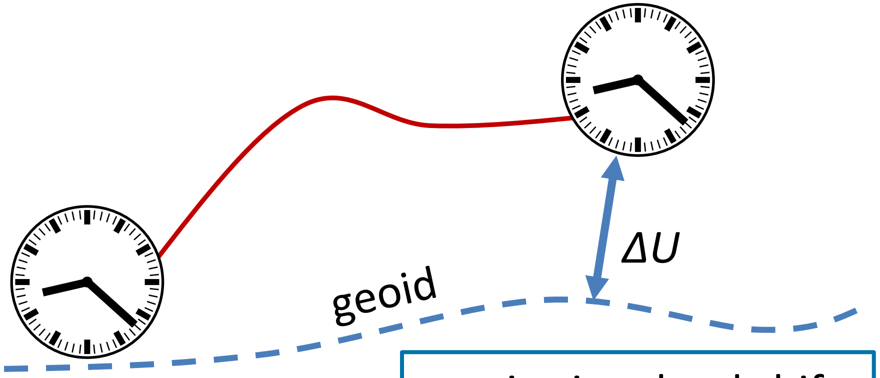
S. Koller *et al.*, Phys. Rev. Lett. **118**, 073601 (2017)

View into the car trailer ▶



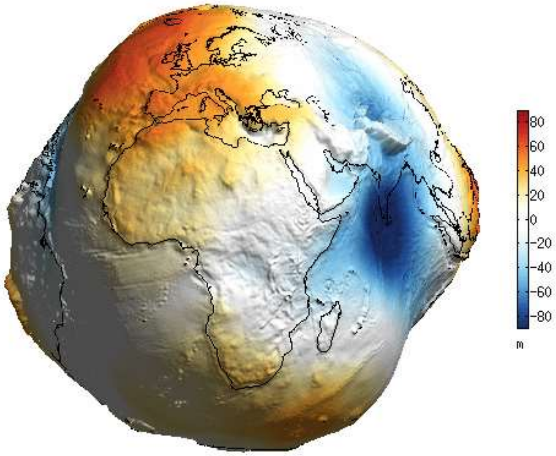
Transportable optical clocks

- Optical clocks as sensors:
  - Directly measure potential differences.
  - Vision: Realize geoid by clocks.



gravitational red shift:
$\Delta\nu/\nu = \Delta U/c^2$

Earth topography (ETOPO2_010arcmin)



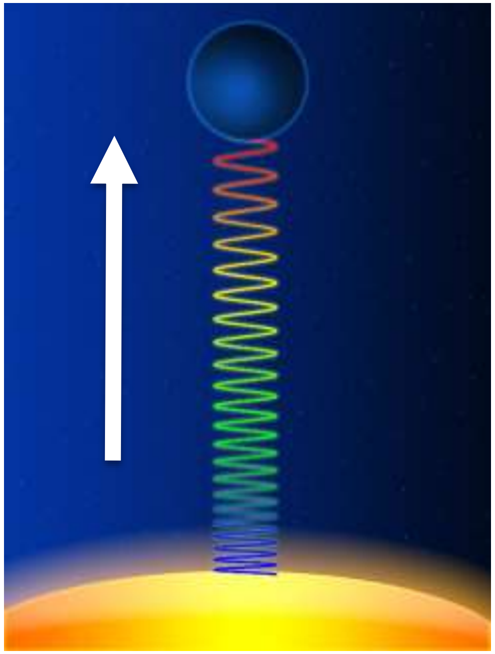
Geoid height (EGM2008, nmax=500)

M. Vermeer, Rep. of the Finnish Geod. Insti. **83**, 1 (1983)

A. Bjerhammar, Bull. Geodesique **59**, 207 (1985)

Geodesy: Why clocks?

Effect: relativistic redshift

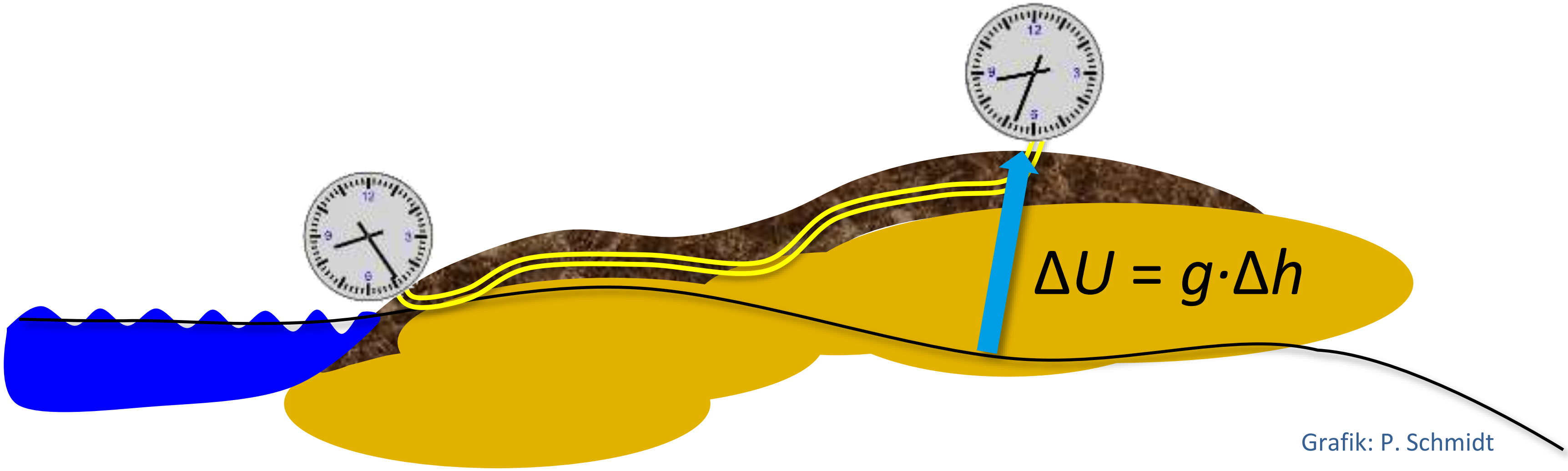


Quelle: Wikipedia

- Photon loses energy when fighting against gravity
- It cannot slow down but it can change its frequency
- The lower pendulum‘s oscillation is perceived as slower by the upper clock
- *Lower clocks run slower!*

Geodesy: Why clocks?

Effect: relativistic redshift



Grafik: P. Schmidt

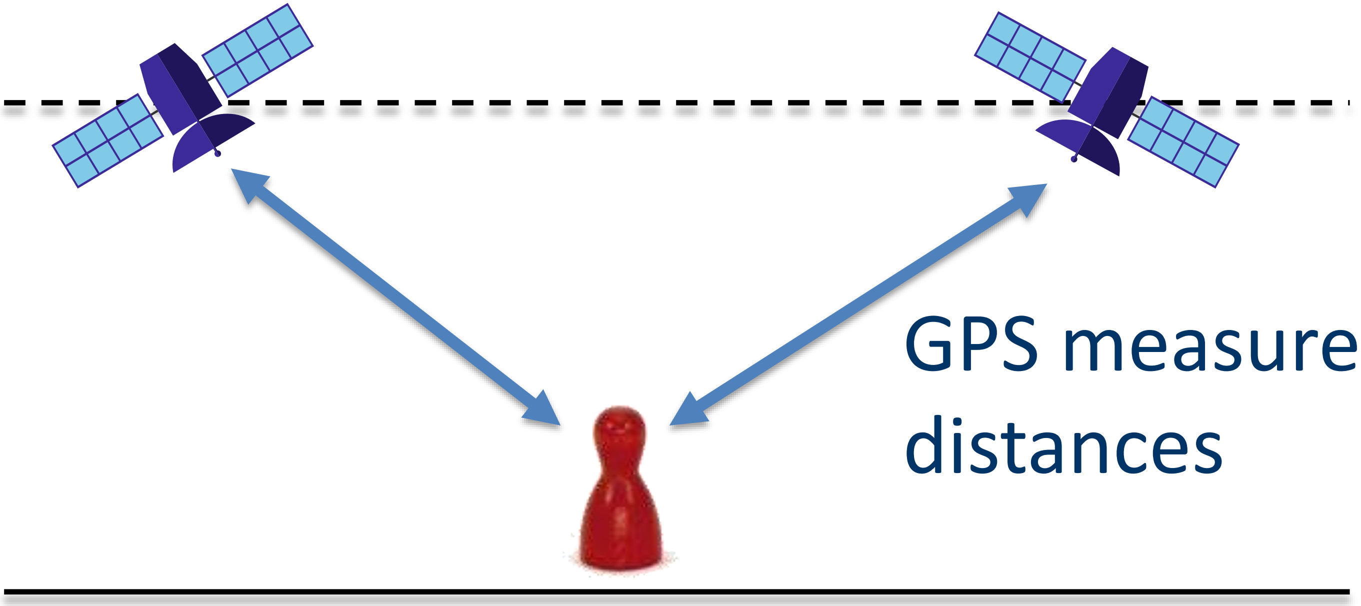
Frequency change Δf proportional to potential difference ΔU

A small effect: 1×10^{-18} per $g \cdot 1$ cm

Rep. Prog. Phys. **81**, 064401 (2018)

Where is the problem?

My GPS tells me the height!



GPS measures distances

Rep. Prog. Phys. **81**, 064401 (2018)

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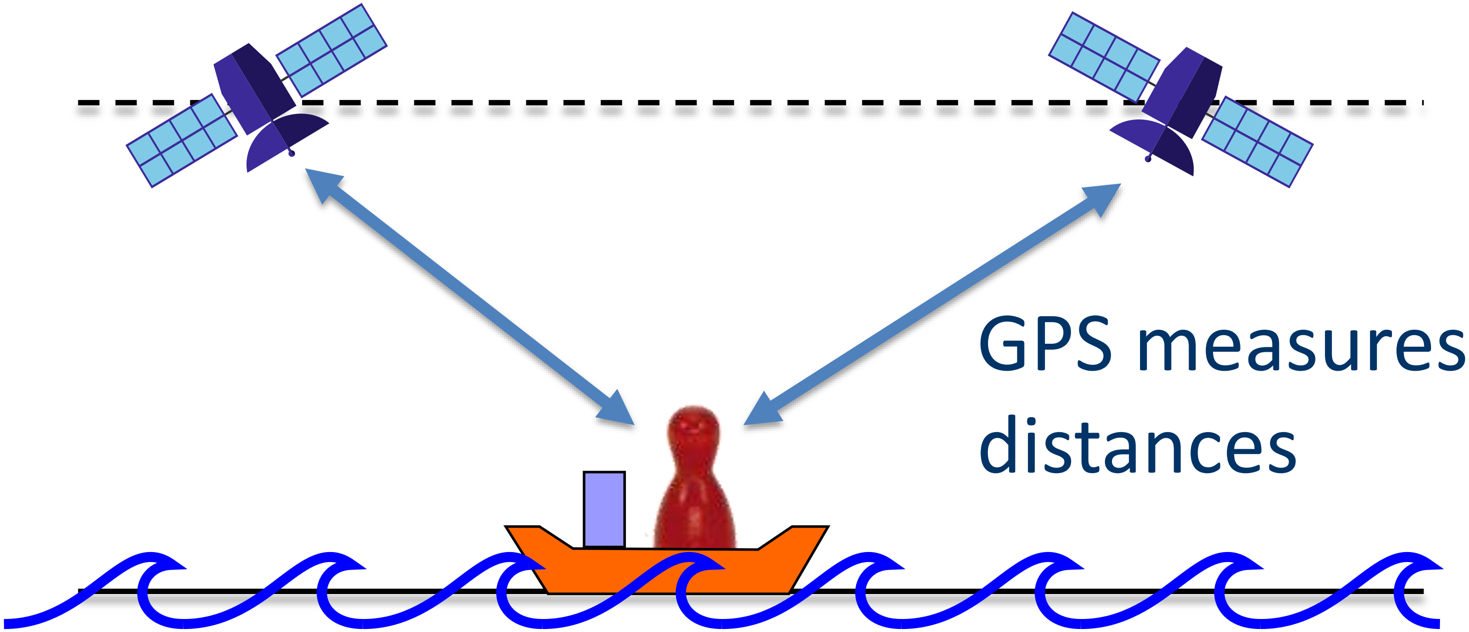

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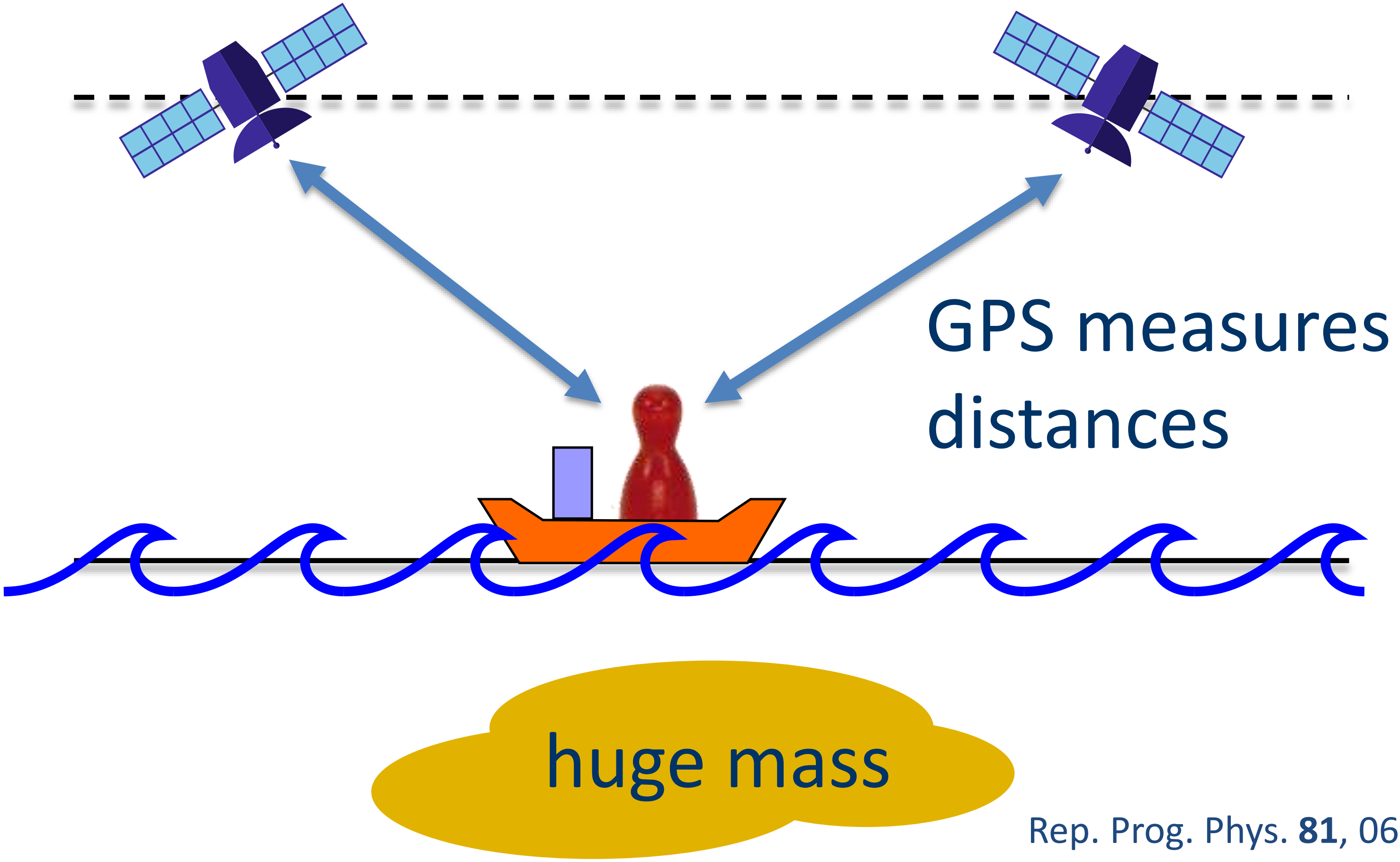

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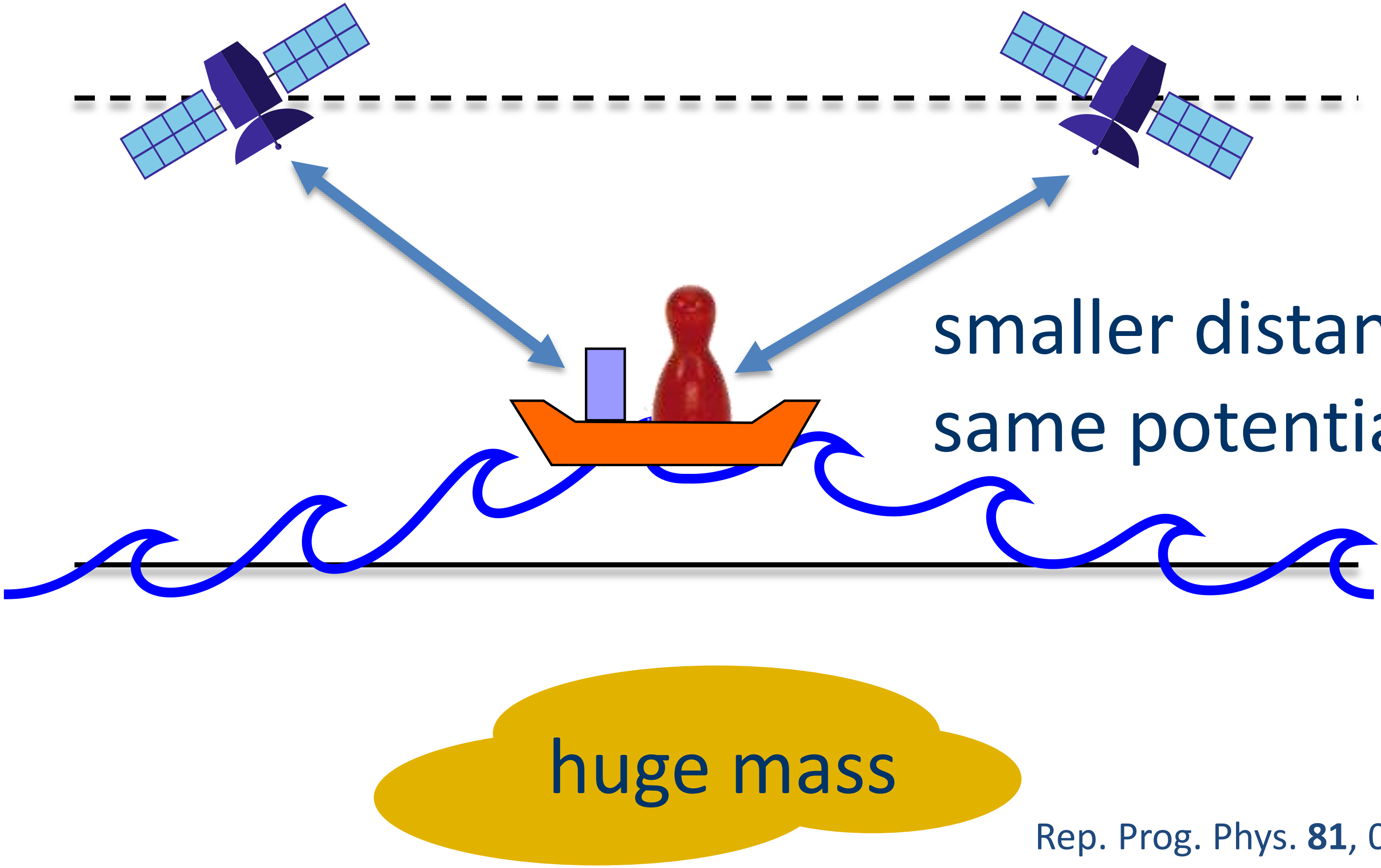


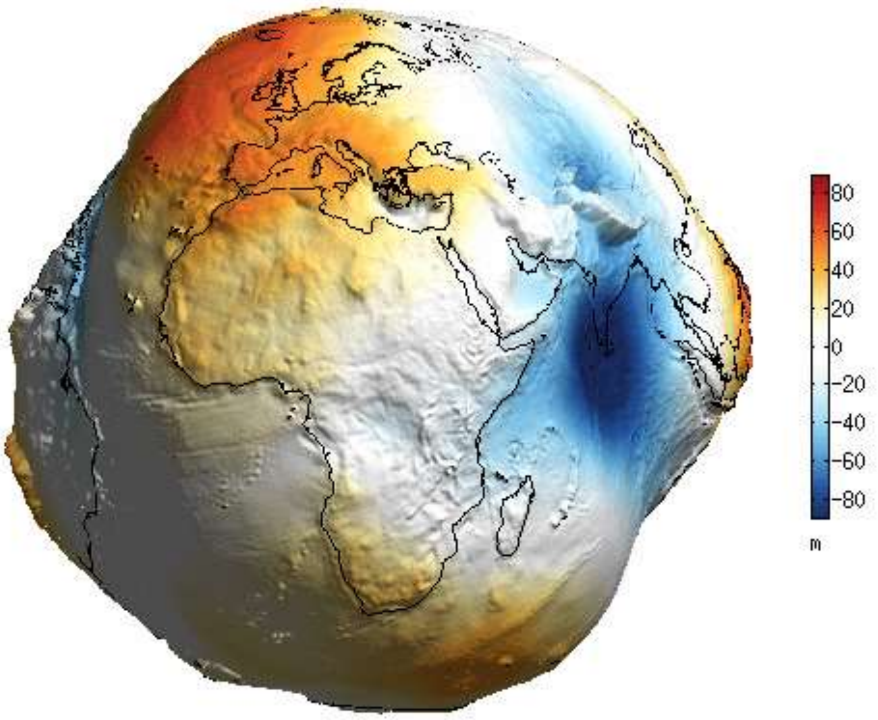
smaller distance,
same potential

huge mass

Rep. Prog. Phys. **81**, 064401 (2018)

Where is the problem?

Surface of equal potential: Geoid

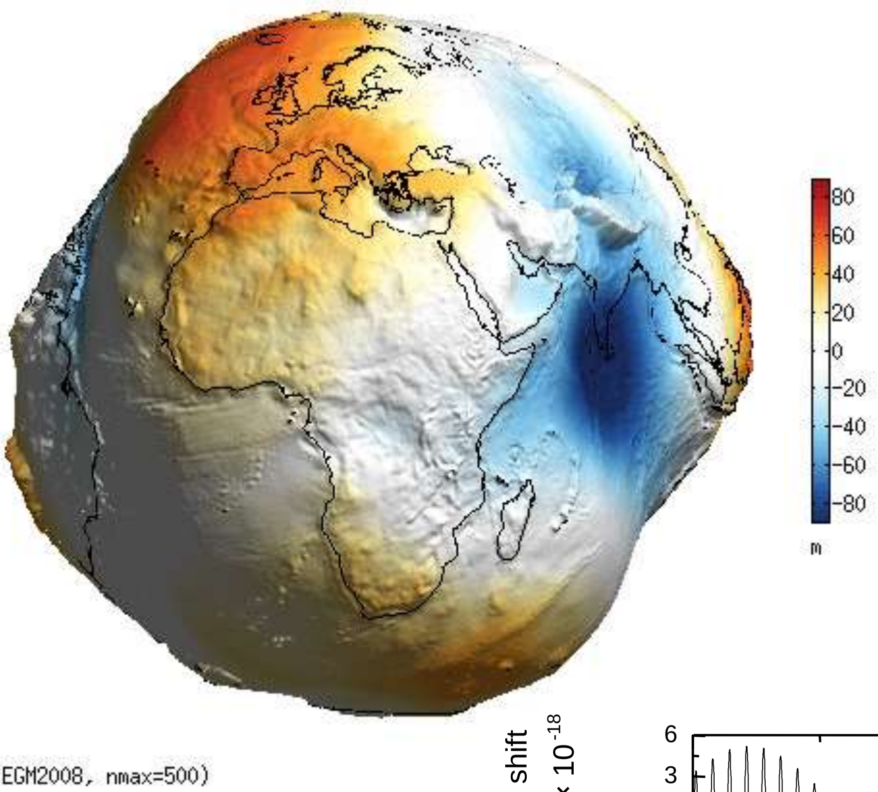


Geoid height (EGM2008, nmax=500)

smaller distance,
same potential

Where is the problem?

Surface of equal potential: Geoid



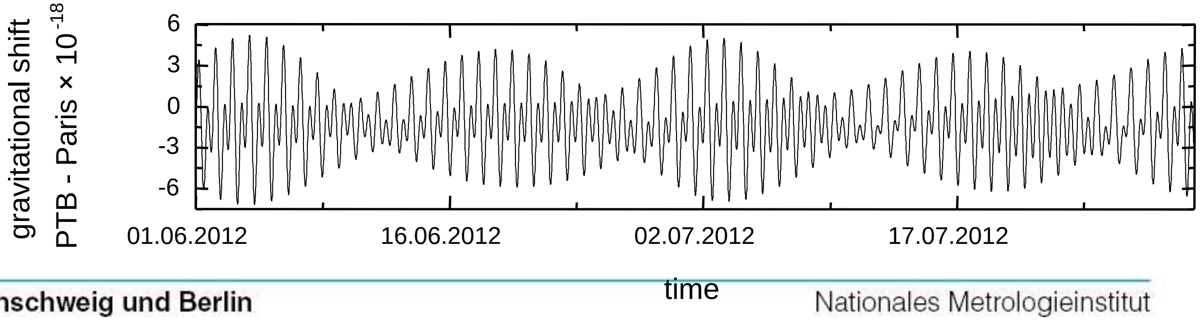
How do I get it?

Levelling = 60 m steps

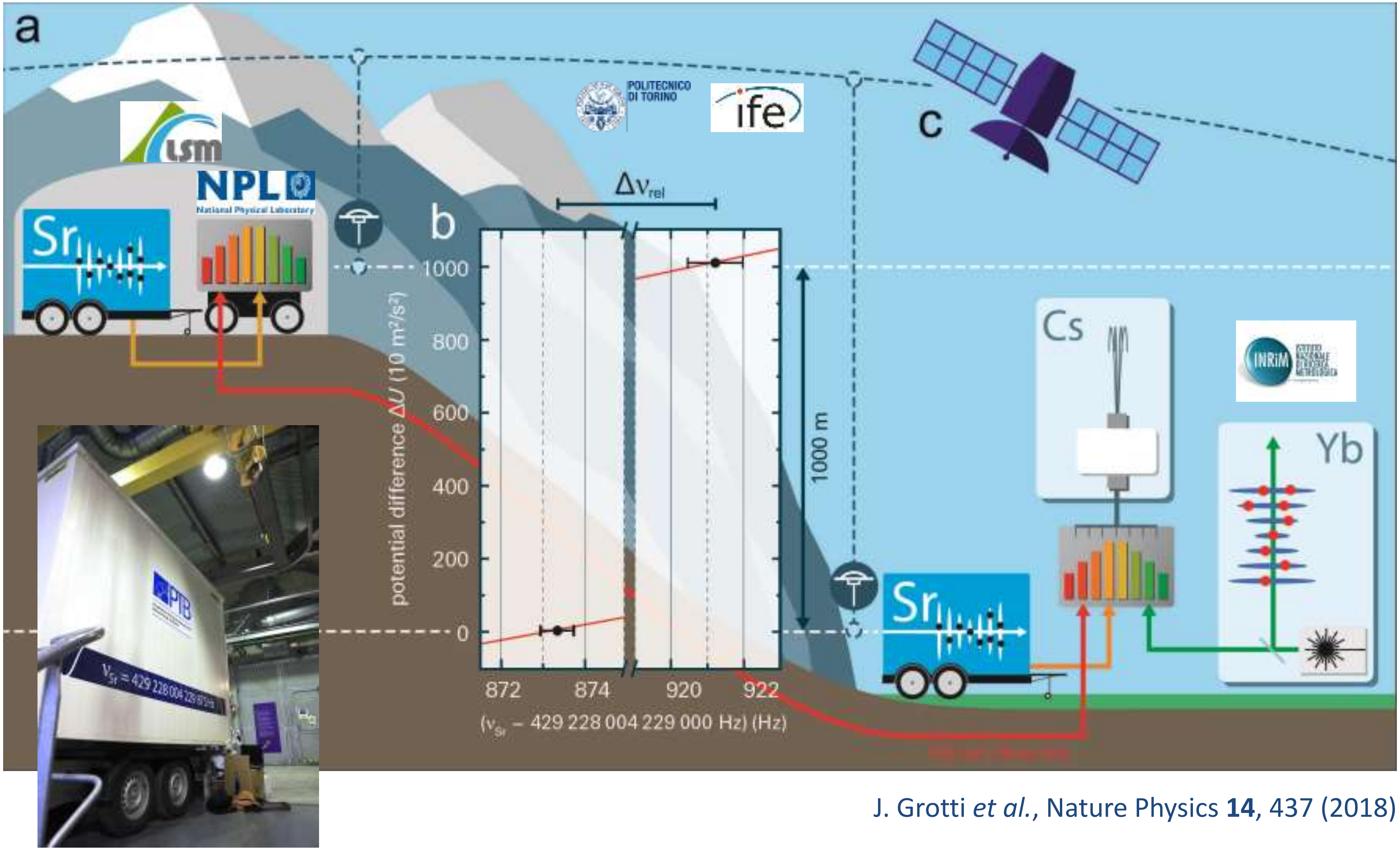
\+ gravimetry

Satellite missions

Grace-FO, low spatial resolution

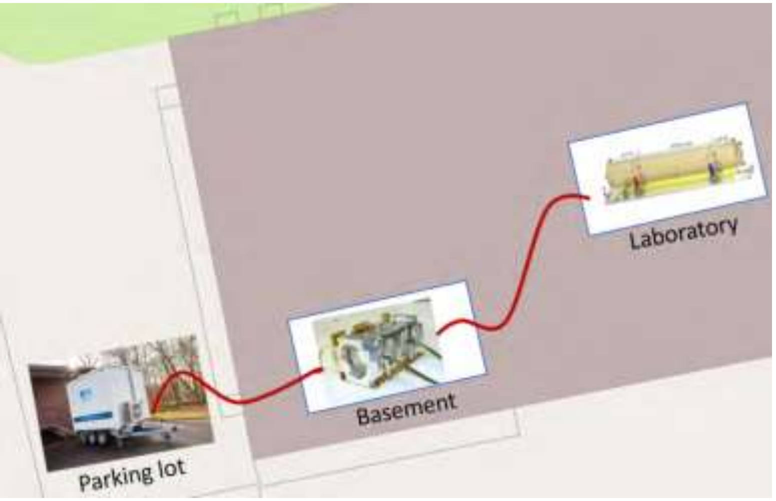


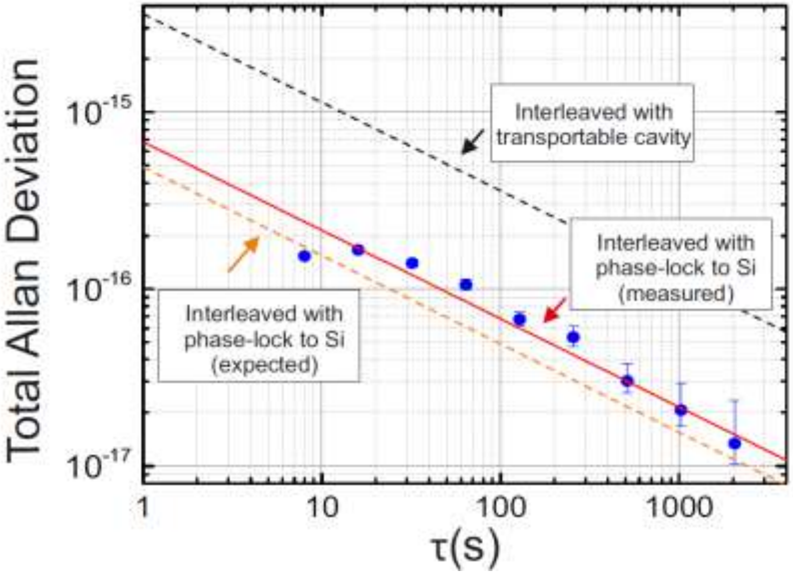
First time off-campus



J. Grotti *et al.*, Nature Physics **14**, 437 (2018)

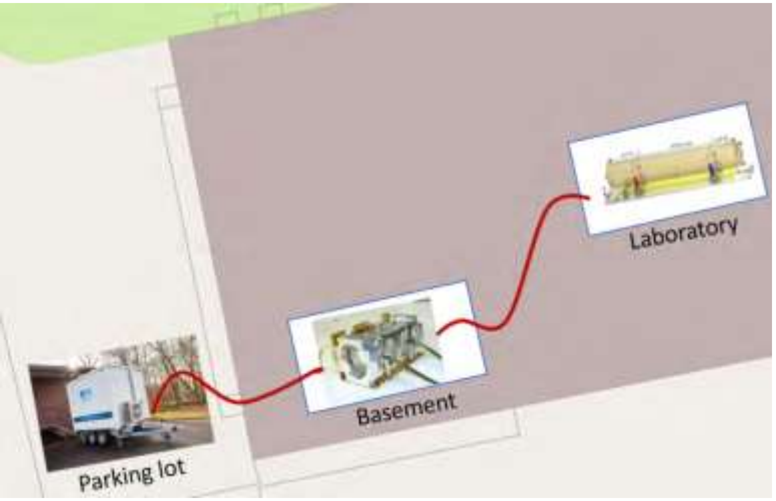
Testing at PTB – know your clock

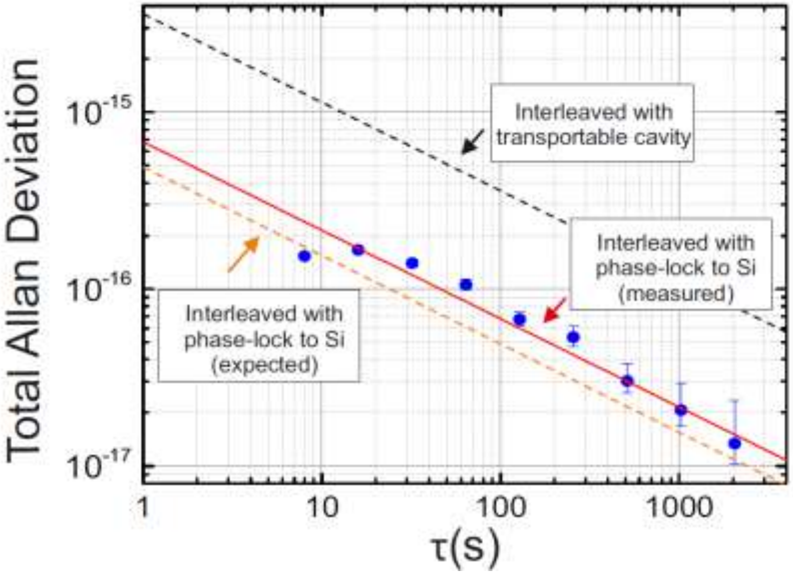




faster averaging using laboratory lasers

Testing at PTB – know your clock





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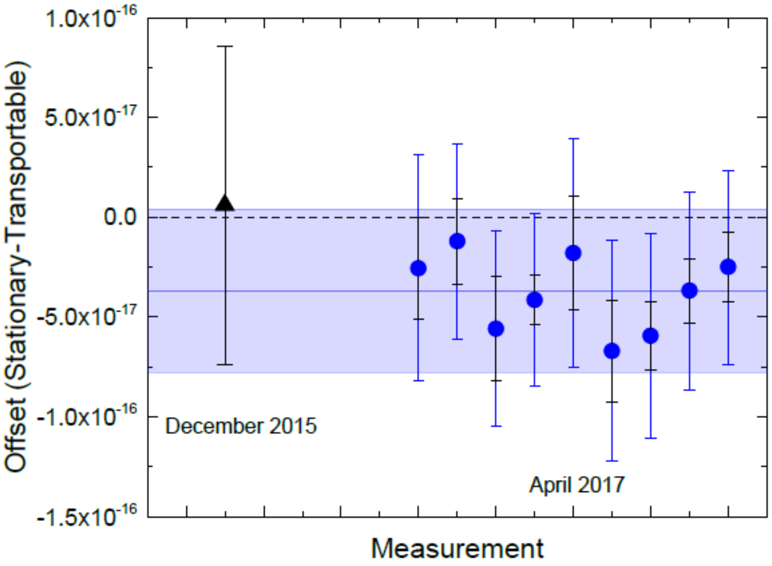
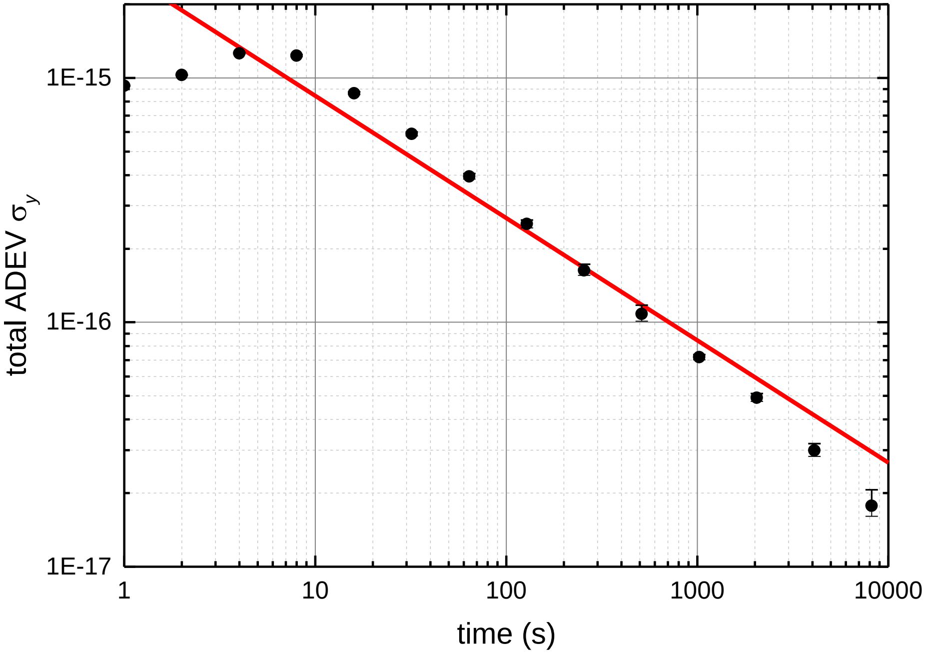


$$\nu_{stat}/\nu_{trans} - 1 = -37(41)\times10^{-18}$$

Second campaign: Paris – PTB



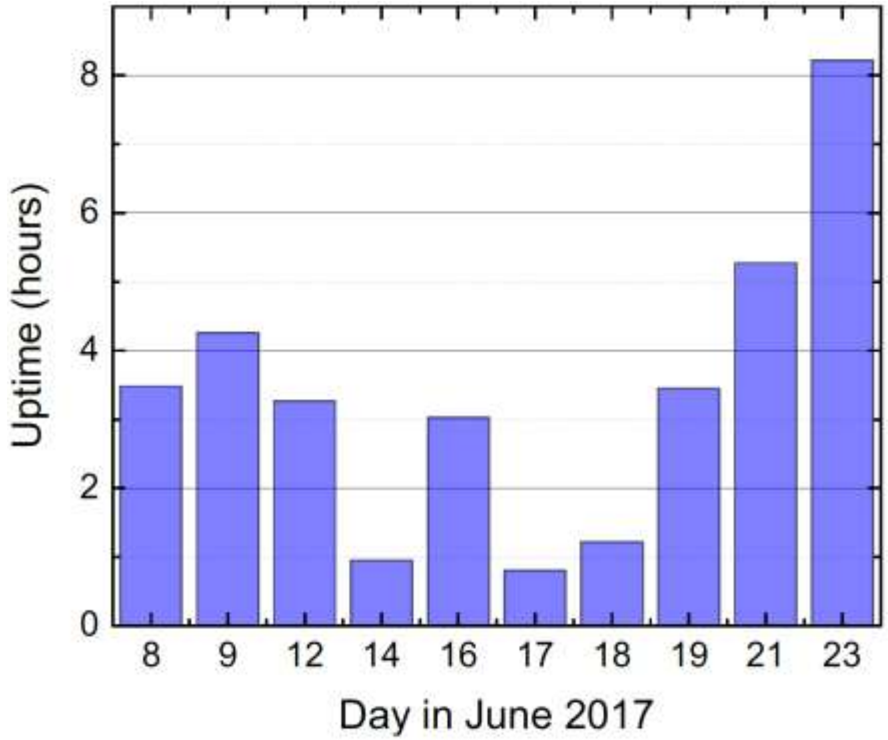
Combined uncertainty ≈ 3×10^{-17} or 30 cm in 3 hours.

Gravity potential correction from geodesy: $-247.2(4) \times 10^{-17}$

unfortunately: ‘anomaly’ in the second half of the campaign

Second campaign: Paris – PTB

Recording data:



Now: Sr clock is in Munich

Day 2: blue MOT

Day 3: atoms in the lattice

Summary II:

- Clock comparisons require knowledge of the red shift
- They can be used to test clocks or theories or …
- Geodesy requires trust in your clock
  – and the best available clocks
- Compact, commercial, simple, and cheap optical clocks may change geodesy

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e-mail: christian.lisdat@ptb.de
www.ptb.de