Optical frequency measurements and frequency combs

Y. Le Coq

LNE-SYRTE – Observatoire de Paris, CNRS, UPMC, Paris, France
yann.lecoq@obspm.fr

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General concepts abouts noise characterization

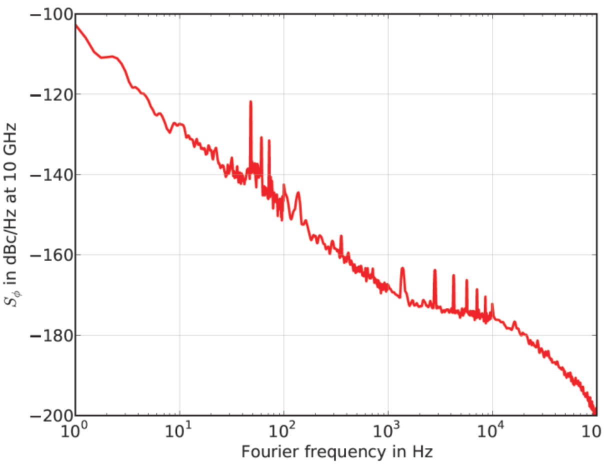
Toolbox of high precision measurement

A signal with some noise : $s(t) = s_0 + x(t)$

Three basic concepts to characterize this signal:

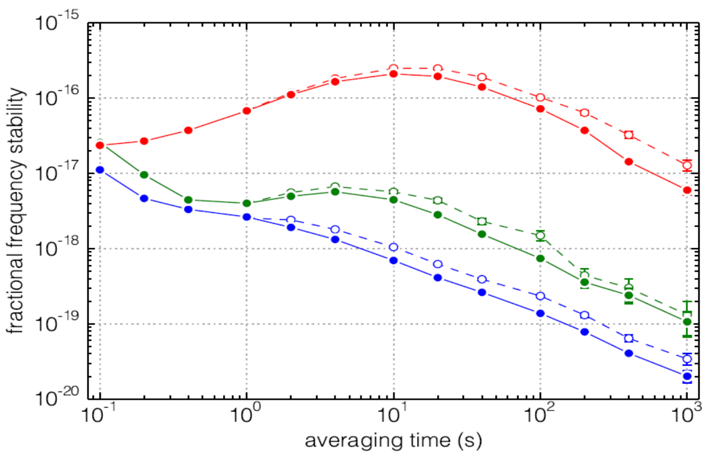
Power Spectral Density of Noise: $S_x(f)$

= typ. Fluctuations of s(t) at a given Fourier frequency f



Signal stability: $\sigma_x(\tau)$

= ftyp. Fluctuation of s(t) at a caracteristic time τ



Accuracy

= How miuch do we know <s(t)> converges toward what we want i.e. s_0

→ systematic error from different parameters evaluation (all you can think about... and more)

→ NB: a good stability makes such evaluation easier !

Power Spectral Density (PSS)

If $x(t)$ is a stochastic physical quantity, of unit [unité]

One defines $$x_T(t) \triangleq \begin{cases} x(t) & \text{pour} \quad |t| \leq T \\ 0 & \text{pour} \quad |t| > T \end{cases}$$ (finite suport, hence its FT exists...)

The PSD of x is: $$S_x(f) \triangleq \lim_{T \to \infty} \frac{\mathbb{E}\left[\left|X_T(f)\right|^2\right]}{2T}$$ in [unit^2.Hz^{-1}]

NOTE: alternatively, on can also define PSD as: $$S_x(f) = \int_{-\infty}^{+\infty} R_{xx}(\tau) e^{-2\pi i f \tau} d\tau$$

with $R_{xx}(\tau) \triangleq \mathbb{E}\left[x(t) \cdot x(t+\tau)\right]$ The autocorrelatipon function of x

(from Wiener-Khintchine theorem, these 2 definitions are <u>equivalent</u>)

For $x(t)$ real, $S_x(f)$ is even, therefore, with no loss on information, one uses the one-sided PSD $\boxed{S_x^{\mathrm{OS}}(f) = S_s(f) + S_x(-f) = 2S_x(f) \quad \text{pour} \quad \mathrm{f} > 0}$ et $S_x^{\mathrm{OS}}(0) = S_x(0)$

<u>This is normally the quantity that is used</u> (eg.: FFT analyzer) !

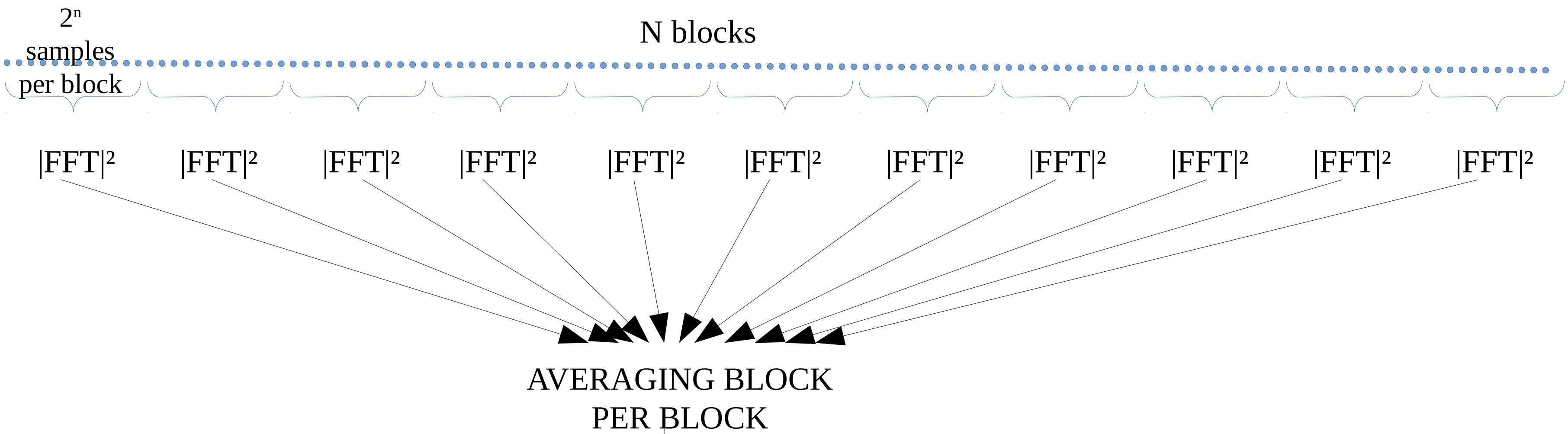
Properties:

$S_{\alpha x} = \alpha^2 S_x$ $\qquad S_{dx/dt}(f) = f^2 S_x(f)$ $\qquad S_{x_1+x_2} = S_{x_1} + S_{x_2}$ For x_1, x_2 indep.

A PSD estimator: avgeraged periodogram

(~Welsch)

If $x(t)$ is a stochastic physical quantity sampled in x_n every Δt
using ergodicity and stationnarity of the noise process:



When N large (variance in $1/\sqrt{N}$)

$$S_x(f_i) \text{ avec } \Delta \mathrm{f_i} = 1/(2^{\mathrm{n}}\Delta \mathrm{t})$$

Notes :

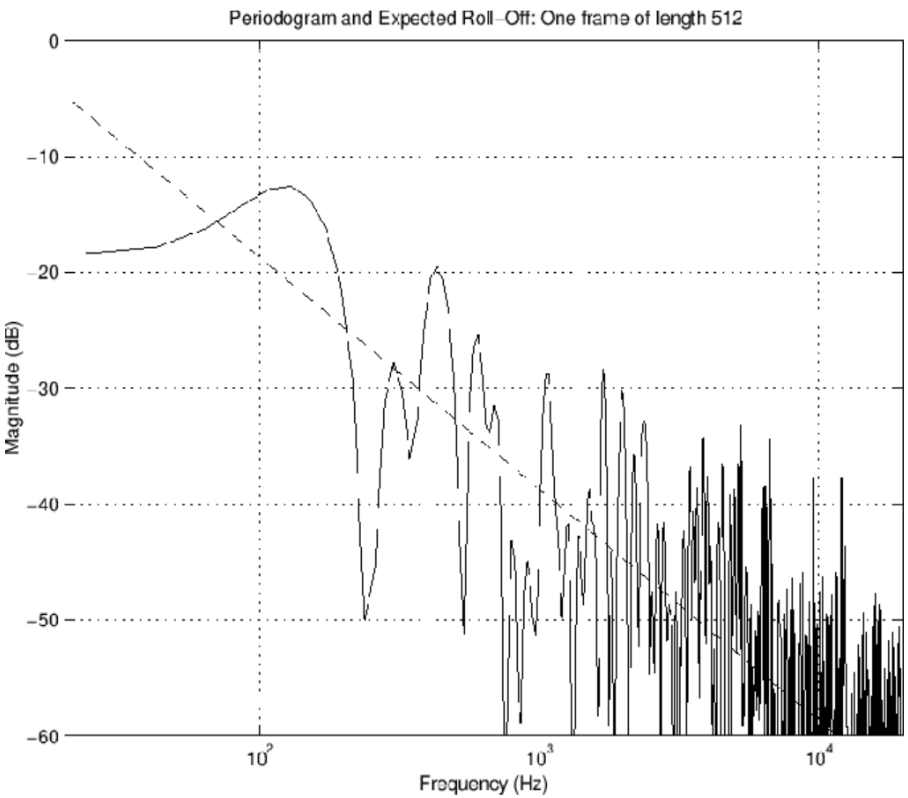
- generaly necessary to use a windowing function (ex.: Hanning) before the FFT to limit spectral overlap
- if a window function is used, one can overlap blocks (Welsh method)

Très bien pour représentation log-lin ou lin-lin,

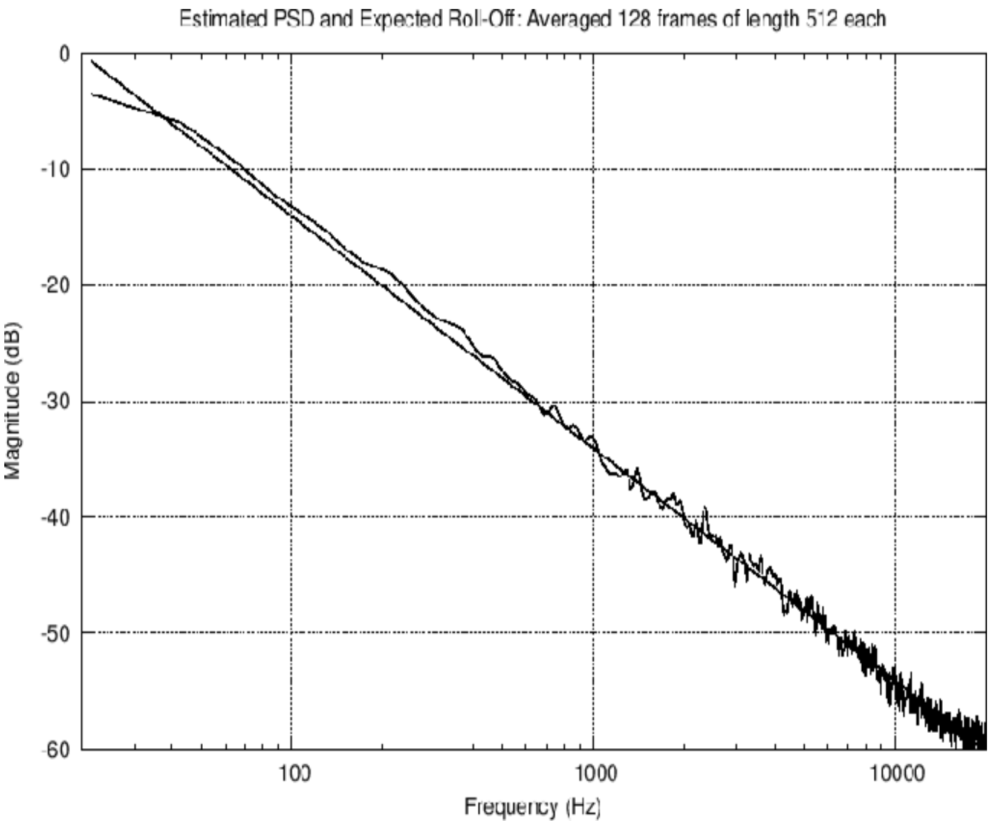
MAIS pas très pratique si représentation log-log sur plusieurs décades :

→ grande résolution (nombre de pts) sur la dernière décade, très faible sur la première

Importance of averaging



|FFT|², 512 points, 1/f noise



|FFT|² 512 points, 1/f noise, averaged 128 times

Note: when plotting in log-log over several decades of Fourier frequencies, the resolution is mediocre on the left, and too high on the right.

Credits: www.dsprelated.com

An exemple of PSD algorithm

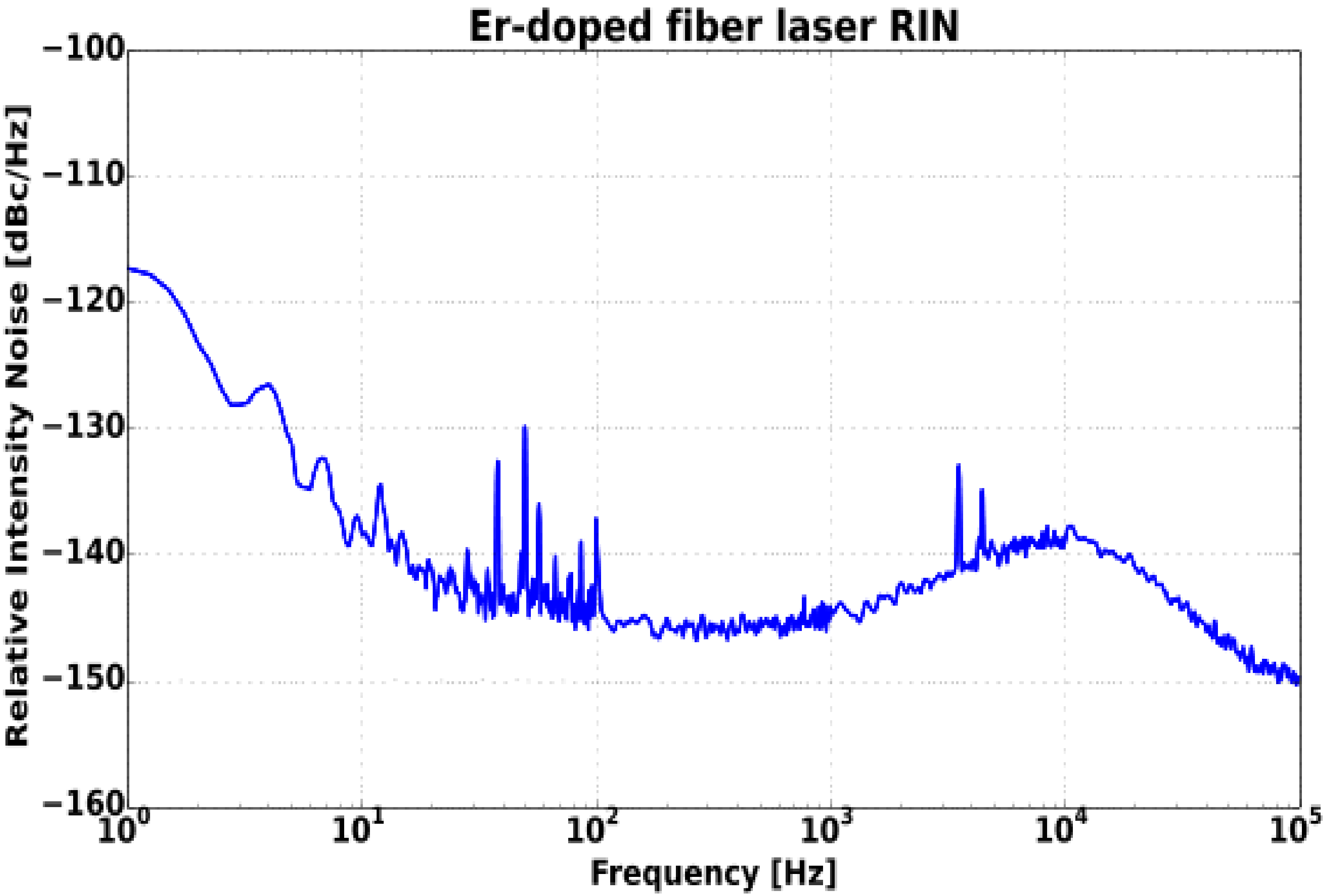
We have N(>1024*1000~1M here) samples of a signal DataA[n], sampled at 2MSPS. We want the PSD over >1 decade of Fourier frequencies (log-log)

```
SAMPLING_FREQUENCY = 2000000.0 # 2MSPS used here
nfft=1024 #1024 points of fft in each decade
decades = 4 # for Fourier frequencies from 100Hz to 1MHz (with still some info <100Hz from the lowest decade)
window = scipy.signal.windows.get_window('blackmanharris', nfft) # blackmanharris windowing is used here

def PSDcore(dataA, nfft, decades, window):

    # single sided power spectral density
    scaling = 2.0 / (SAMPLING_FREQUENCY * (window * window).sum())

    psdA = []

    for decade in range(decades):  # ie 0,1,2,3 here

        if decade > 0:
            dataA = scipy.signal.decimate(dataA, 10)

        fftA = rfft(window * np.reshape(dataA, (-1, nfft)))

        A = np.mean(fftA * fftA.conj(), axis=0).real * scaling * 10**decade
        relative_f = rfftfreq(nfft, 0.5)
        mask = np.ones(f.shape, dtype=bool)
        if decade != decades - 1:
            mask = mask & (relative_f > 0.08)
        if decade > 0:
            mask = mask & (relative_f <= 0.8)

        psdA.append(A[mask])

    psdA = np.concatenate(psdA[::-1])

    return psdA
```

NB.: if data arrives continuously by blocks of N samples, averaging the results from PSDcore(), the estimator of PSD is improving over time

An exemple of PSD algorithm

A typical result of such algorithm



Allan deviation/variance

Statistical tool introduced by David Allan to characterize the fluctuations of the frequzecny of oscillators. Nevertheless, can be applied to any kind of measurement...

$$s(t) = s_0 + x(t)$$ [unit]

Definition :

$$\sigma_x^2(\tau) = \frac{1}{2} < (x_{n+1} - x_n)^2 >$$ (Variance, [unit²])

$$\sigma_x(\tau) = \sqrt{\frac{1}{2} < (x_{n+1} - x_n)^2 >}$$ (Deviation, [unit])

with: $x_n = \frac{1}{\tau} \int_{n\tau}^{(n+1)\tau} x(t) dt$

Utility :

- (much) faster to estimate than the PSD for "long " characteristic times τ (ie low Fourier frequencies)

- defined when the PSD may be isn't (eg. drift)

- defined when the true average $< s(t) >$ maybe isn't (eg. drift or LF colored noise)

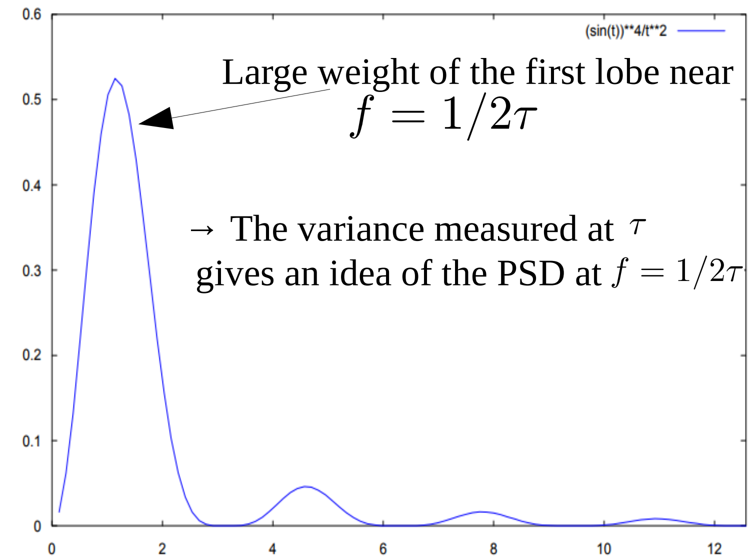
- gives the "useful" time of measurement:

If the AVAR is cte or increases after a time $> \tau$
averaging for longer than τ is useless, or even bad (more noise when you average more...) !

PSD/ AVAR relation: math

One can actually calculate the AVAR from the PSD:

$$\sigma_x^2(\tau) = 2\int_0^\infty S_x(f)\frac{\sin^4(\pi\tau f)}{(\pi f)^2}$$



The other way around is not true... except if you make hypothesis on the noise, typically: $S_x(f) \propto f^\alpha$

Noise type	$S_x(f)$	$\sigma_x^2(\tau)$
Linear drift	$-$	$K\tau^2$
Random walk	$h_{-2}f^{-2}$	$h_{-2}A\tau^1$
Flicker (pink)	$h_{-1}f^{-1}$	$h_{-1}B\tau^0$
white	$h_0 f^0$	$h_0 C\tau^{-1}$
blue	$h_1 f^1$	$h_1 D\tau^{-2}$
violet	$h_2 f^2$	$h_2 E\tau^{-2}$

With:

$K = \Delta^2/2$ for Δ the linear drift

$A = 2\pi^2/3$

$B = 2\ln(2)$

$C = 1/2$

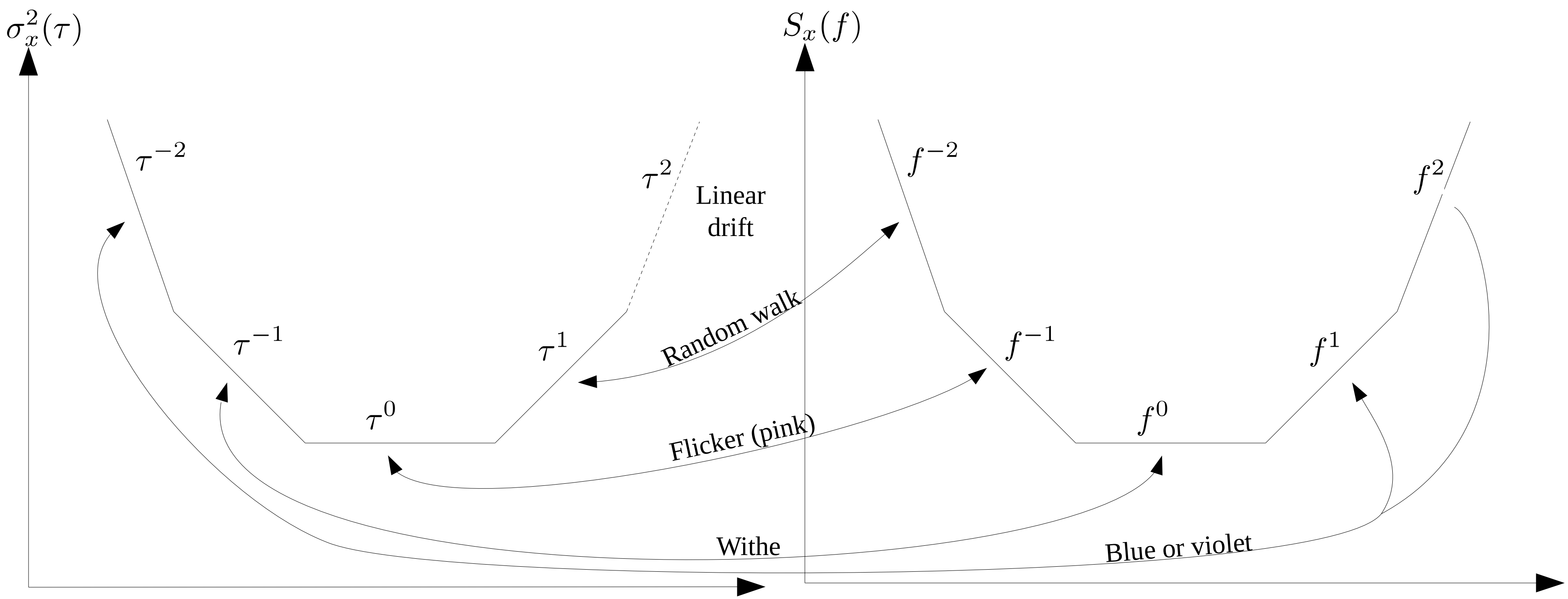
$D = [1.038 + 3\ln(2\pi f_h \tau)]/4\pi^2$

$E = 3f_h/4\pi^2$

With f_h a high-frequency cut-off, that allows convergence...

Note: table largely used in time/frequency when taking about the frequency fluctuations of an oscillator, but in works for any noisy measurement in general

PSD/AVAR: graph



The Allan variance allows identification of the type of noise (power law)

It doesn't distinguish between noises in f^1 and f^2 .
Other 2 sample variances exist for that (eg. MVAR)

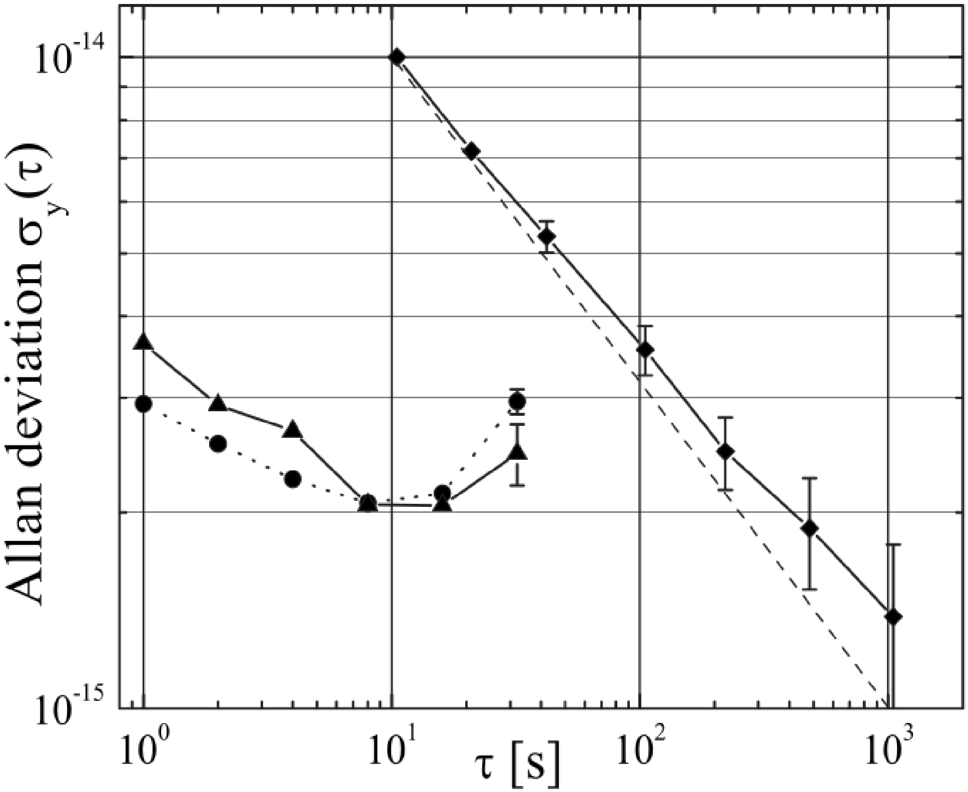
AVAR algorithm

Exemple of software packages

Alavar www.alamath.com

Stable32 www.wriley.com

AllanTools (python module)



Exemple of python code

```
def adev(x, tau, sampl=1.0):
    """Allan deviation"""

    x = np.asarray(x)
    tau = np.asarray(tau)

    # allocate output vectors
    adev = np.zeros(tau.size)
    dadev = np.zeros(tau.size)

    # samples
    n = x.size
    # partitioning
    p = np.floor(tau * sampl).astype(int)

    for i, m in enumerate(p):
        d = x[0:n - n % m].reshape(-1, m)
        y = np.mean(d, axis=1)
        adev[i] = np.sqrt(0.5 * np.mean(np.diff(y)**2))
        dadev[i] = adev[i] / np.sqrt(y.size)

    return adev, dadev
```

Note : error bars in $\sigma(\tau)/\sqrt{N}$ for large N (for smmall N, a bit different, depending on the kind of noise...)

Case of a sinusoidal oscillator

Toolbox for an oscillator…

Oscillator : $s(t) = A_0 \cdot [1 + \alpha(t)] \cdot \cos[2\pi\nu_0 t + \phi_0 + \phi(t)]$

ν_0: GHz range: microwave oscillator

100's THz rang: optical oscillator

$\alpha(t)$: amplitude noise

(generaly neglected wrt phase noise, bounded)

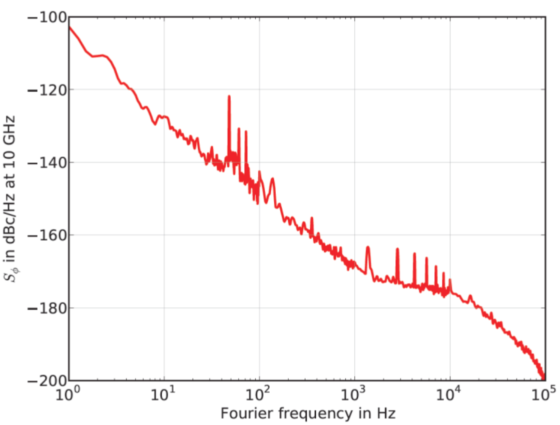
$\boxed{\phi(t)\text{: phase noise (not bounded, always divergent)}}$

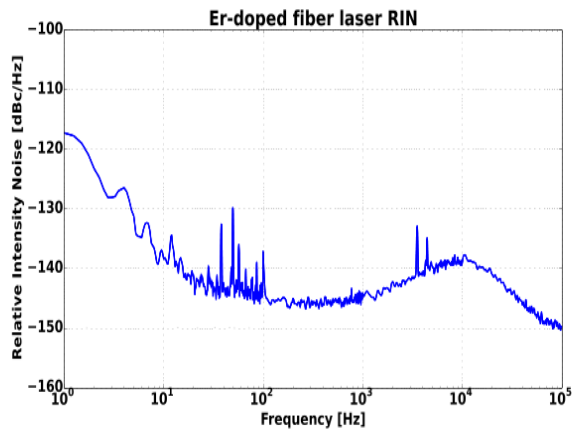
$\nu(t) = \nu_0 + d\phi/dt(t)$ instantaneous frequency

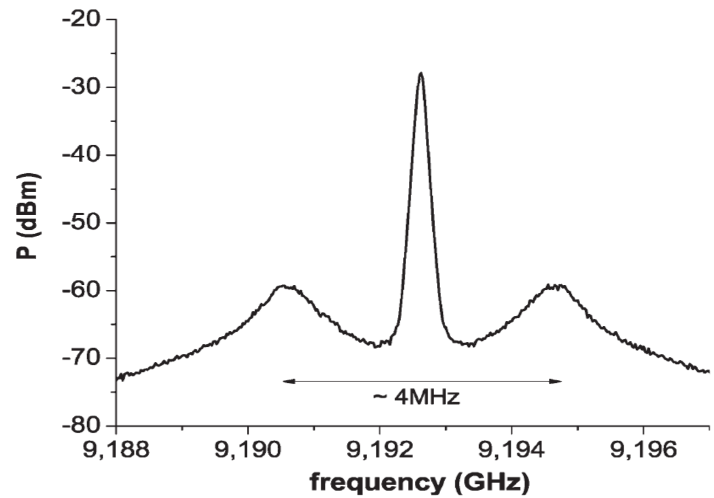
PSD phhase noise: $S_\phi(f)$

PSD ampl. noise(RIN): $S_\alpha(f)$

PSD signal : (spectrum analyz. $S_s(f)$)







+ stabilité de fréquence (en Hz) : $\sigma_\nu(\tau)$
+ stabilité de fréquence relative : $\sigma_y(\tau)$ où $y = \nu/\nu_0$
+ stabilité d'amplitude (relative) $\sigma_{\alpha/A_0}(\tau)$

+ frequency accuracy

The units we use...

$$s(t) = A_0 \cdot [1+\alpha(t)] \cdot \cos[2\pi\nu_0 t + \phi_0 + \phi(t)]$$

Phase noise: $\phi(t)$ [rad] → $S\phi(f)$ [rad^2/Hz] (one-sided PSD)

ou, plus usuellement [dB rad^2/Hz] ou [dB rad/√Hz]

(deux noms pour la même chose, ie 10.log10($S\phi(f)$))

By convention, one uses also $Lc(f) = 10.\log_{10}(S\phi(f)) - 3dB$ [dBc/Hz]

IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology— Random Instabilities ; Standard 1139, revision 1999

Amplitude noise: $\alpha(t)$ [relave, unitless] → $S\alpha(f)$ [Hz^{-1}] (one-sided PSD)

Usually, one also uses for phase noise $Lc(f) = 10.\log_{10}(S\alpha(f))$ - 3dB [dBc/Hz]

Both noises can be expressed in Lc(f) [dBc/Hz] because of a historic relation with $S_s(\nu'-\nu)$ (sideband/carrier ratio on a spectrum analyzer BUT only true under very restrictive conditions of low noise)

Two more concepts...

$$s(t) = A_0 \cdot [1 + \alpha(t)] \cdot \cos[2\pi\nu_0 t + \phi_0 + \phi(t)]$$

Oscillator linewidth:

Many different definitions are possibel and used.
La plus raisonnable, pour bruit d'amplitude négligeable vs. bruit de phase:

Linewidth $\Delta\nu$ defined as:

$$\int_{\Delta\nu/2}^{\infty} S_\phi(f) \cdot df = \frac{2}{\pi}$$

(corresponds to FWHM spectrum linewidth
for white phase noise $S_\phi(f) = S_\nu / f^2 = \mathrm{cte/f^2}$)

Timing jitter

$$S_{\Delta t}(f) = \frac{S_\phi(f)}{(2\pi\nu_0)^2}$$

$$\text{integrated jitter;} = \int_{\mathrm{f_{min}}}^{\mathrm{f_{max}}} \mathrm{S_{\Delta t}(f)dt}$$

Particularly useful when comparing oscillators which oscillates at different carrier frequencies...

A word of advice

Talking about a “very low phase noise oscillator” – without context – is meaningless.

The phase noise level is meaningless only wrt the carrier frequency !!!

Exemple.:

-170dBc/Hz at 10kHz from a de 10GHz carrier is a worl record

-170dBc/Hz at 10kHz from a 100MHz is just normal for a good commercial quarz oscillator costing a few k€/pièce (ex. Rakon LNO100)

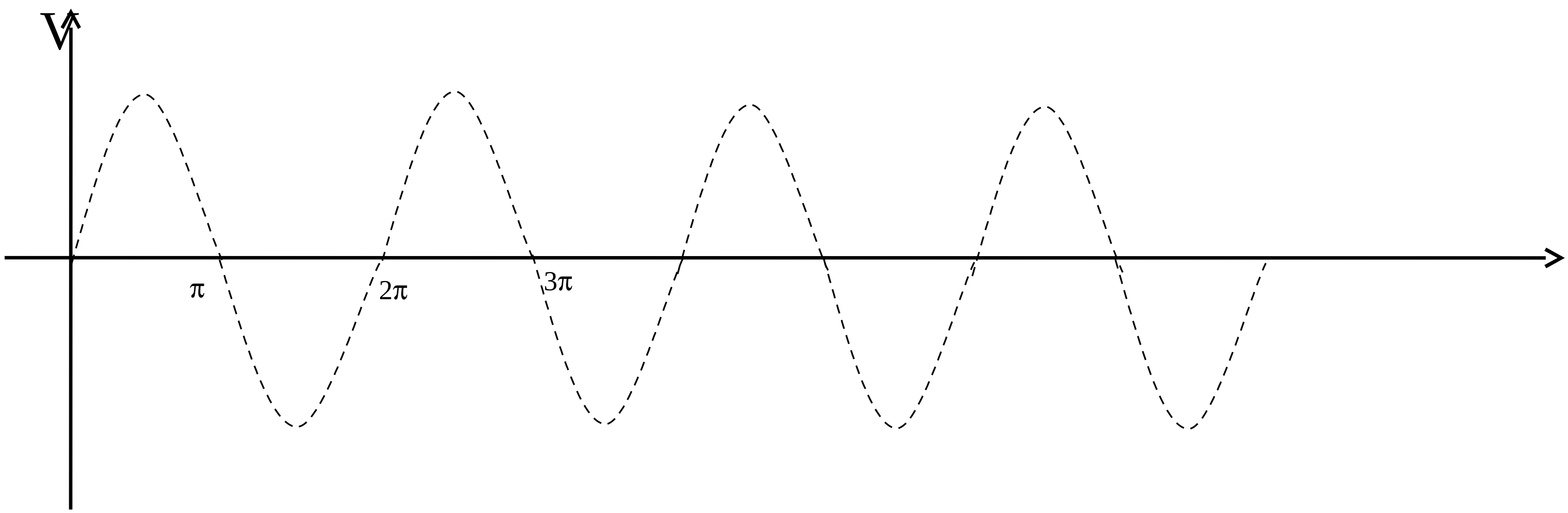
Frequency division: effect on phase noise

fc [Hz] → fc/N [Hz] then $\Delta\phi \rightarrow \Delta\phi/N$ [rad]

$S_\phi(f)$ [dBc/Hz] → $S_\phi(f) - 20.\log10(N)$ [dBc/Hz]

Large reduction of phase noise for large N…

Exemple : division by 2



Frequency division: effect on phase noise

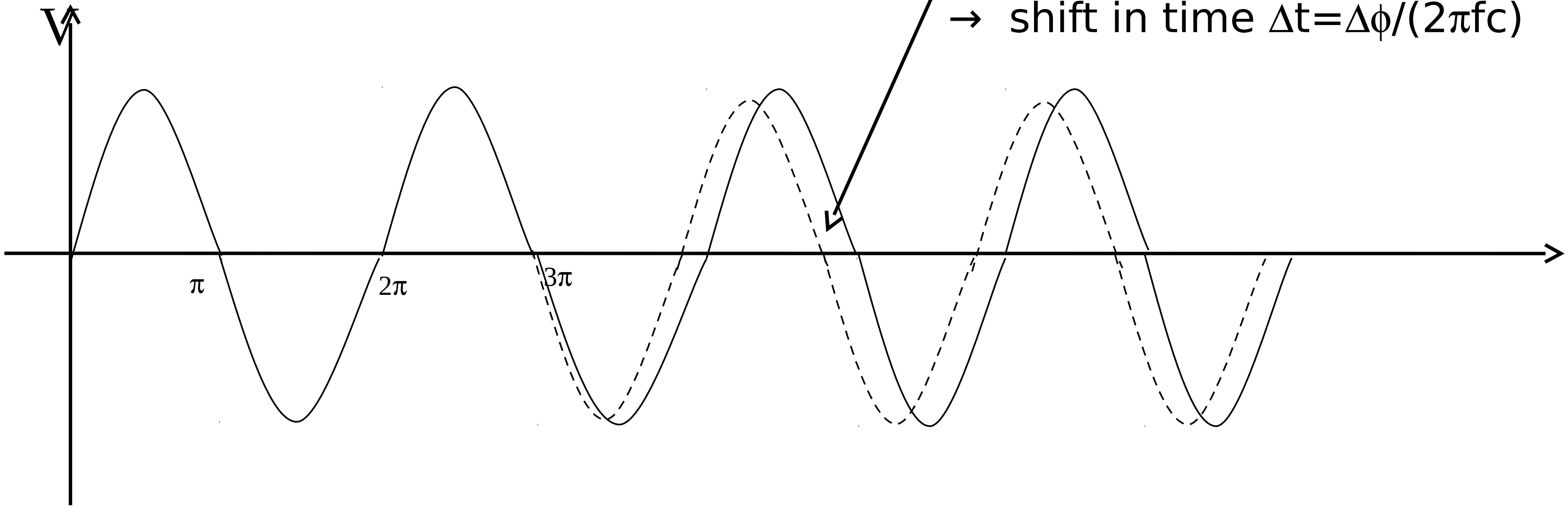
fc [Hz] → fc/N [Hz] alors $\Delta\phi$ → $\Delta\phi$/N [rad]

$S_\phi(f)$ [dBc/Hz] → $S_\phi(f) - 20.\log10(N)$ [dBc/Hz]

Exemple : division by 2

Dephasing $\Delta\phi$

→ shift in time $\Delta t = \Delta\phi/(2\pi fc)$



Frequency division: effect on phase noise

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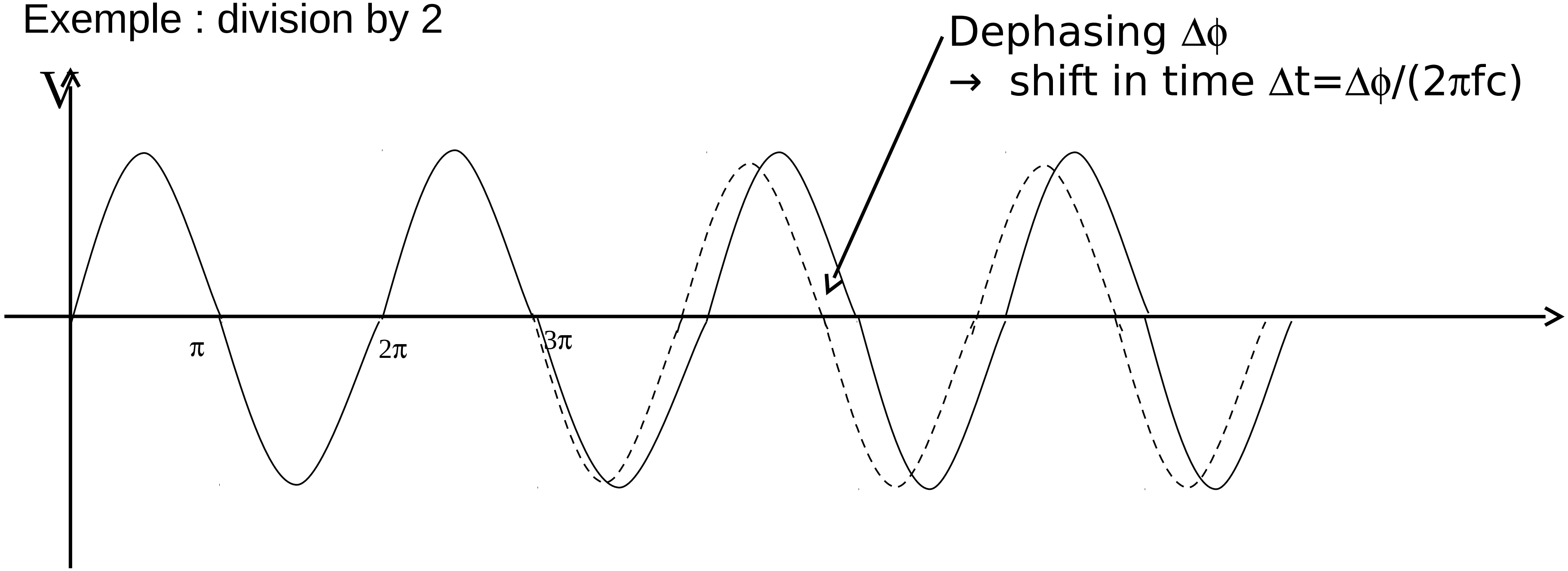
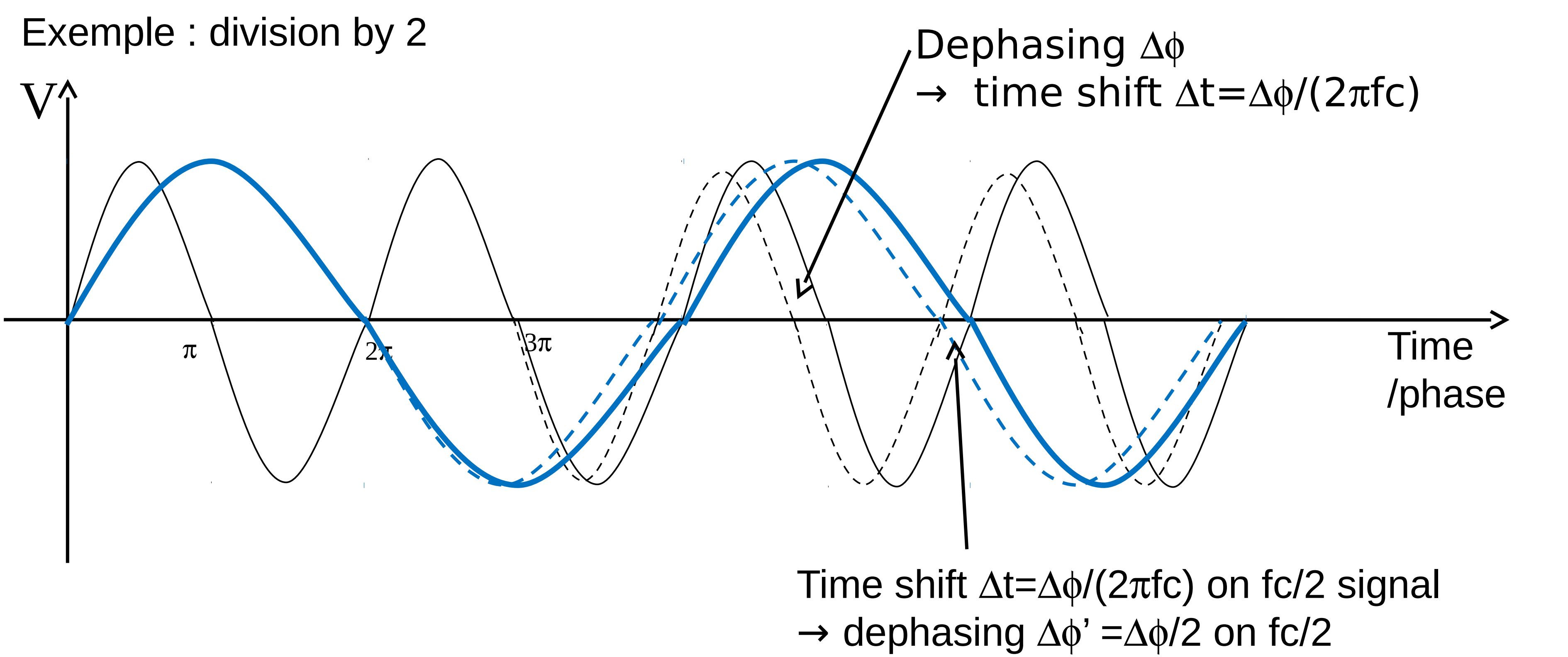
Exemple : division by 2



Division de fréquence : effet sur le bruit de phase

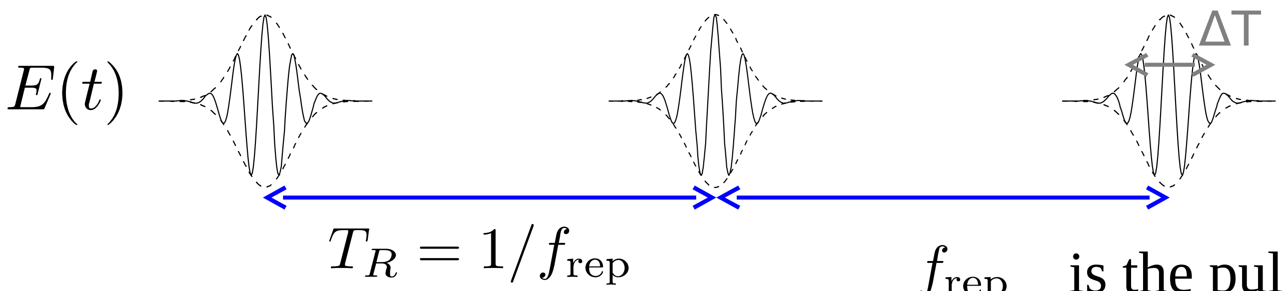
fc [Hz] → fc/N [Hz] then Δf → $\Delta f/N$ [rad]

$S_\phi(f)$ [dBc/Hz] → $S_\phi(f)-20.\log10(N)$ [dBc/Hz]

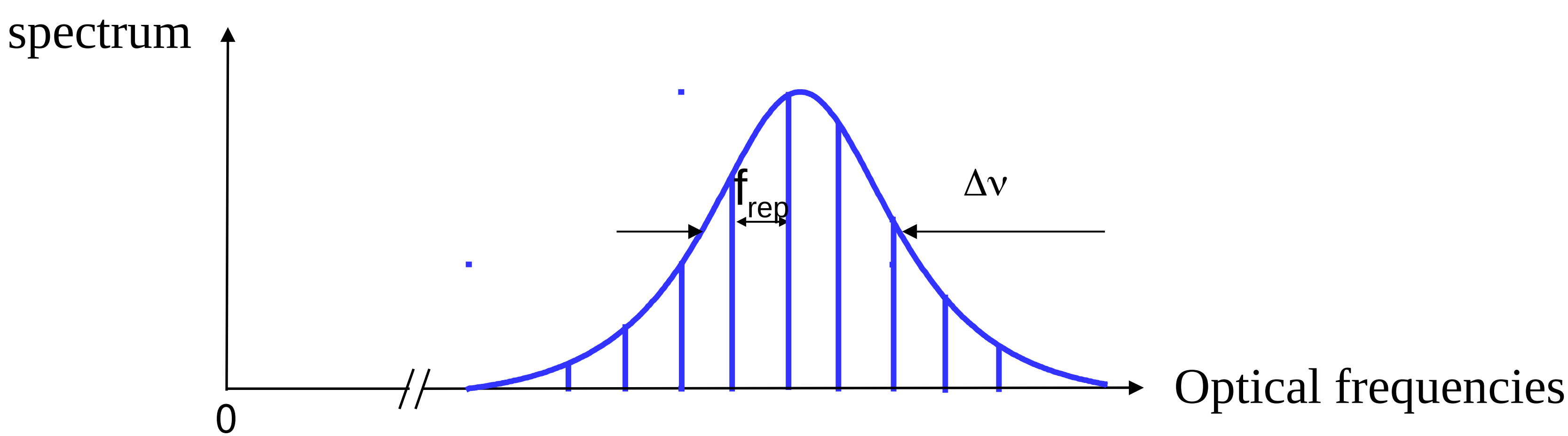


Case of a femto-second laser

Output of a mode-locked Laser



f_{rep} is the pulses repetition rate

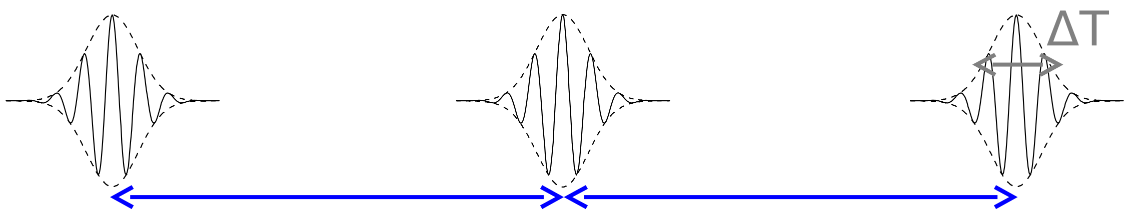


A priori: >10^5 modes, each one a sine-wave-like oscillator !
A lot of possible degrees of freedom, each potentially noisy !!!

BUT: mode-lock laser = controlled dispersion in the laser cavity (GVD=0)
otherwise the pulses don't “hang together” and break apart

Output of a mode-locked Laser

$E(t)$



$T_R = 1/f_{\rm rep}$

$f_{\rm rep}$ is the pulses repetition rate

Pulses inside the cavity with $\boxed{v_g \neq v_\phi}$

→ the pulses "hang together", so there must be no group velocity dispersion (GVD), but there can be phase velocity dispersion !

Zero GVD means:

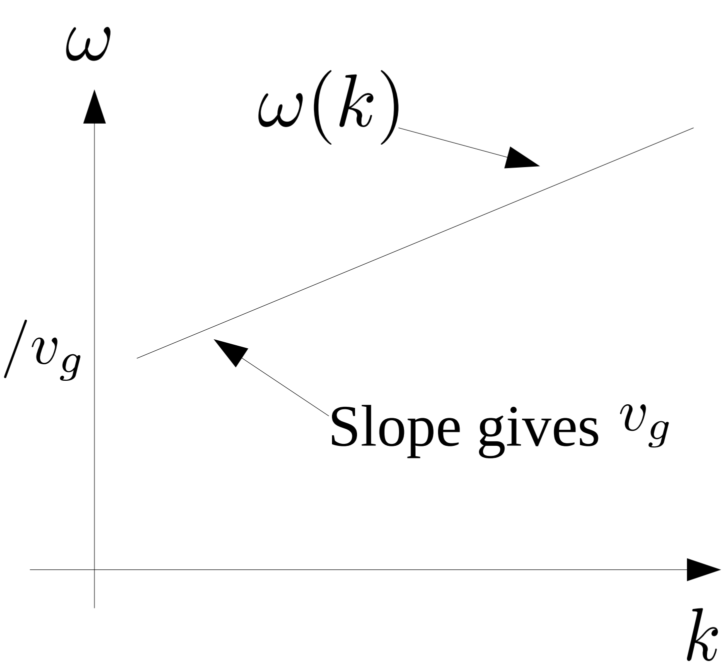
$$\frac{\partial k(\omega)}{\partial \omega} = \frac{1}{v_g} = \text{const.}$$

$$\frac{\partial^m k(\omega)}{\partial \omega^m} = 0, \forall m > 1$$

This is satisfied for: $k(\omega) = n(\omega)\omega/c = k_0 + \omega/v_g$

Which corresponds to: $\boxed{n(\omega) = c\left(\frac{1}{v_g} + \frac{k_0}{\omega}\right)}$

(on average over 1 round trip of the cavity)



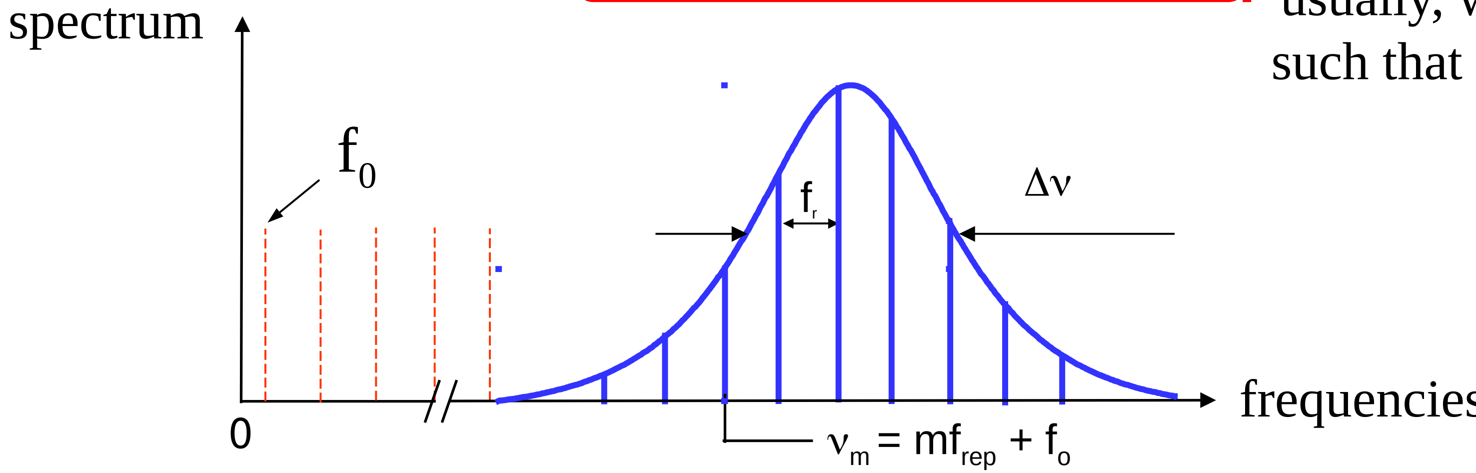
Output of a mode-locked Laser

If we plug $n = c\left(\frac{1}{v_g} + \frac{k}{\omega}\right)$ into the freq. of the FP cavity modes $\omega_N = N\frac{2\pi}{nL}$

and solve for the frequencies, we get: $\omega_N = N\frac{2\pi v_g}{L} - k_0 v_g$

This is the celebrated formula $\boxed{\nu_N = N \times f_{\mathrm{rep}} + f_0}$

Note : usually, we choose N such that $0 < f_0 < f_{rep}/2$



The comb is a « ruler » in optical frequency domain, with one mode every f_{rep}

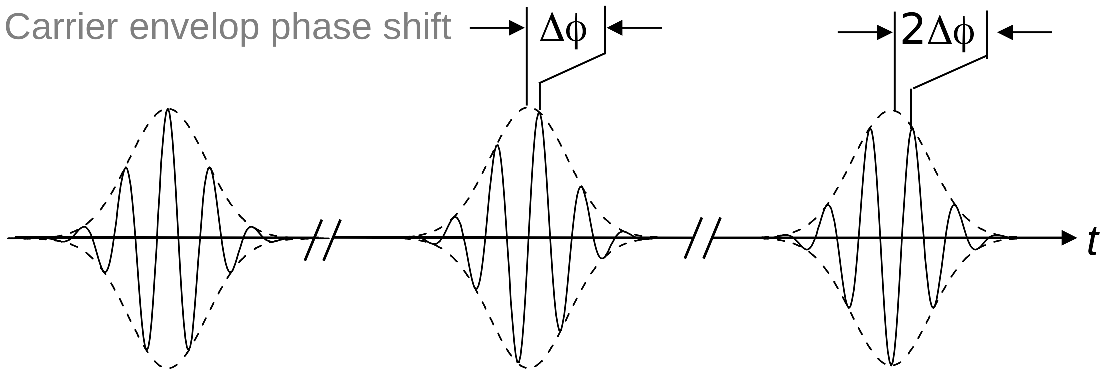
There is a **fixed phase relationship** between the different modes !

→ phase noise of f_{rep} and of f_0 (plus ~pulse amplitude noise) is sufficient to characterize the noise properties of the femtosecond laser …

(note consequence on Schalow-Townes limit for mode ν_N << cw with same power...)

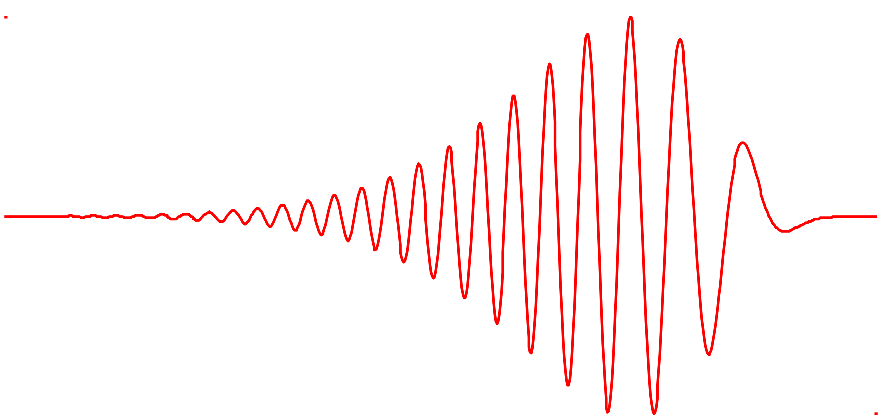
Output of a mode-locked Laser

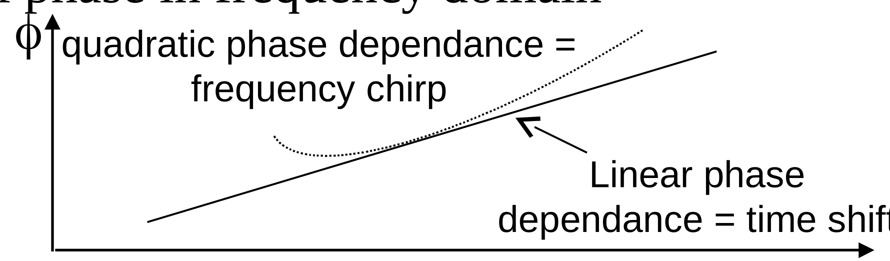
- In time domain (Fourier transform), this corresponds to:

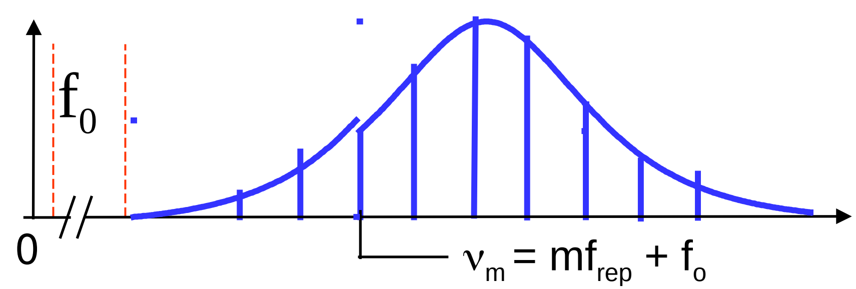


$$\boxed{f_0 = f_r \times \frac{\Delta\varphi}{2\pi}}$$

- After getting out of the cavity, the pulse can experience further dispersion (GVD≠0 : blue and red part propagate at different speed)
  → chirped pulse in time domain // spectral phase in frequency domain





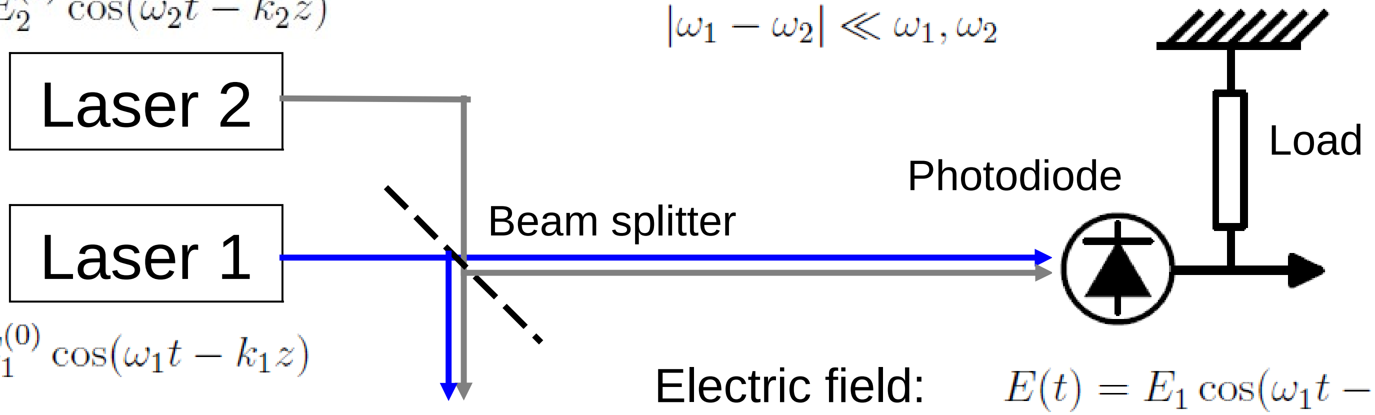


Optical frequency comb

Beat note between two lasers

$E_2^{(0)}\cos(\omega_2 t - k_2 z)$

$|\omega_1 - \omega_2| \ll \omega_1, \omega_2$



$E_1^{(0)}\cos(\omega_1 t - k_1 z)$

Electric field: $E(t) = E_1\cos(\omega_1 t - \phi_1) + E_2\cos(\omega_2 t - \phi_2)$

Poynting vector:

$$\Pi(t) \propto |E(t)|^2 = E_1^2\cos^2(\omega_1 t - \phi_1) + E_2^2\cos^2(\omega_2 t - \phi_2) + 2E_1E_2\cos(\omega_1 t - \phi_1)\cos(\omega_2 t - \phi_2)$$

$$= \frac{E_1^2}{2}[\underbrace{1}_{\text{cst.}} + \underbrace{\cos(2\omega_1 t - 2\phi_1)}_{\text{fast}}] + \frac{E_2^2}{2}[\underbrace{1}_{\text{cst.}} + \underbrace{\cos(2\omega_2 t - 2\phi_2)}_{\text{fast}}]$$

$$+E_1E_2[\underbrace{\cos((\omega_1 - \omega_2)t - (\phi_1 - \phi_2))}_{\text{slow}} + \underbrace{\cos((\omega_1 + \omega_2)t - (\phi_1 + \phi_2))}_{\text{fast}}]$$

Average optical power over detector time constant: $|\omega_1 - \omega_2| \ll 1/\tau_{det} \ll \omega_1, \omega_2$

$$P_{opt}(t) \propto E_1^2 + E_2^2 + 2E_1E_2\underbrace{\cos((\omega_1 - \omega_2)t - (\phi_1 - \phi_2))}$$

Beat note between the 2 lasers at frequency: $|\omega_1 - \omega_2|$

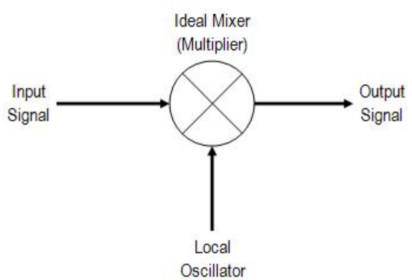
Current through the load: $I(t) = \eta P_{opt}(t)$

Beat note RF power: $P_{RF} = < R_{load} \times I^2(t) > \propto R_{load} \times \eta^2 |E_1|^2 |E_2|^2 \propto R_{load} \times \eta^2 P_{1opt} P_{2opt}$

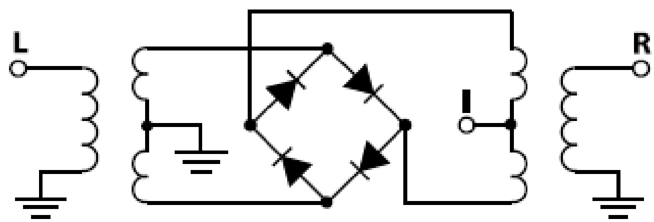
Beat note between two lasers

- Fast photodiodes and electronics are typically limited to <100 GHz.
- 100 GHz = 0.1 THz is a tiny fraction UV/VIS/IR range (~1000 THz).
- → in practice beat note measurements are **narrow band** at the scale of the UV/VIS/IR spectrum.

- Detecting a beat note is similar to using a mixer in the RF or microwave domain. However, a mixer usually gives access to both $(\omega_1-\omega_2)$ and $(\omega_1+\omega_2)$.



Electrical Schematic



IF(t) =LO(t)xRF(t)

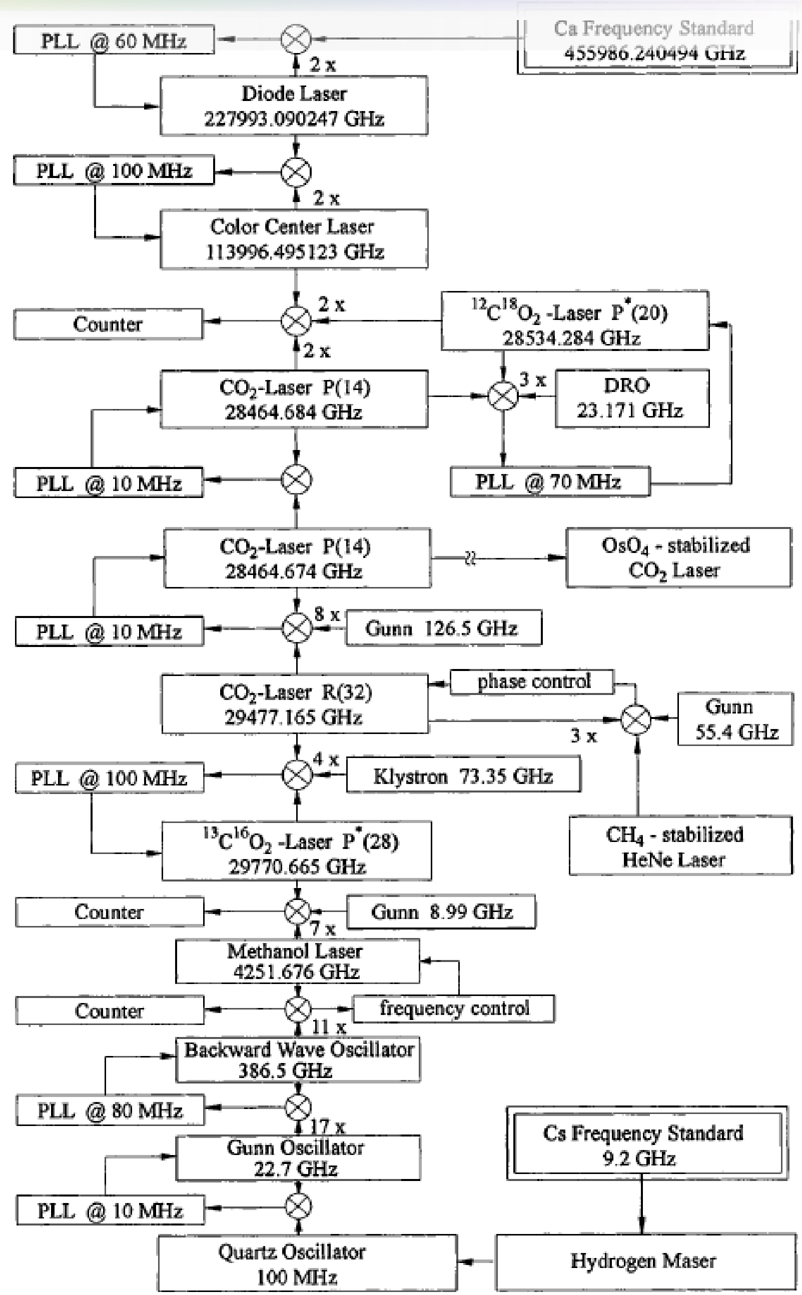
$$\cos(\omega_1 t).\cos(\omega_2 t) = 1/2.[\cos(\omega_1-\omega_2)t+\cos(\omega_1+\omega_2)t]$$

- Other methods to “move” in the optical frequency domain:
  - Second Harmonic Generation, Sum and Difference Frequency Generation.
  - Generally **narrow band** (crystals, coatings, geometry,… specific to a narrow wavelength range).
- No straightforward way to link optical frequency to microwave frequency.

Optical Frequency Measurements (old technique)

- Harmonic frequency chain: Ensemble of subsystems each allowing a frequency multiplication by a factor 2 or 3.
  - Complexity.
  - Inconvenient frequencies (THz, mid-IR).
  - 3 or 4 chains in the world at PTB, SYRTE, NRC, JILA.
  - Continuous operation <3h.

- The setup has to be redesigned if one changes the frequency to be measured.



*H. Schnatz et al. PRL **76**, 18 (1996)*

Optical Frequency Measurements with Optical Combs

- Harmonic chain replaced with:

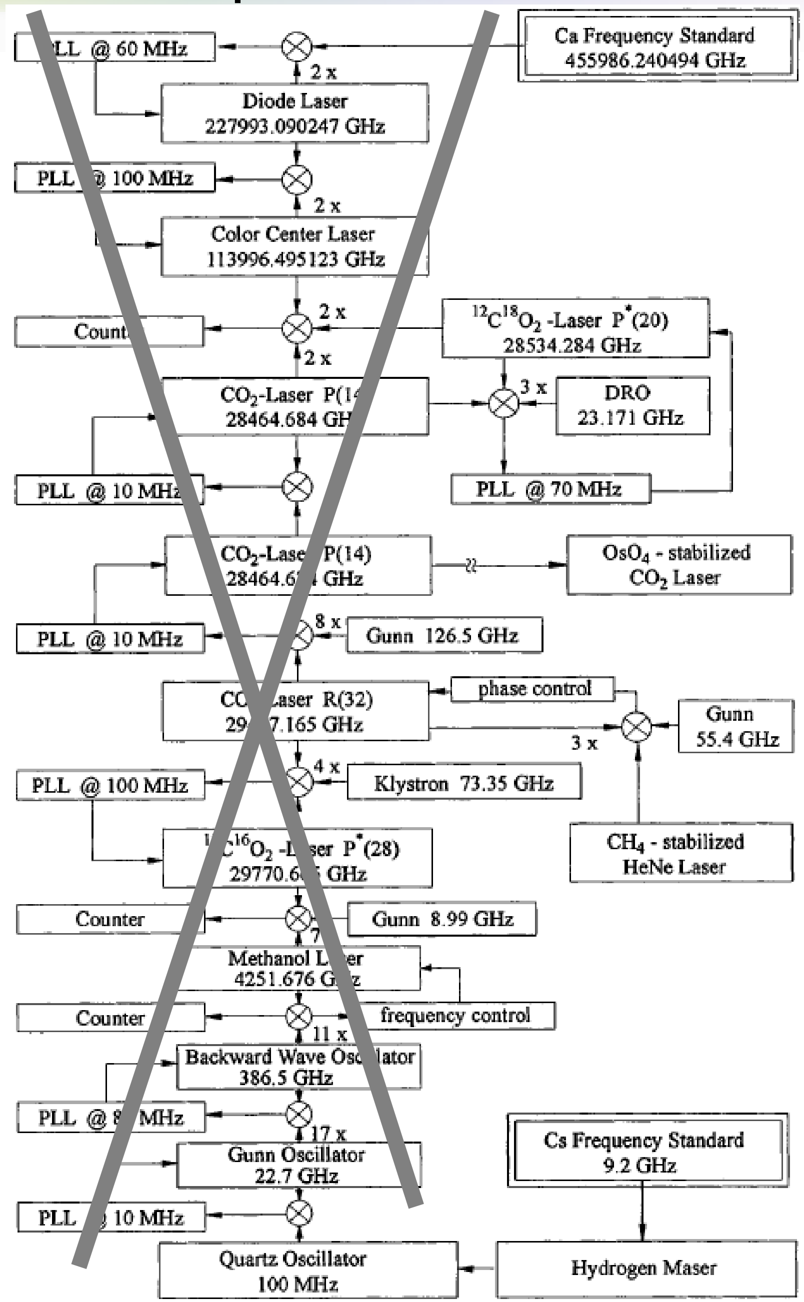
$$\boxed{\text{1 Laser}}$$

$$\boxed{\nu_N = N f_{rep} + f_0}$$ N large !

- Extremely simple and cost effective in comparison.
- No inconvenient frequency.
- Measurement of several frequencies at the same time.
- Continuous operation for weeks.
- Covers the visible-near IR range.
- Commercially available systems.

Nobel Lecture: Defining and measuring optical frequencies
J. L. Hall, Rev. Mod. Phys. 78, 1279 (2006)

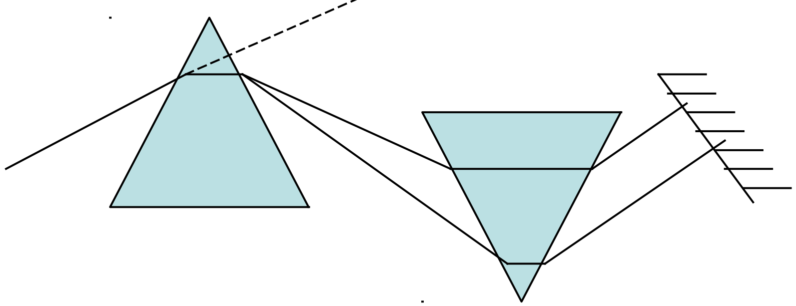
Nobel Lecture: Passion For Precision
T. W. Hänsch, Rev. Mod. Phys. 78, 1297 (2006)



Requirements for mode-locked operation

- Gain medium with broad gain curve to support a large number of modes (the largest known is Ti:Sapphire by far, but many other exists and researchers are working on making and characterizing some more…)

- Controlled dispersion to allow short pulses to remain short through round trip propagation in the cavity (glass is GVD> 0 in visible spectrum)
  - Prism pairs to compensate normal dispersion

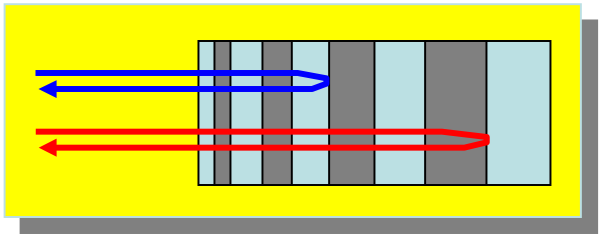


R.L. Fork et al. Opt. Lett. 9 150 (1984)

Also double pair for ring cavity (but harder to align…)

  - Grating pairs (mostly for compressors/stretchers, with CPA for exemple)
  - Combining gratings and prism for shortest pulses (historical)
  - (doubly) chirped mirrors

(Large oscillations of GDD otherwise, due to Gire-Tournois multiple interf.)



(Note GDD is expressed in ps², which is really ps/PHz…)

Requirements for mode-locked operation

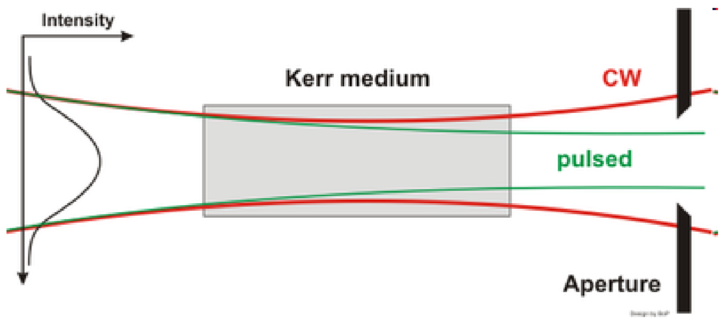
- Mechanism for (passive) mode-locking: Coupling between longitudinal modes so that they oscillate with a well defined phase with respect to each others
  Any intra-cavity effect which will make high instantaneous intensity (pulsed) operation favorable will do. Examples:

  → Saturable absorber (often quite slow, carbon nanotubes ?),

  → Non-linear polarisation rotation,

- → Non-linear fiber loop mittor

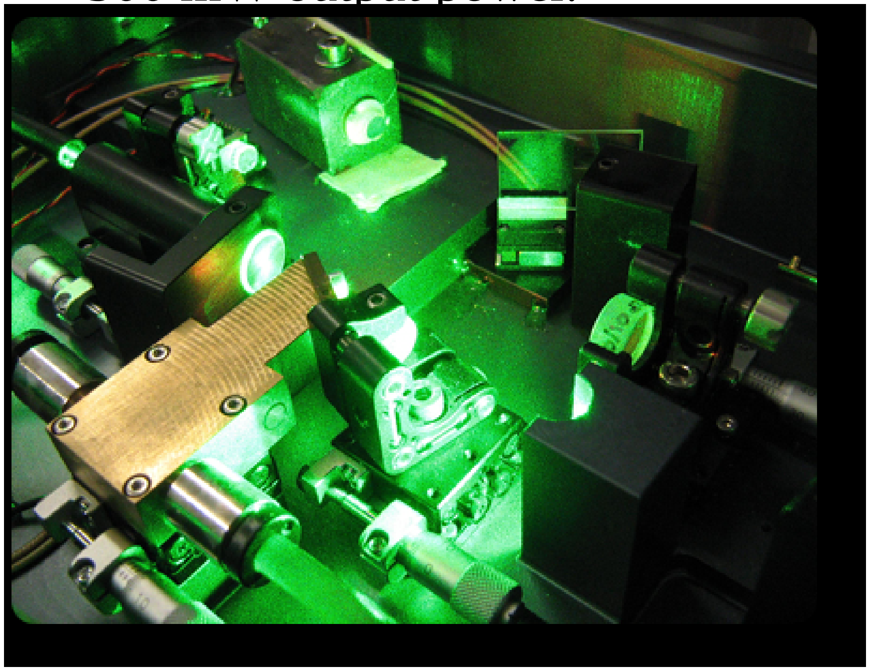
  → Kerr-lens effect (very fast)

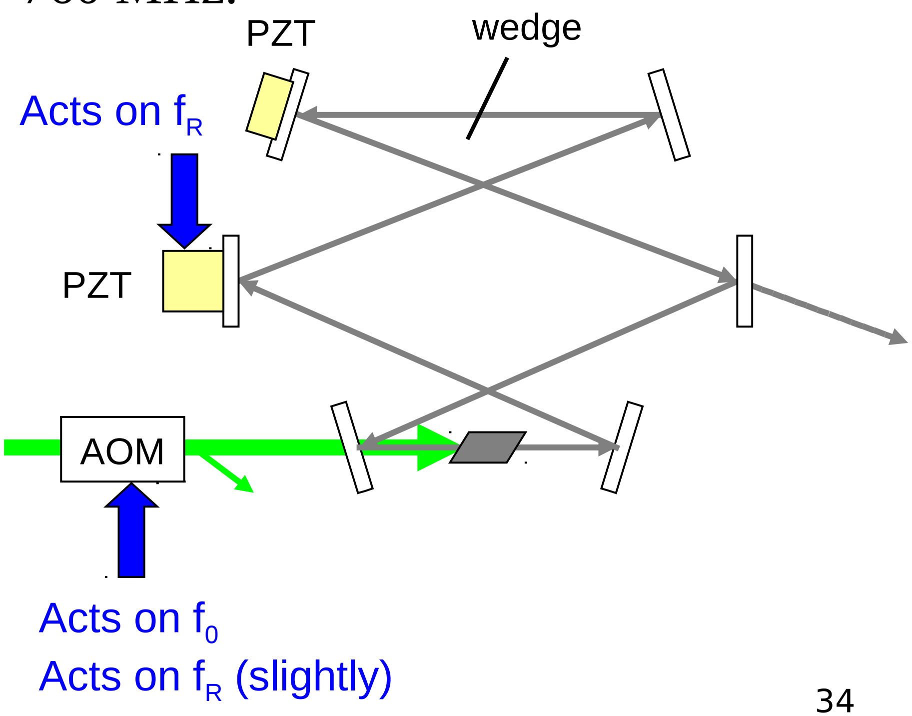


- Startup by noise, more or less easily (self start, kick start, also synchronous pumping)

Example of a femtosecond Ti:Sa laser

- Gain medium is a Titanium doped sapphire crystal.
- Kerr-lens effect is making pulsed operation favorable
- Pumped with ~6 W of laser light at 532 nm.
- Cavity made of 6 chirped mirrors.
- Intra cavity wedge for coarse tuning f_0.
- Fast and slow piezo actuators.
- Fast pump power control through an AOM.
- 30 fs pulses at 850 nm. Repetition rate: ~760 MHz.
- ~500 mW output power.





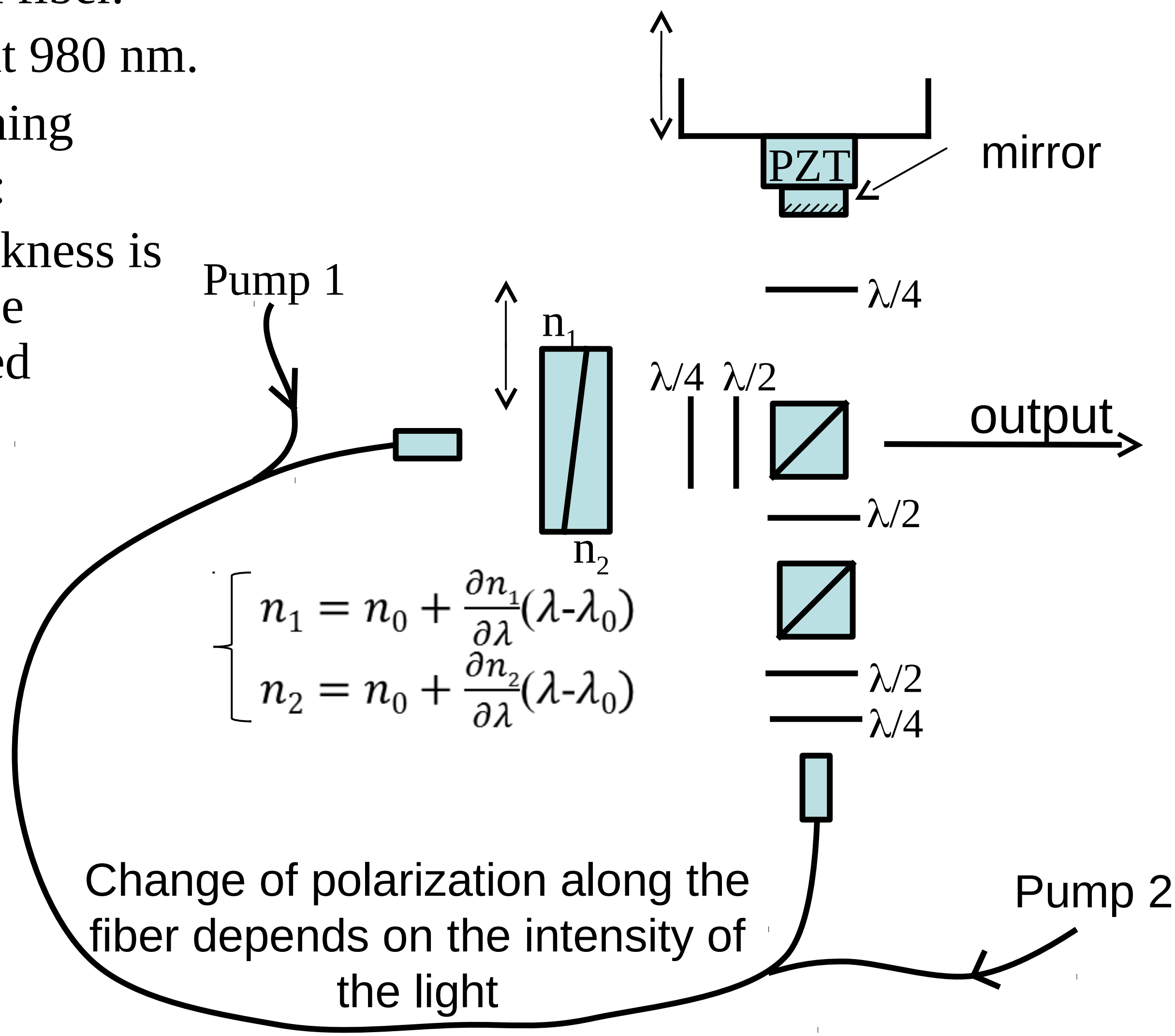
Example of a Er-doped fiber laser with non lin. pol. rotation

- Gain medium is a Er-doped fiber.
- Pumped with diode lasers at 980 nm.

Double wedge for f_0 coarse tuning without too much f_{rep} coupling:
when translated total glass thickness is identical and n-index very close

→ group delay is conserved
BUT, dn/dλ different
→ group delay dispersion is changed, hence f_0 too

- Actuation is possible via
  → PZT (mainly frep)
  → Pump power



- Non-linear polarization rotation in the fiber is making pulsed operation favorable when λ plates are adjusted accordingly

Example of a Er-doped fiber laser with “figure 8” topology

- Gain medium is a Er-doped fiber.
- Pumped with diode lasers at 980 nm.

- Very robust, no tuning at all for mode locking (but usually low power), full PM possible

- Possible to add an EOM for feed back on f_{rep}

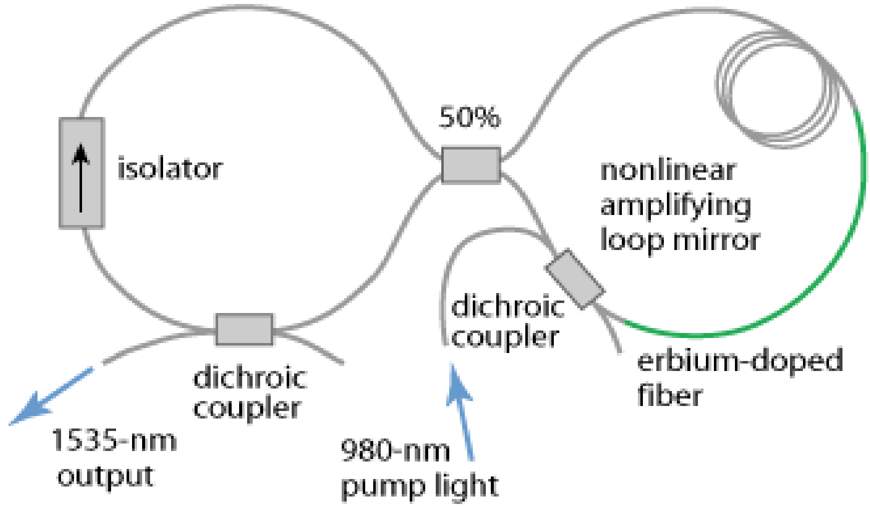
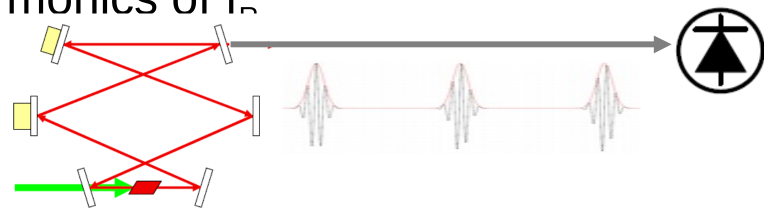


Figure from Encyclopedia of Laser physics (www.rp-photonics.com)

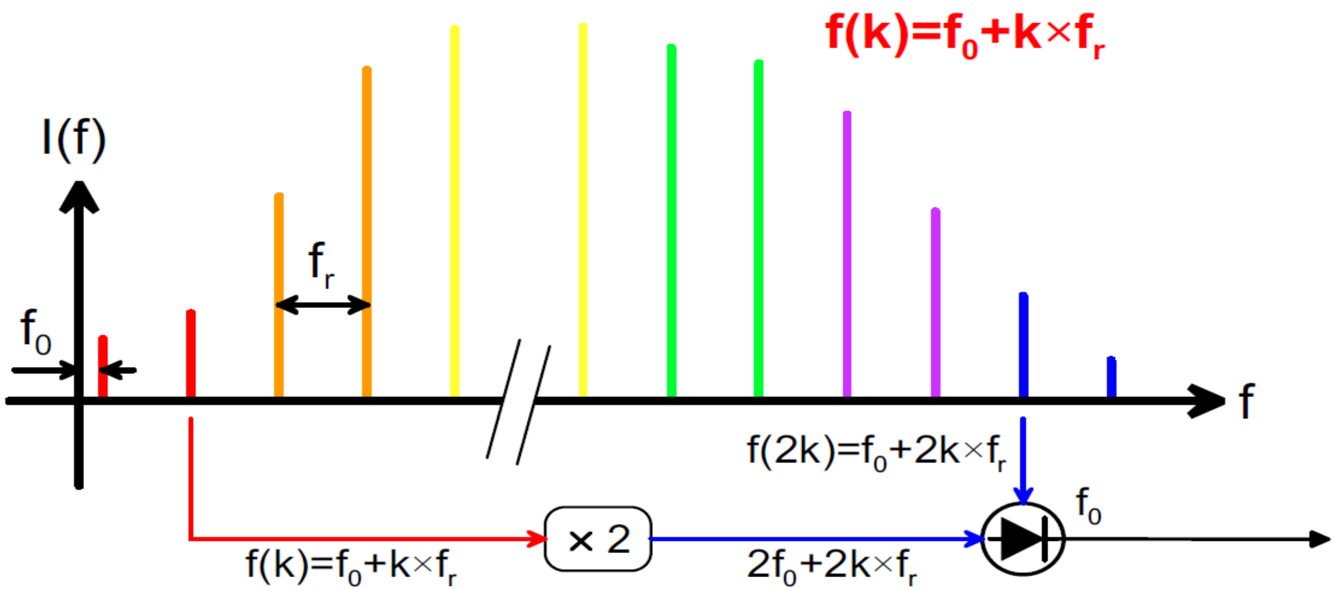
Measuring the comb parameters f_R and f_0

- Measurement of f_R: detect the pulse train with a fast photodiode
  - Note: gives directly access to harmonics of f_R



  -

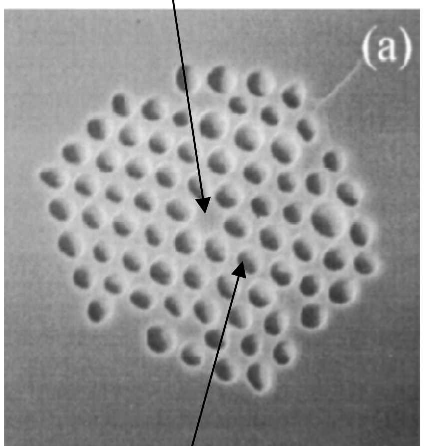
- Measurement of f_0 with an octave spanning comb: The self-referencing method
  - Note: many modes contribute to the signal at f_0 (requires phase maching condition between each couple freq beats red-doubled and blue part of the pulse arrive at the same time on the detector



Generation of an octave spanning optical frequency comb

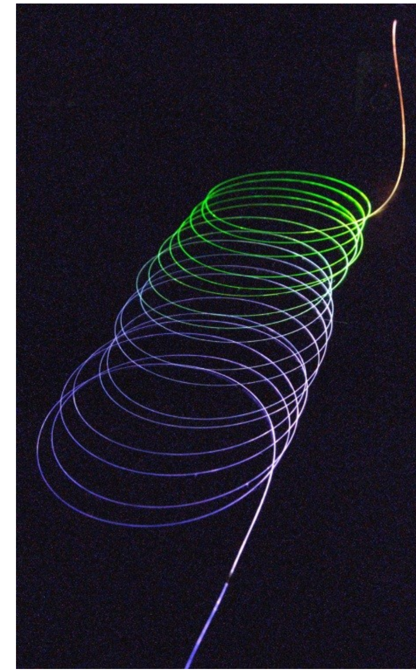
Photonic crystal fiber

Core: 2μm



Dispersion management allows propagation of short pulses

- Short input pulses with high peak power
- Small core: high peak intensity
- Pulses remain short through propagation
- ➔ Large non-linear effects
  - Time domain: Self phase modulation, cross-phase modulation, stimulated Raman emission,…
  - Frequency domain: Four wave mixing

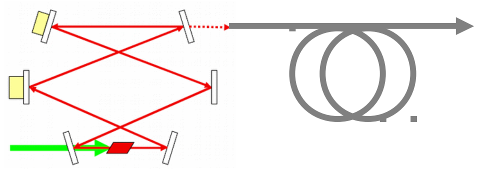


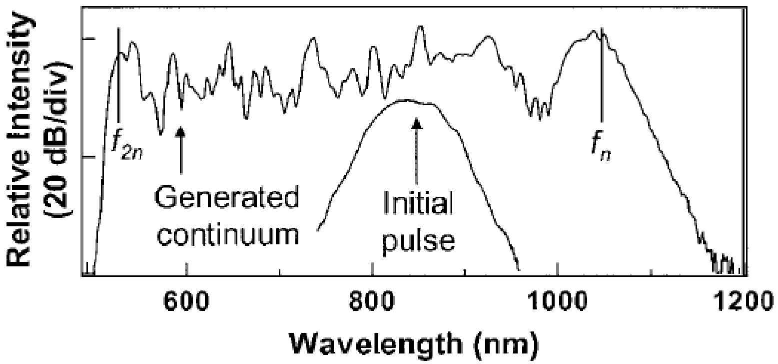
- Coherent spectral broadening
  - Note: pulses are broadened ➔Spectral coherence but no Fourier limited pulses

Octave spanning optical frequency combs

- Ti:Sa, Er-doped fiber broadened with photonic crystal fiber

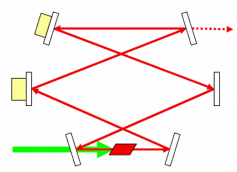
S. Diddams et al., Phys. Rev. Lett. 84, 5102 (2000)

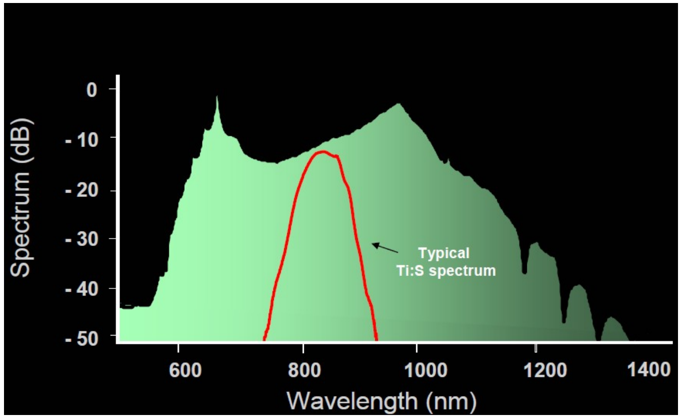




- Ultra broadband Ti:Sa

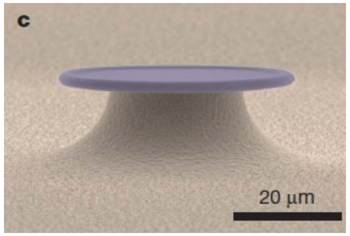
T. M. Fortier et al., Opt. Lett. 31, 1011 (2006)

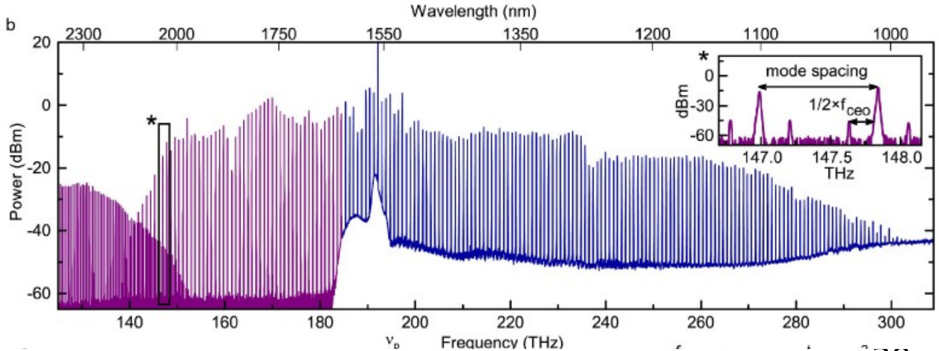




- Toroidal micro resonators

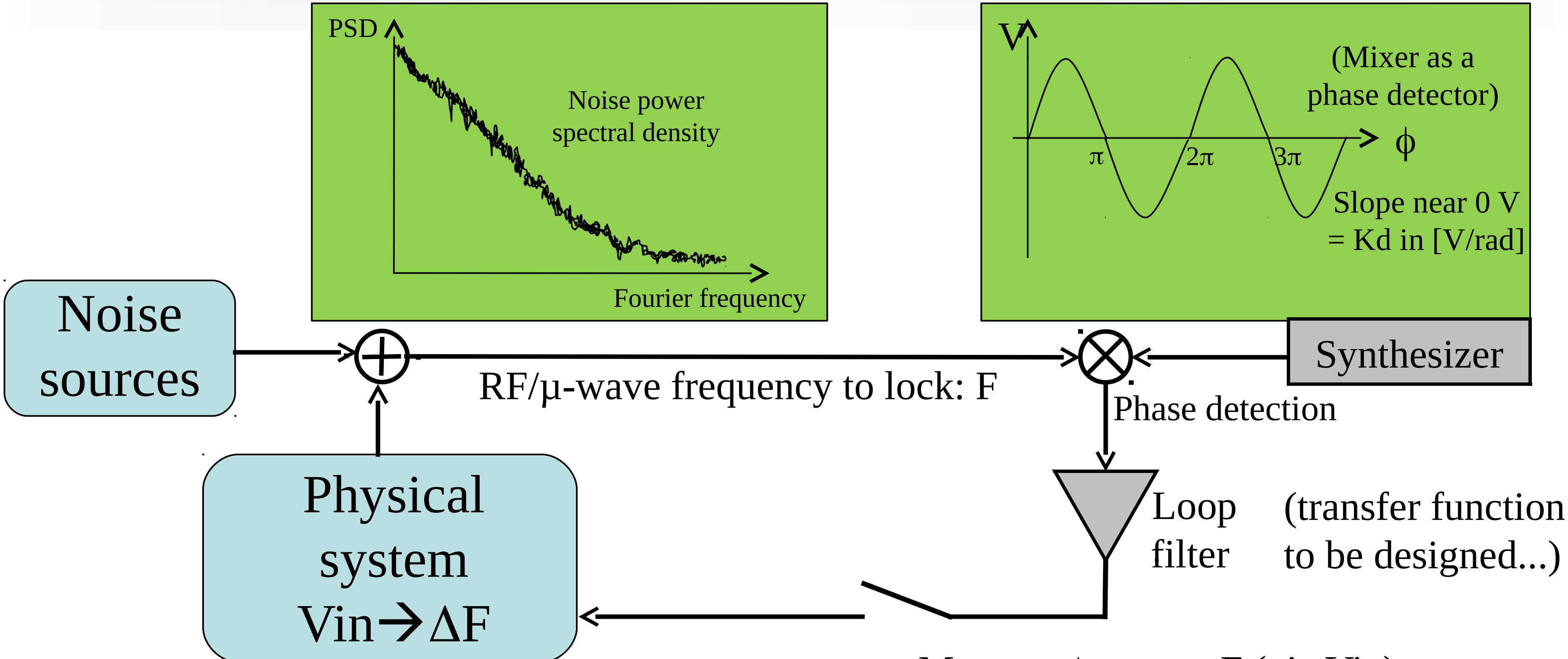
P. Del'Haye et al., arxiv 0912.4890v1 (2009)





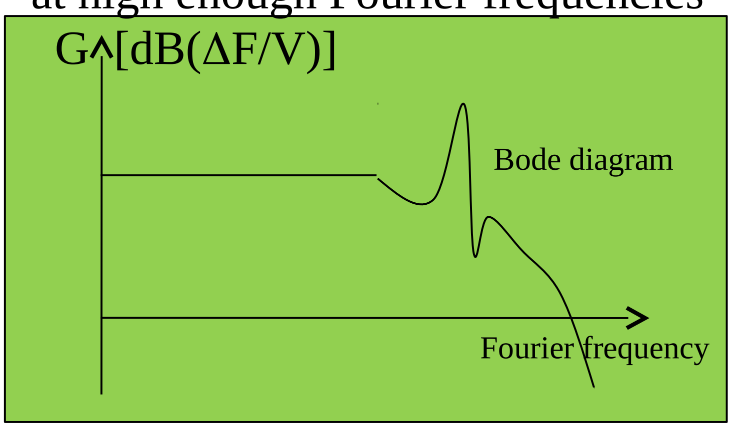
Phase locking and stuff...

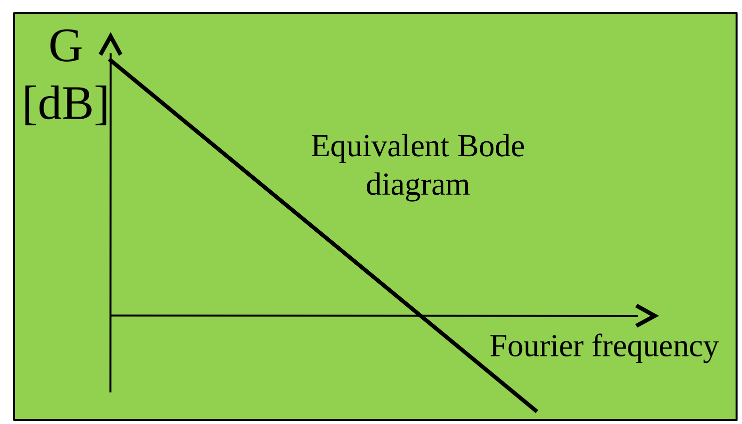
Phase-locking (a comb or anything else…)



Physical actuator ALWAYS gets mad at high enough Fourier frequencies

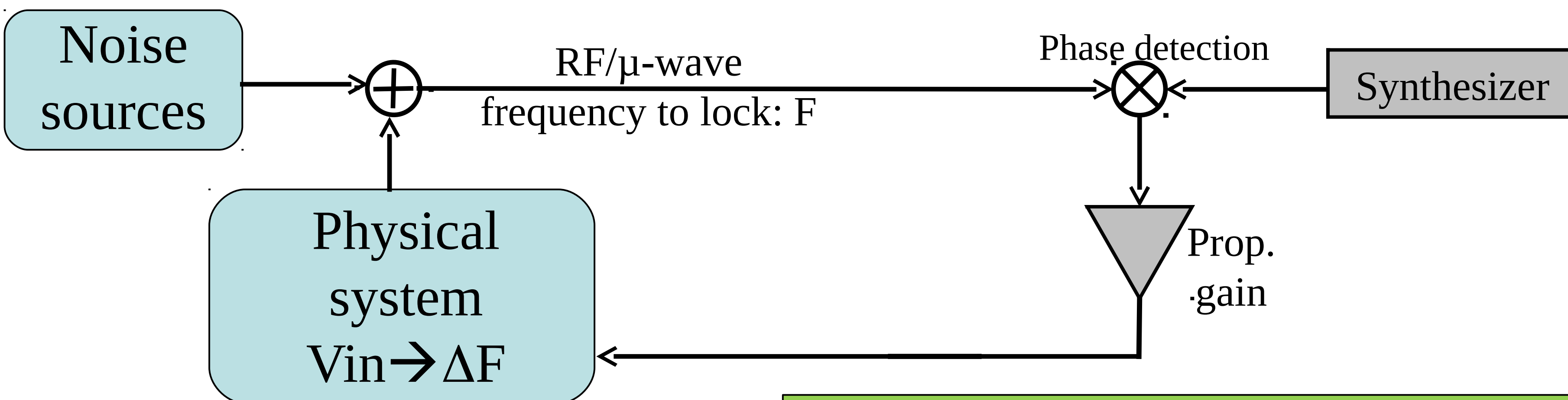
Measure ϕ, act on F (via Vin)
→ equivalent to a pure integrator



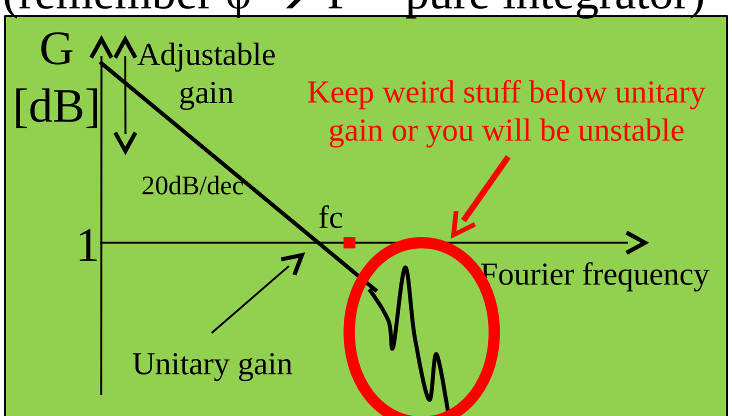


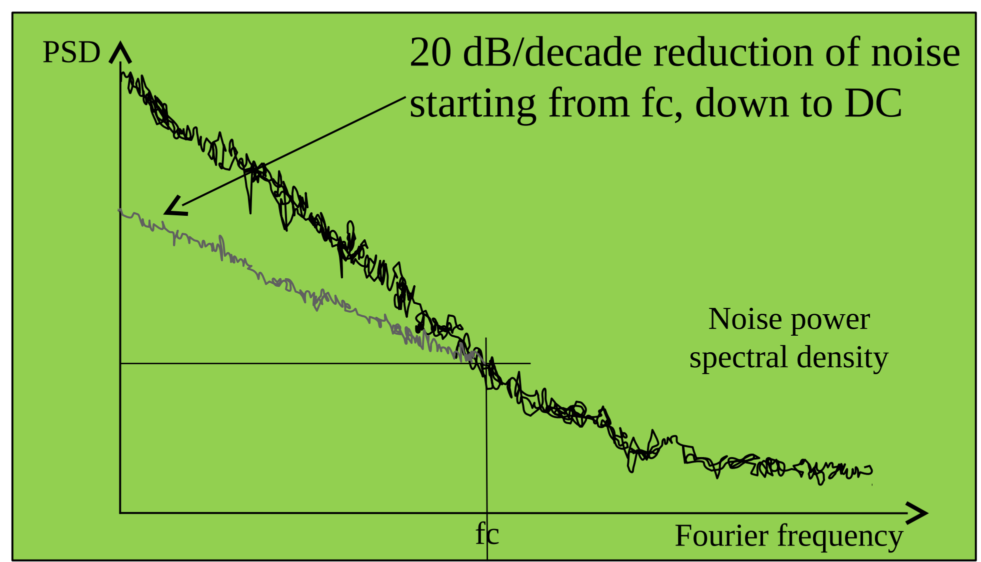
Phase-locking : P corrector

Golden rule of locking: in open loop Bode diag., cross unitary gain with no more than 20dB/decade (unstable otherwise)



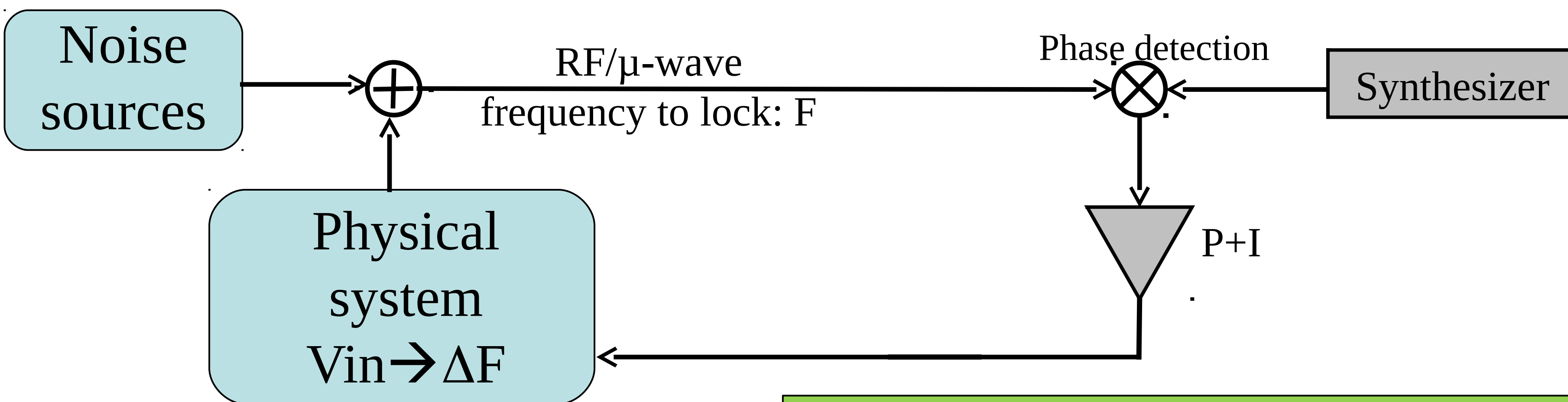
Open loop Bode diagran
(remember ϕ → F = pure integrator)



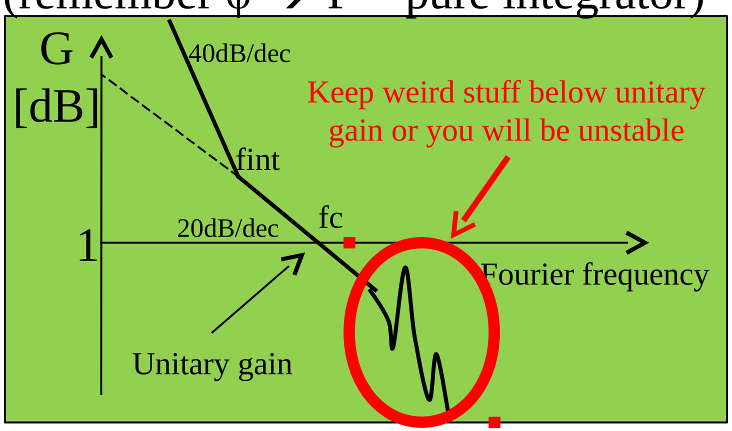


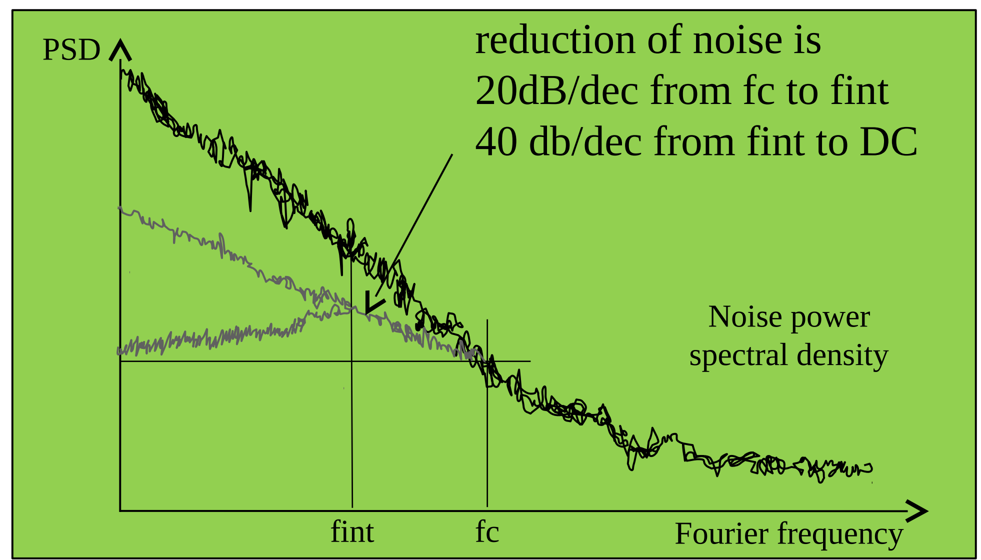
Phase-locking : PI corrector

Rule of thumb : fint < fc/3 to fc/10, otherwise unstable (too much residual dephasing at fc (unitary gain) due to integrator



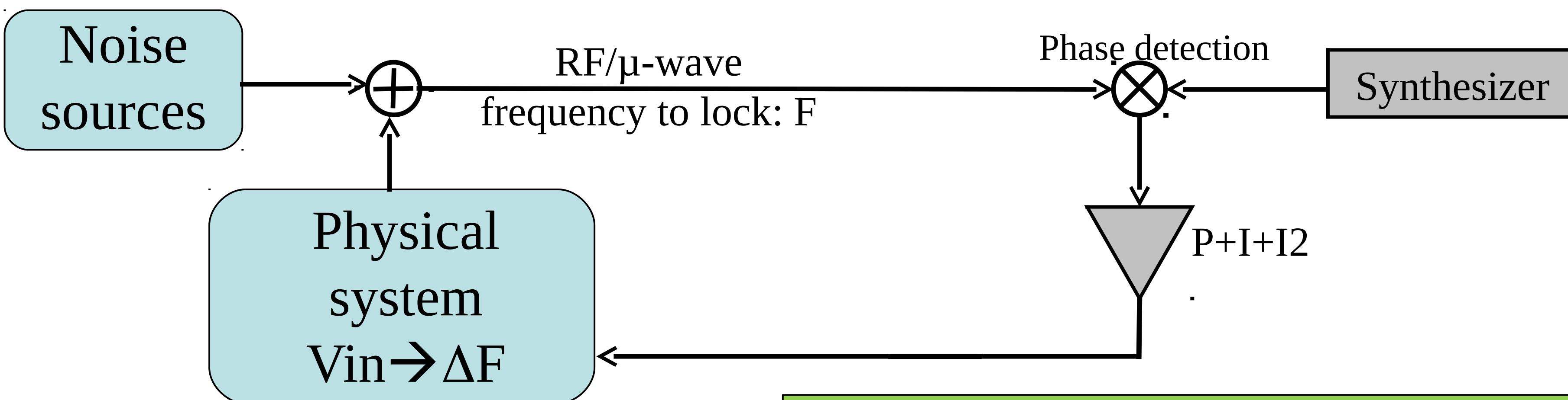
Open loop Bode diagran
(remember ϕ → F = pure integrator)



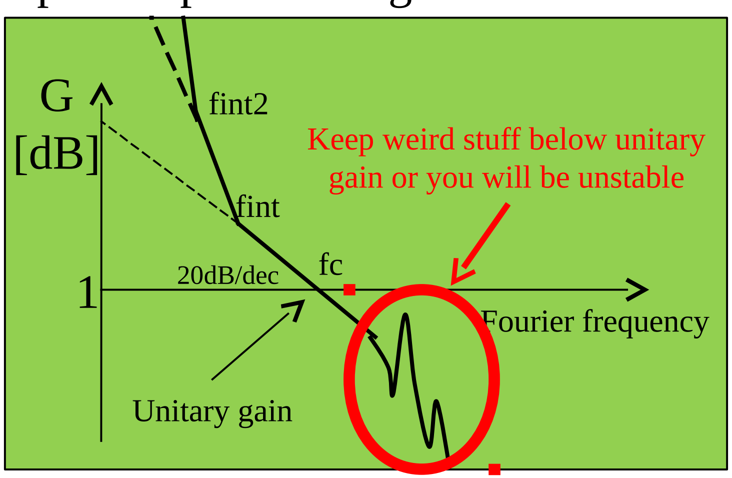


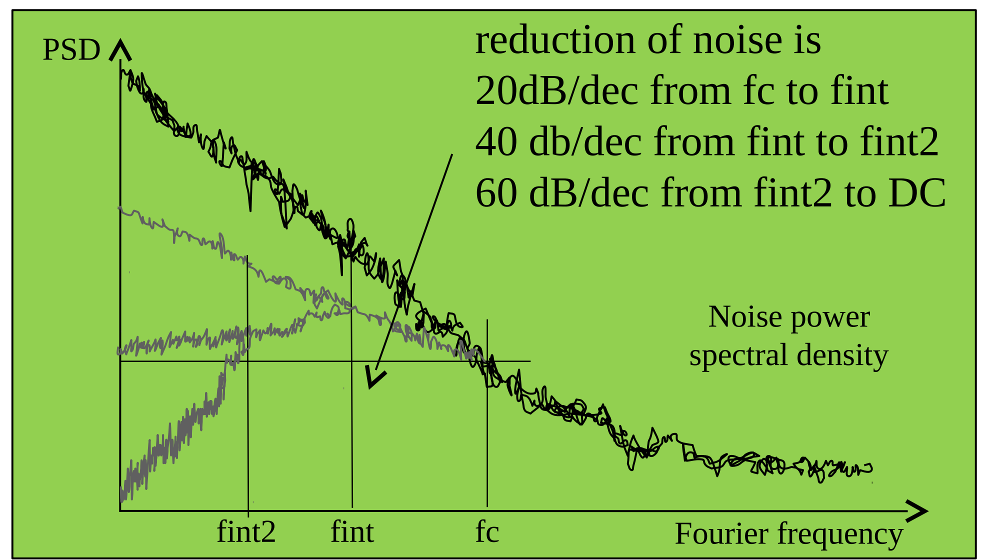
Phase-locking : PI2 corrector

Rule of thumb : fint2 < fint/3 to fint/10, otherwise unstable

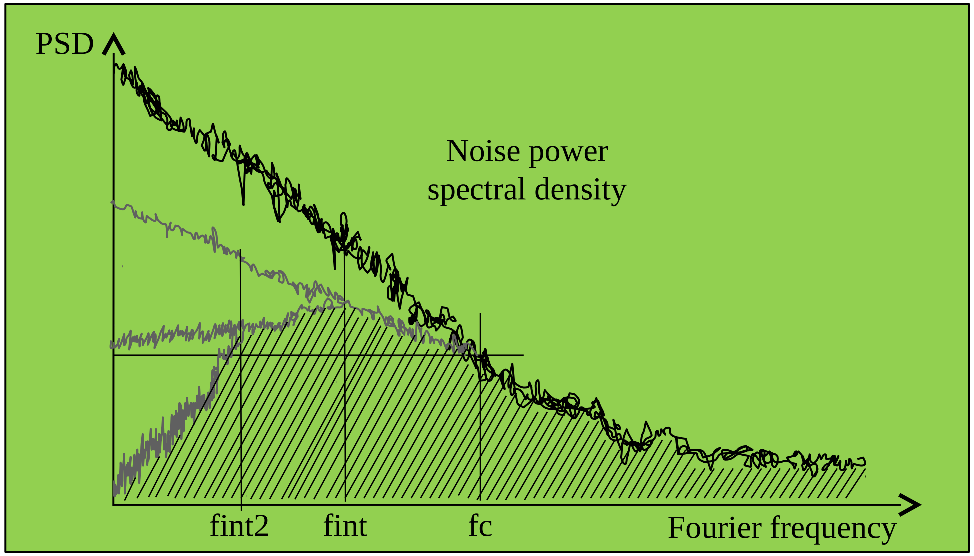


Open loop Bode diagran



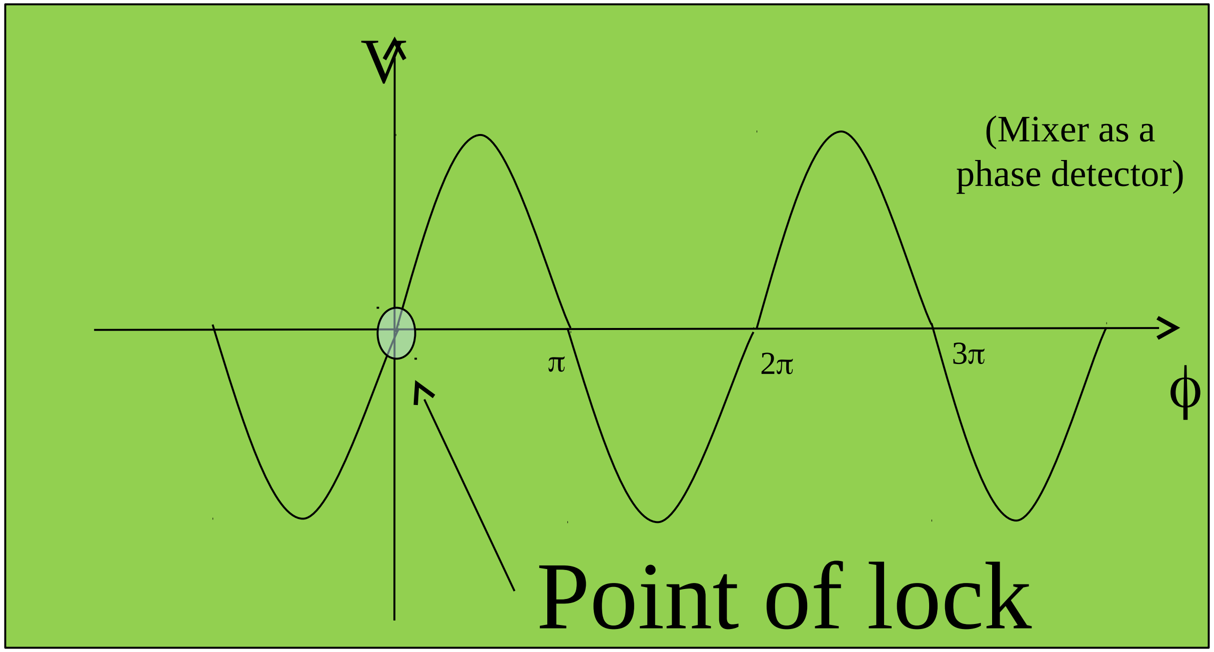


Phase-locking : why you may need a divider

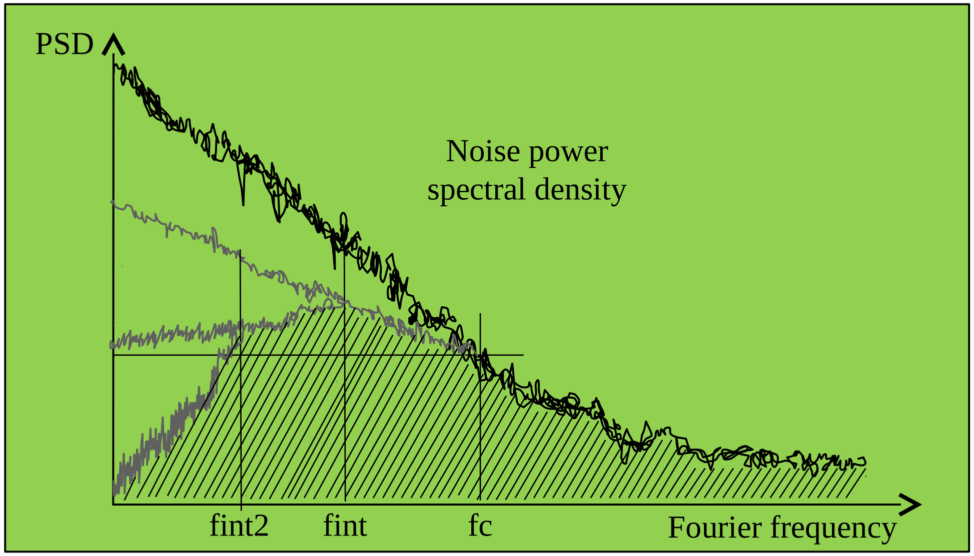


When loop is on:

- If $\sqrt{(\int PSD)} << 2\pi$ ➔ OK
(small excursions around 0)



Phase-locking : why you may need a divider

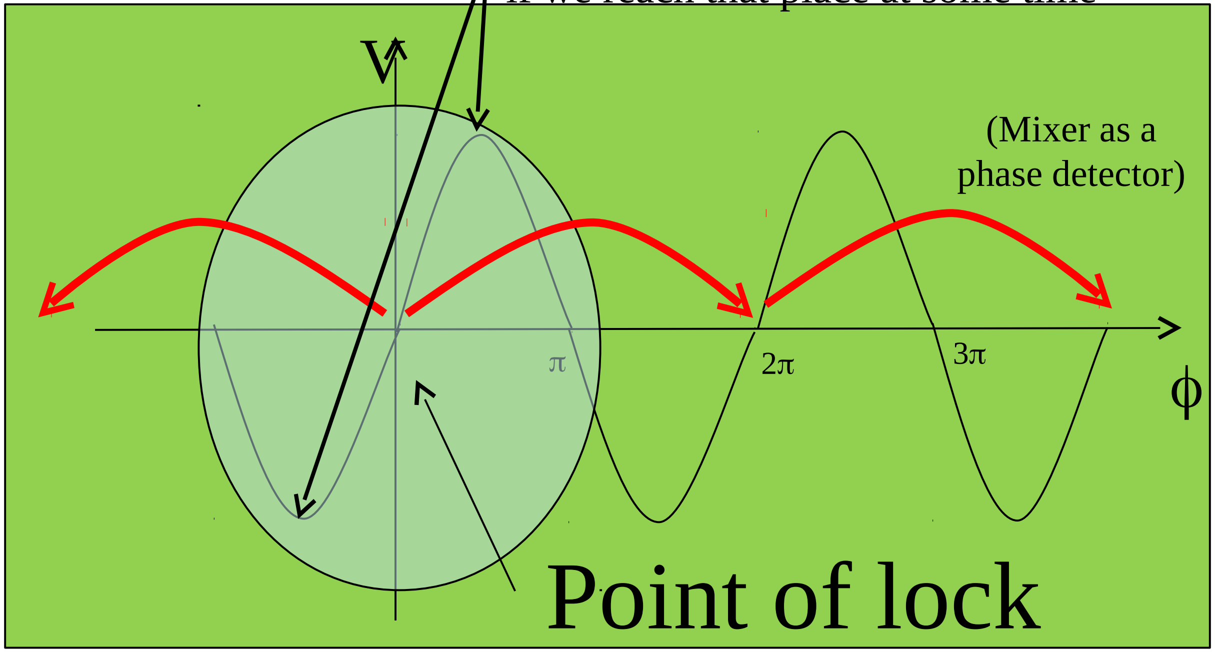


When loop is on:

- If √(∫PSD)<<2π ➔ OK

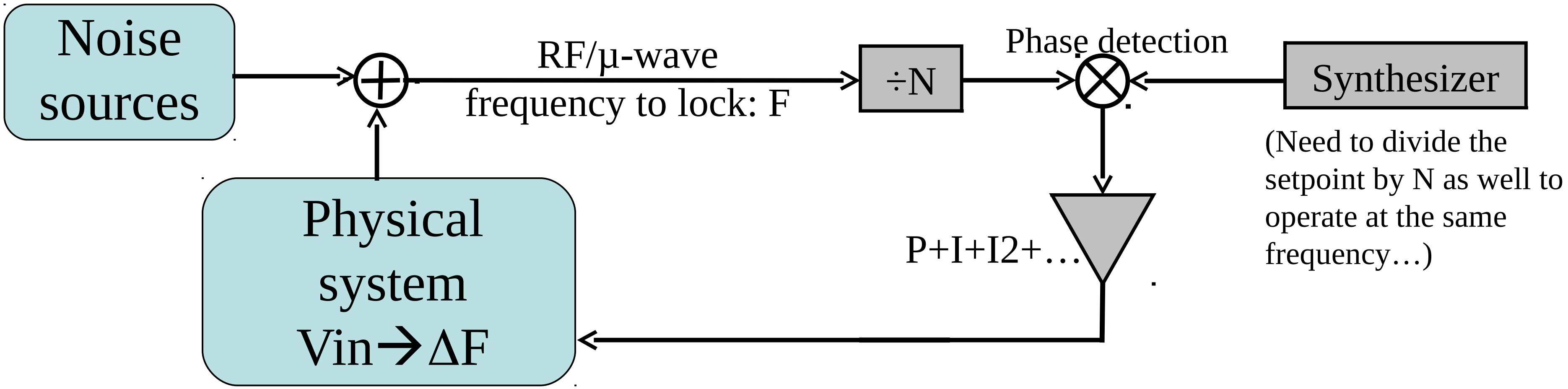
-If not ➔ cycle slips
(BAD LOCK)

Change of slope -> get out of lock (temporarily)
if we reach that place at some time

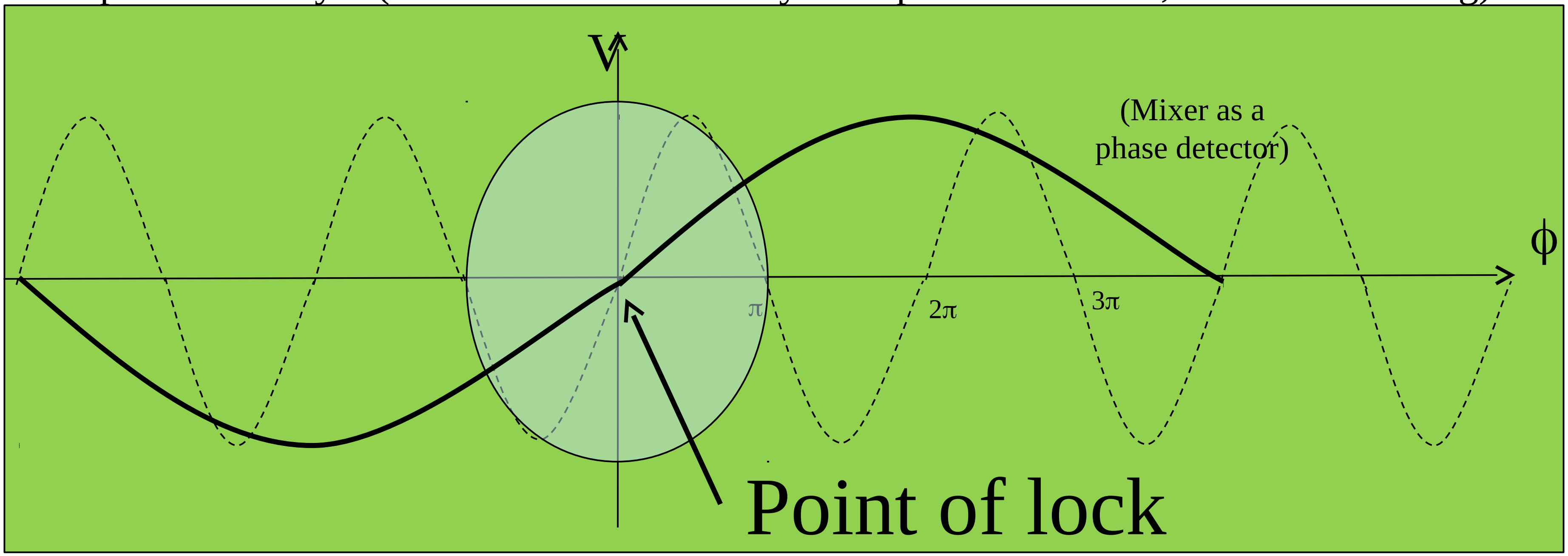


Note : mathematically, cycle-slips allways happen, but maybe only once in a few months (negligible impact) if you are good enough…

Phase-locking : adding a divider

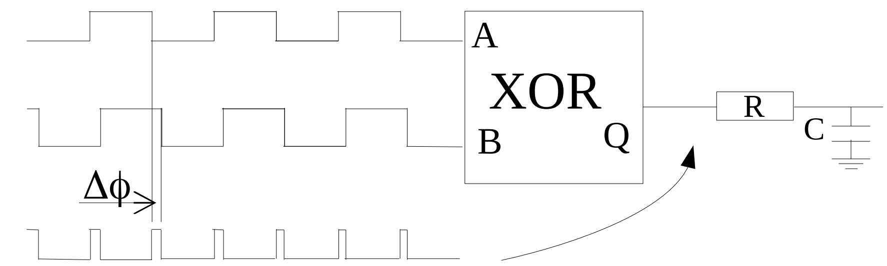


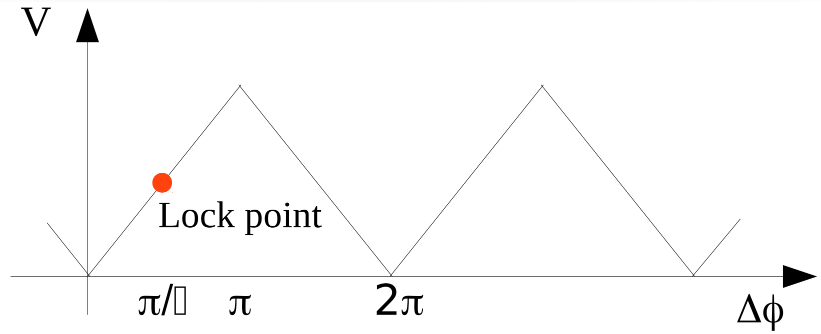
Exemple : divide by 4 (residual noise induces cycle slips if no division, not when dividing)



Phase-locking : digital systems

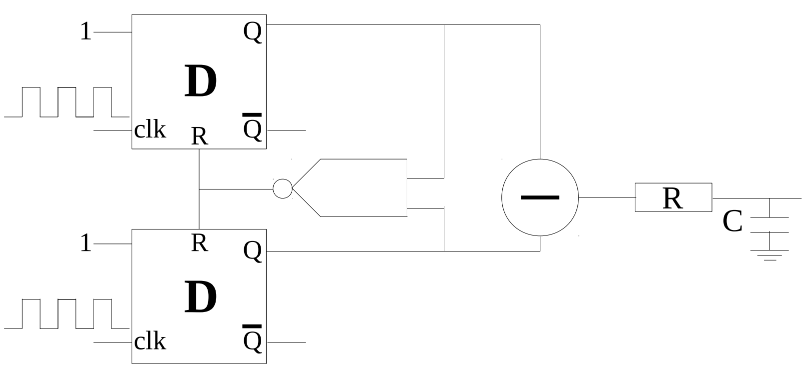
Digital Phase detector (simple)

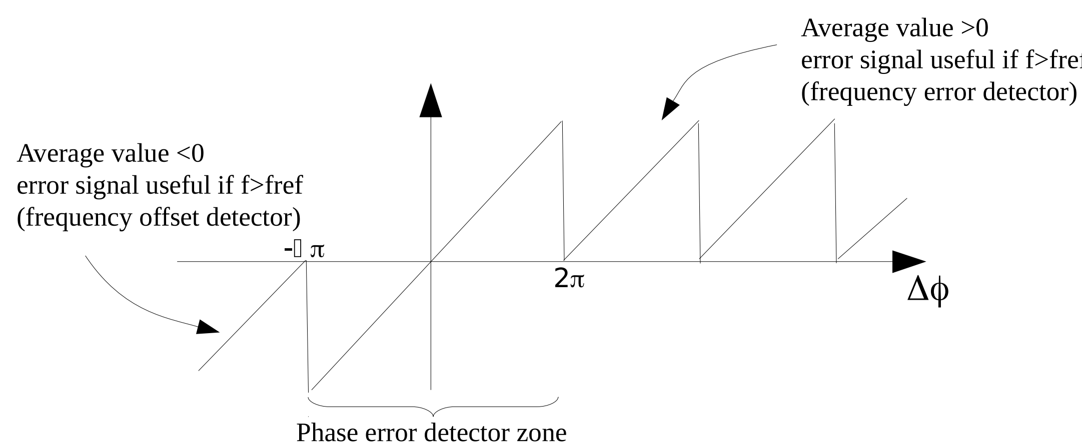




With appropriate offset, locks at quadrature ; same cycle slip issues than DBM.
Needs an analog to digital front-end (with good SNR) !

Digital Phase-Frequency detector

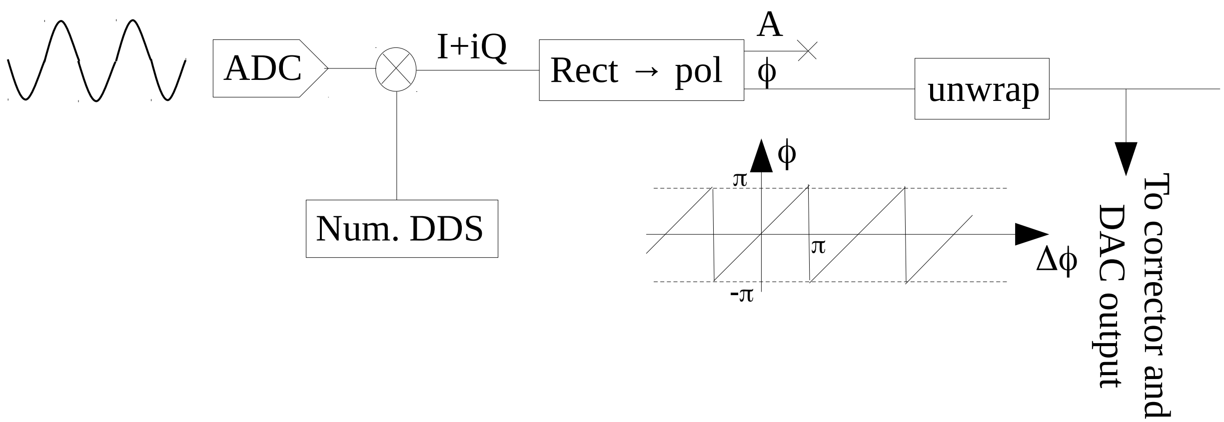


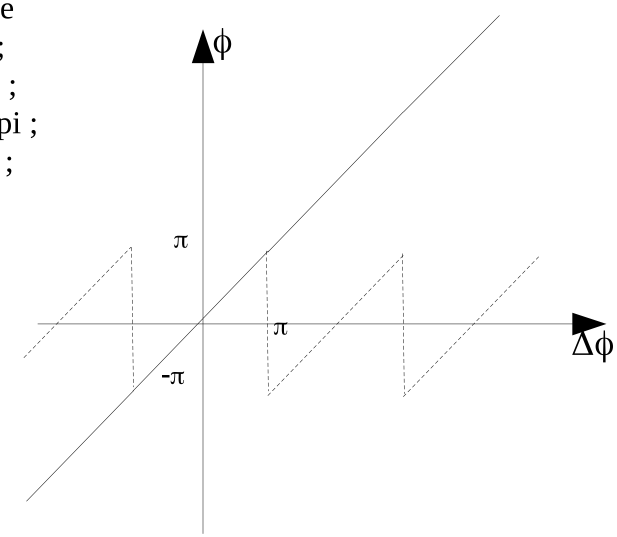


Same cycle slip issues than DBM, BUT natural auto-relock with large capture range.
Many different implementations exist, to address technical issues (blind zone at center, limited operation frequency,...)
Needs an analog to digital front-end (with good SNR) !

FPGA-based phase tracker with unwrap

```
// unwrap pseudo code
d = old_phase-phase ;
if (d>pi), acc += 2*pi ;
if (d<-pi), acc += -2*pi ;
phase += acc*accON ;
```





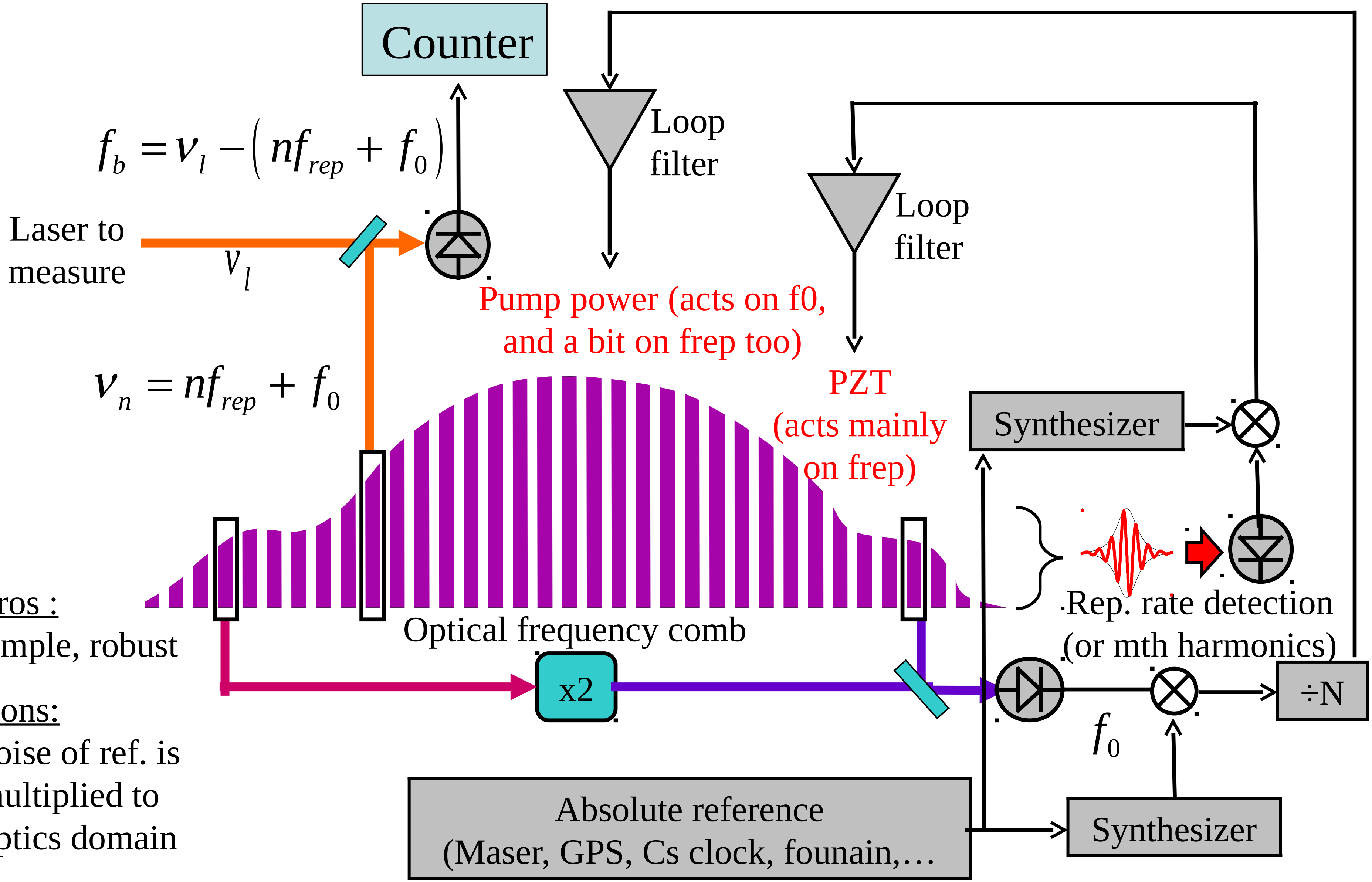
Continuous phase tracking (limited only in BW by sampling rate of unwrapping) with potentially very large dynamics (limited only by number of bits per sample at unwrap output)

Limited in lock BW by delay in FPGA (typ. >400kHz with Redpitaya,)

https://github.com/jddes/Frequency-comb-DPLL

Phase-locking : application to frequency comb

1) Frequency multiplier scheme :

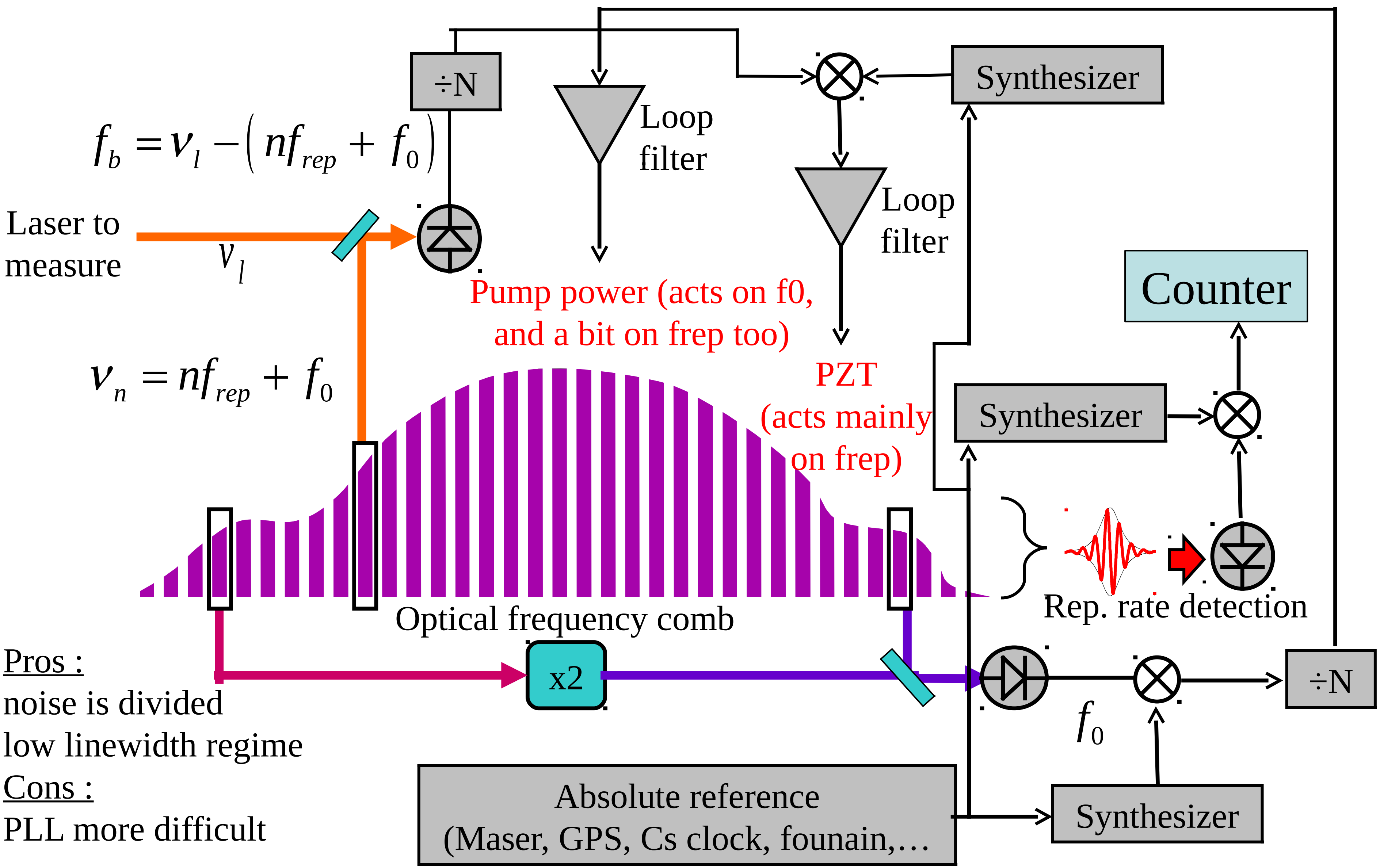


Pros :
simple, robust

Cons:
noise of ref. is multiplied to optics domain

Phase-locking : application to frequency comb

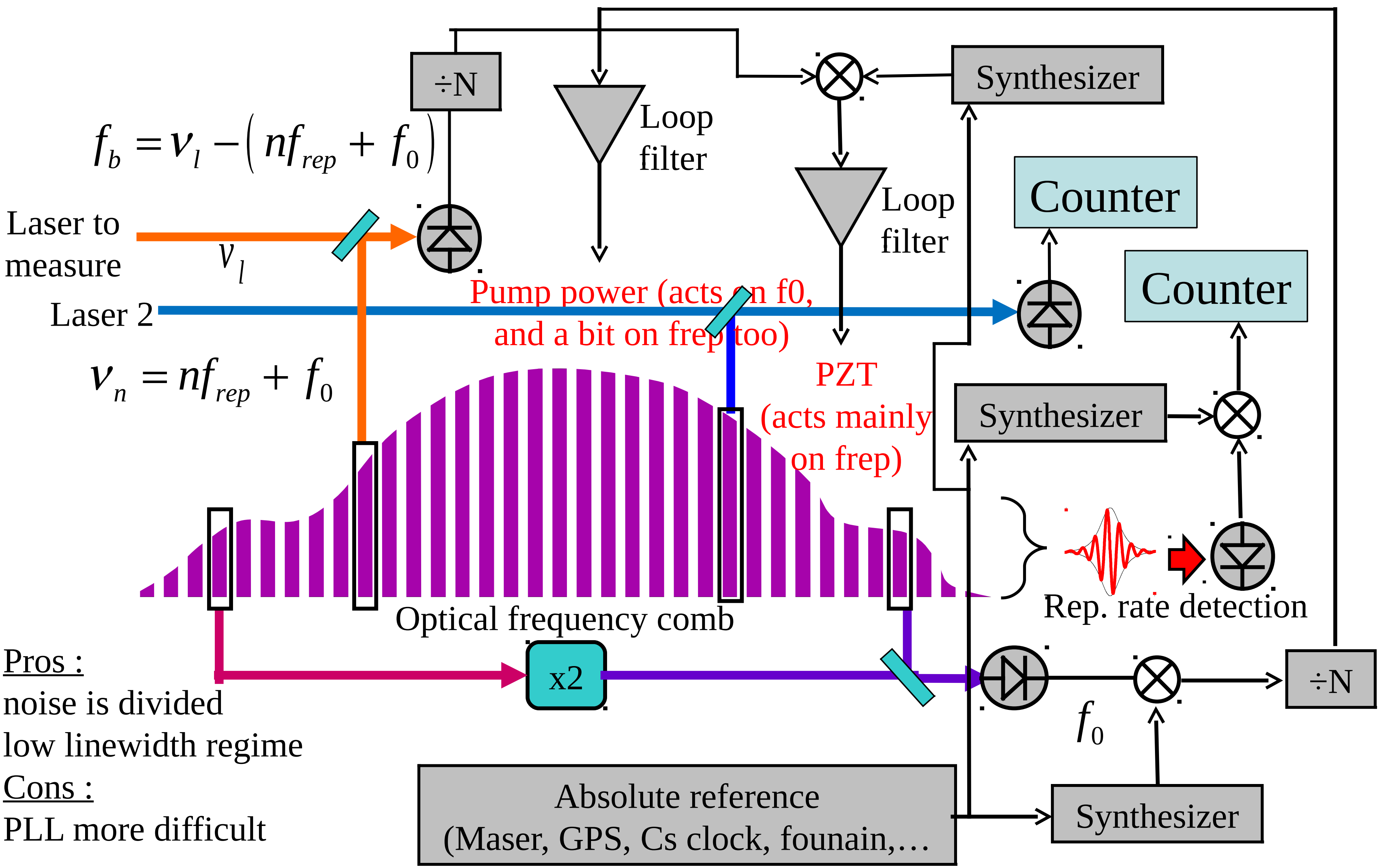
1) Frequency divider scheme :



<u>Pros :</u>
noise is divided
low linewidth regime
<u>Cons :</u>
PLL more difficult

Phase-locking : application to frequency comb

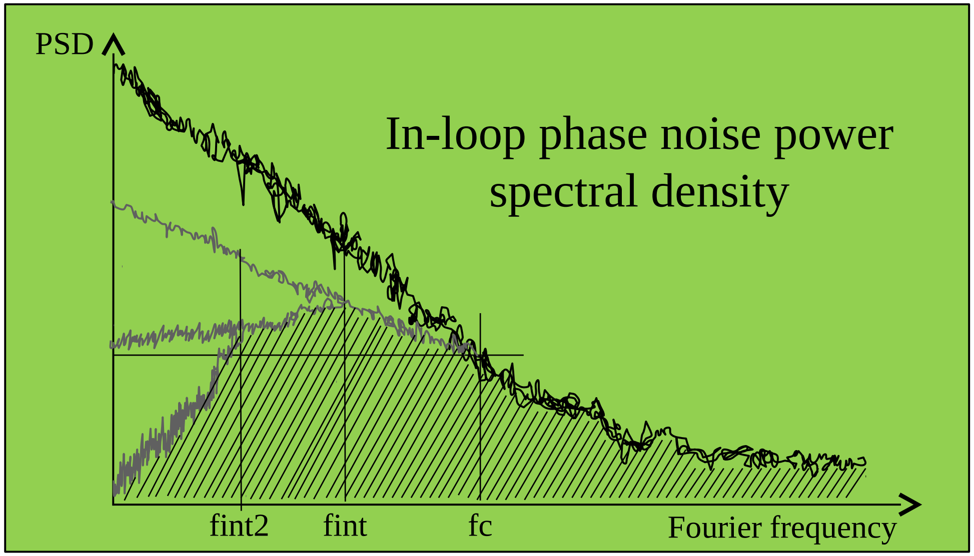
1) Frequency divider scheme :



Pros :
noise is divided
low linewidth regime
Cons :
PLL more difficult

Phase-locking : the low linewidth regime

In the frequency divider scheme :



If $\sqrt{(\int PSD)} << 2\pi$ (before the divider)

→ low-linewidth regime/ultra-stable regime

→ linewidth of every tooth of the comb identical to that of the optical reference

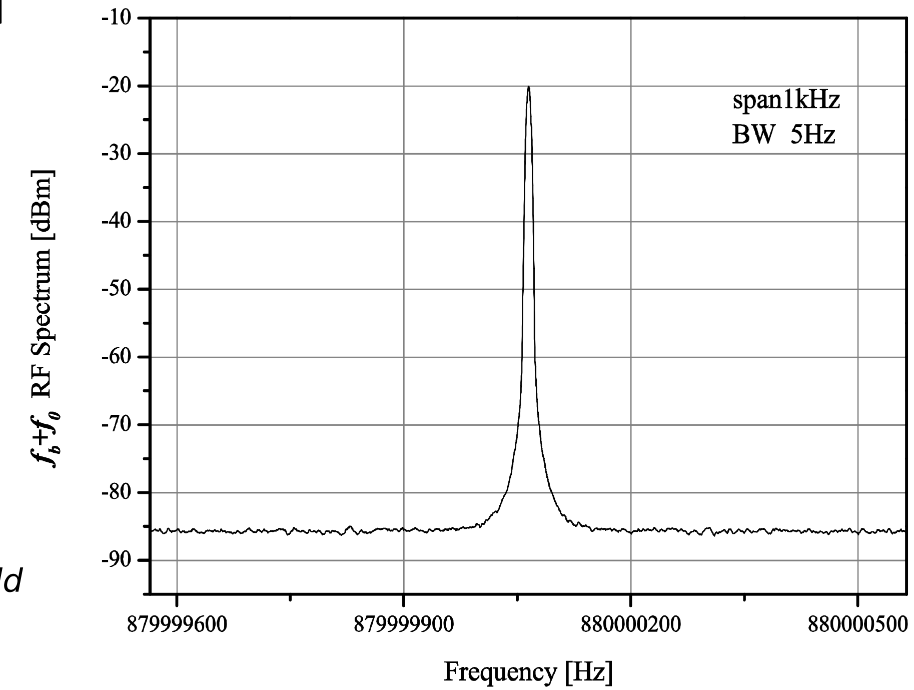
→ doesn't work in freq. mult. (noise of µw reference xN makes a large optical linewidth)

→ Better counting of 2nd optical reference

→ Narrow filter can save you if S/N is bad (you need >~23dB S/N **in the measurement BW** to count or track ; 10 dB in 100kHz is 30dB in 1kHz…)

→ Allow direct spectroscopy of narrow atomic transition

T.M. Fortier et al. "*Kiloherz-resolution spectroscopy of cold atoms with an optical frequency comb*", **Phys. Rev. Lett., 97**, 163905 (2006)

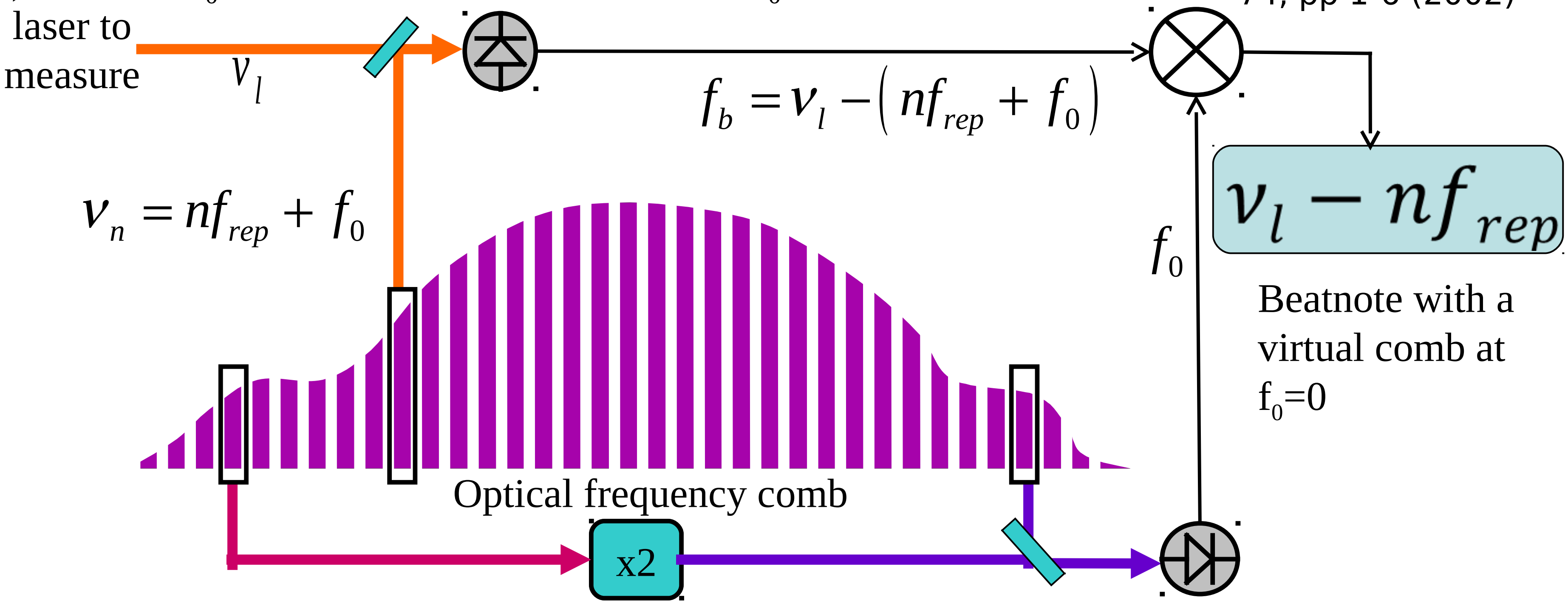


Removing PLLs : the transfer oscillator technique

A clever use of arithmetics allows comparisson of combs-related beatnotes where the free-running noise of the comb vanishes (can replace or be combined with PLLs)

H. Telle et al., Applied Physics B 74, pp 1-6 (2002)

1)Remove f_0 to make a virtual comb with f_0=0 :



Beatnote with a virtual comb at f_0=0

1bis) One can also use an AOM, driven by f_0 to shift the comb to f_0=0 (much more expensive, but comb at f_0=0 is real instead of being virtual, which can be usefull, for exemple in DFCS)

S. Koke et al., Nature Photonics 4, 462 (2010) (Steinmeyer) ; also at UWA (Luiten)

Removing PLLs : the transfer oscillator technique

A clever use of arithmetics allows comparisson of combs-related beatnotes where the free-running noise of the comb vanishes

H. Telle et al., Applied Physics B 74, pp 1-6 (2002)

2)Use arithmetics for removing f_{rep} :

exemple 1 : two optical frequencies to compare (frequency ratio of two clocks)

$$f_{b1} - f_0 = \nu_{l1} - n_1 f_{rep}(t) \qquad \times n_2$$
$$f_{b2} - f_0 = \nu_{l2} - n_2 f_{rep}(t) \qquad \times n_1$$
$$=$$
$$n_2.(f_{b1} - f_0) - n_1.(f_{b2} - f_0) = n_1.\nu_{l1} - n_2.\nu_{l2}$$

Comparisson between ν_{l1} and ν_{l2} is independant of the noise of the comb !

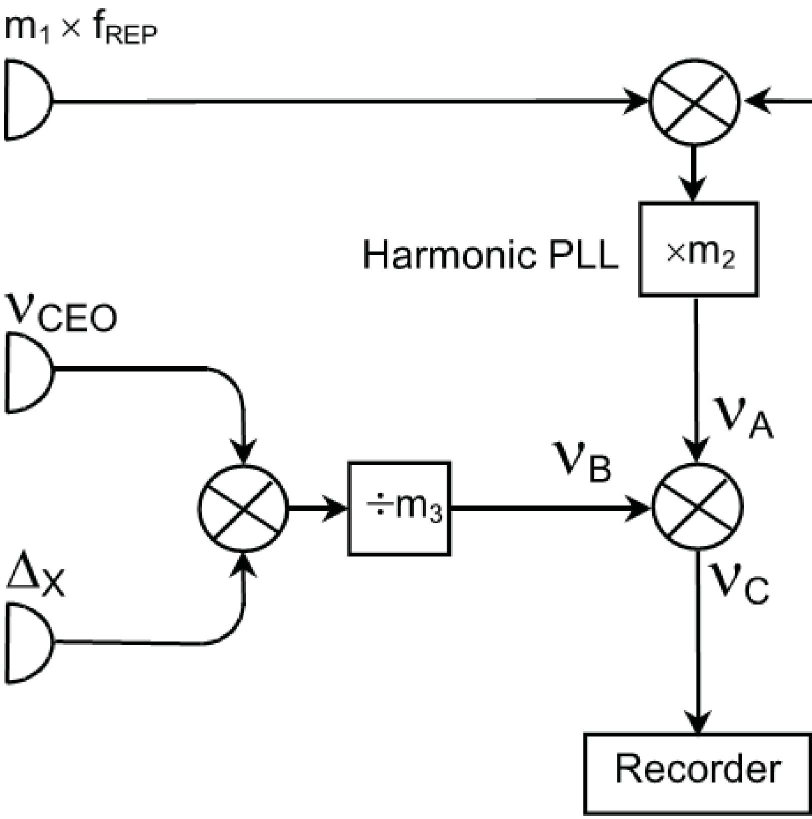
One can realize the arithmetic operation in hardware (with 1 or 2 DDS, which output frequency is really $f_{out} = f_{clock\text{-}in} \times (\text{ctrl word})/2^{\#digits}$)

or directly in software from counters' data, BUT one needs very well synchronised counter channels (otherwise, $f_{rep}(t)$ and $f_{rep}(t+\Delta t)$ don't cancel each other out…). Also be aware of #digits in calculation: double floats in C ($\sim10^{-15}$) are not enough unless you apply some tricks…

Removing PLLs : the transfer oscillator technique

Exemple 2 : compare optical and microwave frequencies

H. Telle et al., Applied Physics B 74, pp 1-6 (2002)



$$\begin{cases} \nu_C = \nu_A - \nu_B = m_2 f_{LO} - \left(m_1 m_2 f_{rep} + \dfrac{\nu_{ceo} + \Delta_x}{m_3} \right) \\ m_1 m_2 f_{rep} + \dfrac{\nu_{ceo} + \Delta_x}{m_3} = \dfrac{1}{m_3} \nu_x \end{cases}$$

$$\nu_C = m_2 f_{LO} - \frac{\nu_x}{m_3}$$

The difference in order of magnitude between optics and RF domain is much larger than in the optics-optics case, therefore more advanced technique needs to be used for realizing the arithmetics…

PLLs vs. transfer oscillator technique

	transfer oscillator	PLL
Pros	- No lock therefore no unlock - No lock therefore no lock-related cycle-slips - Virtual comb is extremely good even though real comb is noisy	- Real comb and not virtual - If narrow linewidth real combs, DFCS - Easy to add a new wavelength (1 simple beatnote)
Cons	- Usually still requires a (loose) lock to stay inside filters - Only a virtual comb → direct spectroscopy is not feasible at low linewidth - Adding a new wavelength is a bit more complicated (RF circuit, DDS,…)	- High bandwidth, low noise, high dynamics PLLs are not trivial - Sometimes, the comb may go unlock (automatic relock may help) - Perf. limited by how good you are at making PLLs

Note : you can also do both, or a bit of both (ex.: remove f_0 only)…

Nasty “details” about the first digits

Nasty “details” of absolute frequency measurements

The comb gives you an simple arythmetic realtion between various frequencies (all measured frequencies are >0 by definition)

BUT

Some parameters (signs, multiplying integers) are not known a priori and need to be determined (with a bit of intelligence)

Exemple :

A comb measuring an optical frequency ν_{cw} against absolute RF references by counting (or locking to a fixed frequency) f_{rep}, f_0, f_b :

$$f_b = |\nu_l - \nu_N| = \pm(\nu_{cw} - N.f_{rep} \pm f_0)$$

$$\Rightarrow \boxed{\nu_l = N \times f_{rep} \pm f_0 \pm f_b}$$

Have to be determined

How to determin signs

Assume : $\boxed{\nu_l = N \times f_{rep} \pm f_0 \pm f_b}$

With f_0 and f_b phase-locked to f_0^{ref} and f_b^{ref} by feed-back to the comb, while measuring frep with a counter (*i.e.* « frequency divider scheme »)

THEN

IF (f_0^{ref} ↗ makes f_{rep} ↘) THEN the formula is with $+f_0$

IF (f_0^{ref} ↗ makes f_{rep} ↗) THEN the formula is with $-f_0$

IF (f_b^{ref} ↗ makes f_{rep} ↘) THEN the formula is with $+f_b$

IF (f_b^{ref} ↗ makes f_{rep} ↗) THEN the formula is with $+f_0$

How to determine N

Assumes the signs are determined to be:

$$\boxed{\nu_l = N \times f_{rep} + f_0 + f_b}$$

- **<u>Method 1:</u>**

If ν_l is known *a priori* with an accurracy better than f_{rep}, f_0 and f_b (ex.: you have just measured it with a very good wavemeter, or it's coming from a well known atomic/molecular reference which someone has measured previously for you, like Sr or Ca+ or…)

➔ $\boxed{N = \text{round to nearest integer } [(\nu_l^{a\ priori} - f_b - f_0)/f_{rep}]}$

- **<u>Method 2:</u>**

Similar to sign determination: by shifting f_b^{ref} : $f_b^{ref} \rightarrow f_b^{ref} + \Delta f_b^{ref}$

one induces : $f_{rep} \rightarrow f_{rep} + \Delta f_{rep}$:

$$\boxed{\nu_l = \cancel{N x f_{rep}} + \cancel{f_0^{ref}} + \cancel{f_b^{ref}} = Nx(\cancel{f_{rep}} + \Delta f_{rep}) + \cancel{f_0^{ref}} + (\cancel{f_b^{ref}} + \Delta f_b^{ref})}$$

➔ $N = \text{round to nearest integer } [-\Delta f_b^{ref}/\Delta f_{rep}]$

☠ when noise of measurement is too high (and/or drift too fast inbetween measurements…) this is (generally) not sufficient

ex.: f_{rep}~250 MHz measured at ±100μHz (4×10^{-13}), ν_l~250THz (hence, N~10^6),

Δf_b^{ref}=1MHz induces Δf_{rep}~1Hz

→ $\Delta N/N \sim \sqrt{2}.100\mu Hz / \Delta f_{rep} \sim 140\times10^{-6}$

$\Delta N >> 1$ → it DOESN'T WORK !!! (here, we would need ~10^{-15} for $\Delta N<1$)

How to determine N

Assumes the signs are determined to be: $\boxed{\nu_l = N \times f_{rep} + f_0 + f_b}$

- **Method 3:**

Make 2 measurements for 2 consecutive N (easy by looking at the beat line moving)

$\nu_l = N \times f_{rep,1} + \cancel{f_0} + \cancel{f_b} = (N+1) \times f_{rep,2} + \cancel{f_0} + \cancel{f_b}$

$\rightarrow \boxed{N = f_{rep,2} / (f_{rep,1} - f_{rep,2})}$

Required measurement stability :

ex.: f_{rep}~250 MHz measured at ±100μHz (4×10^{-13}), ν_l~250THz (hence, $N \sim 10^6$),

$\rightarrow |f_{rep,1} - f_{rep,2}| \sim 250\text{Hz}$

$\rightarrow \Delta N/N \sim 100\mu\text{Hz}/250\text{MHz} + \sqrt{2}.\ 100\mu\text{Hz}/250\text{Hz} \sim 6\times10^{-7}$

$\Delta N < 1$ → it WORKS (without crazy requirements !!!)

(note for a few extra margin, one can increase the difference in N)

- **Method 4:** If you are really rich and have 2 combs, you can use method 2 or 3 with the 2 measurements done simultaneously with each comb...

A word about counting frequencies

- Π-counter *vs.* Λ-counters :
  - In Λ-counters, an averaging process smoothes the data to reduce the noise → the bandwidth of the measurement is extremely reduced !
  - With Π-counters, you should give the measurement BW of your data acquisition for meaningful comparissons.
  - Juxtaposing 1s-gate time data from a Λ-counter (like an HP53131 in gate time mode for exemple) DOESN'T give you an Allan variance !!!
  - It doesn't give you exactly a MVAR either !
- Dead-time *vs.* no-dead-time counters : beware of minimizing dead-times (compared to gate times) or, better, find a counter without dead-times (ex. K+K messtechnik)
- Be carefull when comparing numbers measured with one counter to another
- The only truth is in the power spectral density of (frequency or phase) noise; counting and AVAR-like calculation only gives easily access to an estimation (more or less fair) of the PSD at a given Fourier frequency

➔ See S. Dawkins et al. "Considerations on the Measurement of the Stability of Oscillators with Frequency Counters", IEEE UFFC 54, 918 (2007) for more details

Other Applications of Optical Frequency Combs

- Ultra low noise microwave generation with fiber femtosecond laser

The Stability of an Optical Clock Laser Transferred to the Interrogation Oscillator for a Cs Fountain
B. Lipphardt et al., IEEE Trans. Instrum. Meas. 57, 1119R (2008)

Ultralow noise microwave generation with fiber-based optical frequency comb and application to atomic fountain clock, J. Millo et al., Appl. Phys. Lett. 94, 141105 (2009)

T.M. Fortier et al., Nat. Phot. 5, 425 (2011)
X.Xie, R. Bouchand et al., Nat. Phot. 11, 44 (2017) → Lowest noise microwave source

- Transfer of spectral puirity from near-IR to other wavelengths

D. Nicolodi et al., Nat. Phot. 8, 219 (2014) (NIR) *B. Argence et al., Nat. Phot. 9, 456 (2015) (MIR)*

- Astrocombs: Use of OFC as reference for high resolution spectrometers used in astronomical telescopes

Nature 452, 610 (2008) *Eur. Phys. J. D 48, 57–66 (2008)* *Science 321, 1335 (2008.)*

- Direct Frequency Comb Spectroscopy, CEAS, dual combs spectroscopy

Direct frequency comb spectroscopy ,
Matthew C. Stowe et al., Adv. Atom. Mol. Opt. Phys. 55, 1-60 (2008)

- Molecular fingerprinting

Molecular fingerprinting with the resolved modes of a femtosecond laser frequency comb,
S.A. Diddams et al., Nature 445, 627 (2007)

- Extension to the DUV/XUV range

A deep-UV optical frequency comb at 205 nm
Peters, E. et al., Optics Express 17, 9183 (2009)

Efficient output coupling of intracavity high-harmonic generation
Yost et al., Opt. Lett. 33, 1099 (2008)

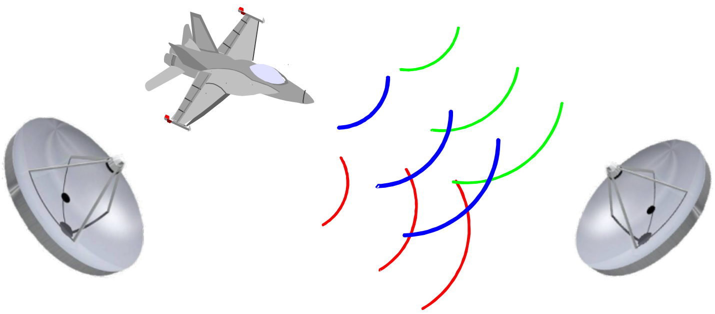
A few extra goodies

Transfer of optical spectral purity to the microwave domain (Low phase noise microwave signal generation)

Low-noise μwave : motivation

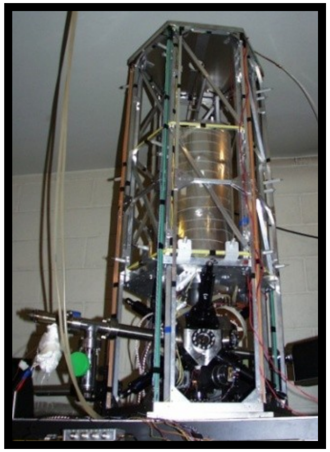
<u>Existing very low f-noise μ-wave sources (~10GHz):</u>

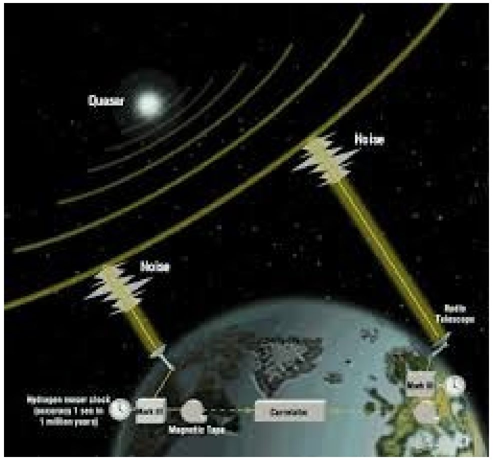
- Room temp Sapphire osc. (Raytheon, formerly Poseidon Australia):
  -40dBc/Hz @ 1Hz, -170dBc/Hz @ 100kHz from carrier
- Cryogenic Sapphire oscillator (UWA, FEMTO-ST, ULISS):
  -100dBc/Hz @1Hz, -140dBc/Hz @ 100kHz from carrier
- Opto Electronic Oscillator (JPL/OEwaves):
  -40dBc/Hz@1Hz, -160dBc/Hz @ 10kHz (large resonances after that)



<u>Applications:</u>

- atomic frequency standards
- radar
- VLBI
- synchronization of particle accelerators
- time reference distribution
- telecommunication
- …





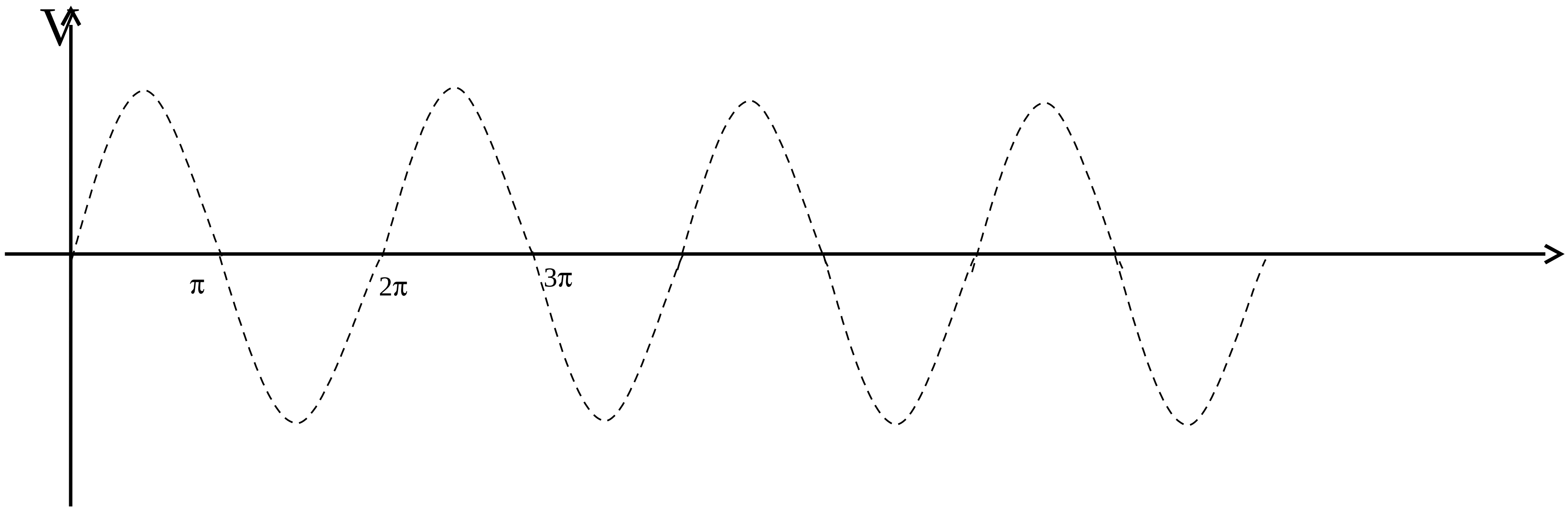
Frequency division, effect on phase noise

fc [Hz] → fc/N [Hz] then $\Delta\phi \rightarrow \Delta\phi/N$ [rad]

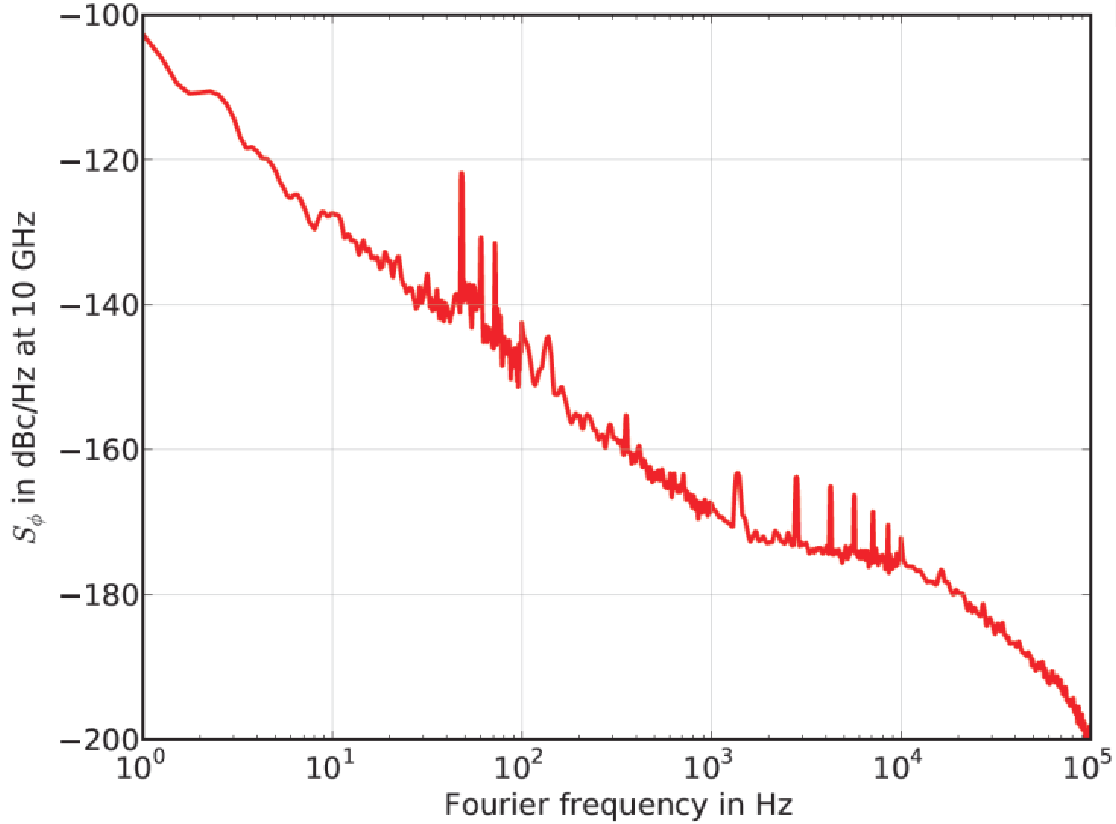
$S_\phi(f)$ [dBc/Hz] → $S_\phi(f) - 20.\log10(N)$ [dBc/Hz]

Large noise reduction
if N is large…

Exemple : divide by 2

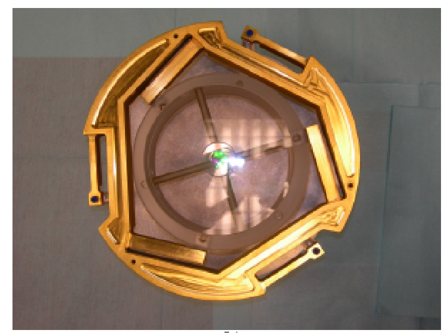


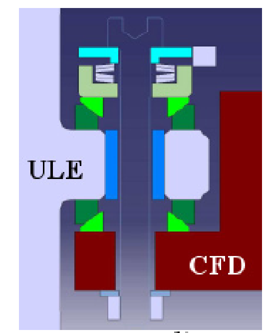
USL transferred to μ-wave (projection)



A robust 4.5x10^{-16} (@1s) level USL cavity (designed following space industry standards and methods)
→ 10cm long cavity with rings

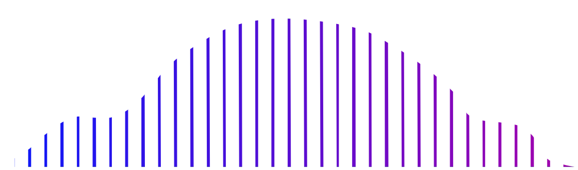
Prototype designed for transport +/-10g and operation at zero-2g
Currently existing lab prototype





Φ-noise of a 10 GHz carrier obtained by frequency division of the space-prototype USL at 200THz (SODERN/CNES/SYRTE), by a frequency comb, assuming perfect division

200 THz (λ=1.5μm)



Opt. Freq. comb

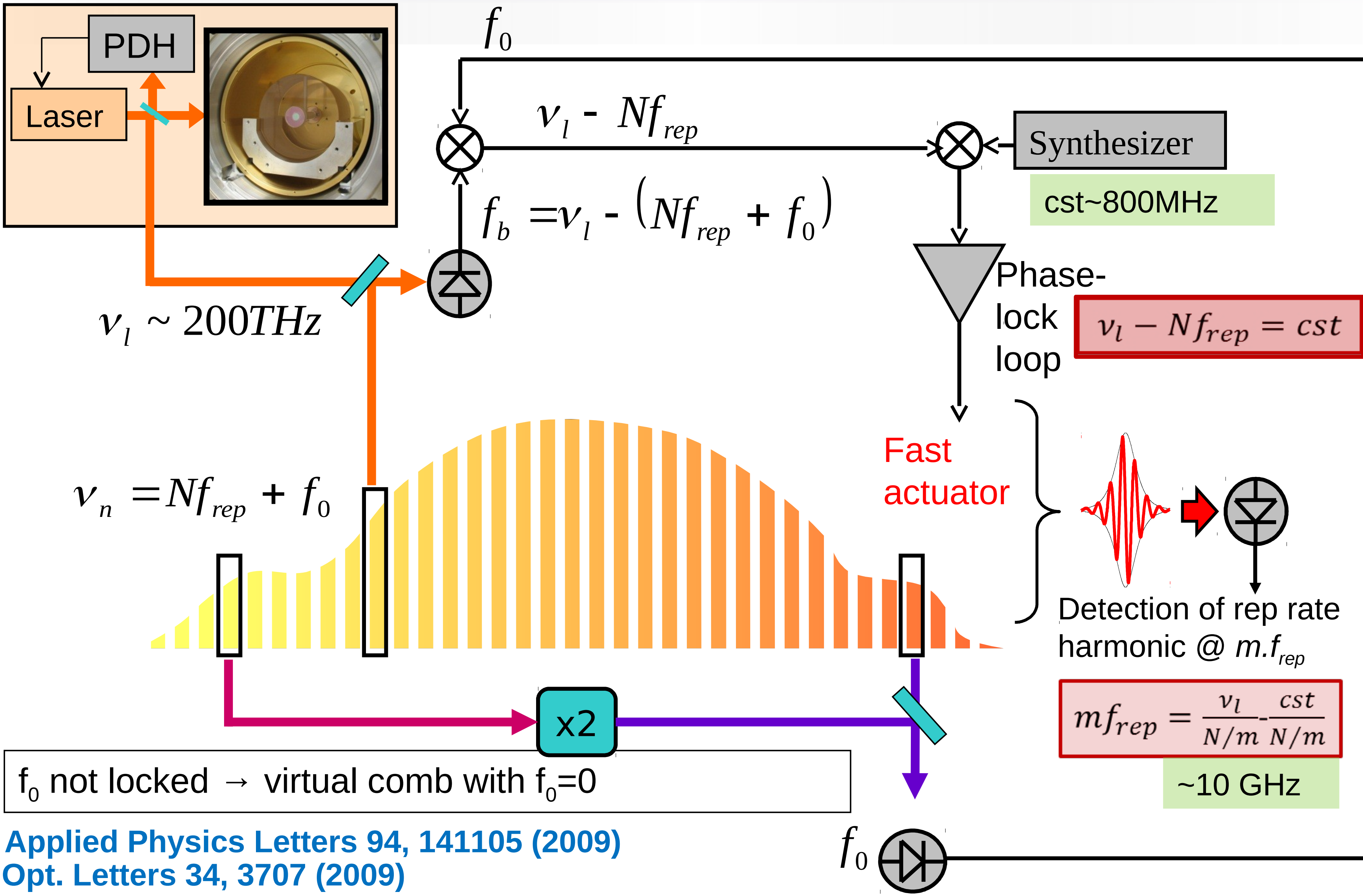
Φ-noise
→ -20.log(20000)
= -86dB (!!!)

10 GHz

Opt. Express 20, 25409 (2012)

cnes

Low noise μ-wave generation with comb (optical frequency divider scheme)



f_0 not locked → virtual comb with f_0=0

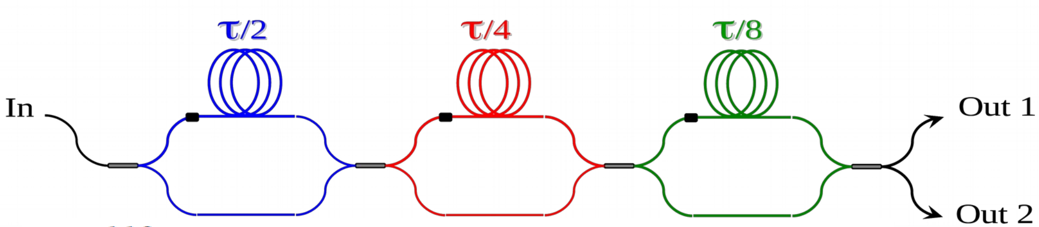
Applied Physics Letters 94, 141105 (2009)
Opt. Letters 34, 3707 (2009)

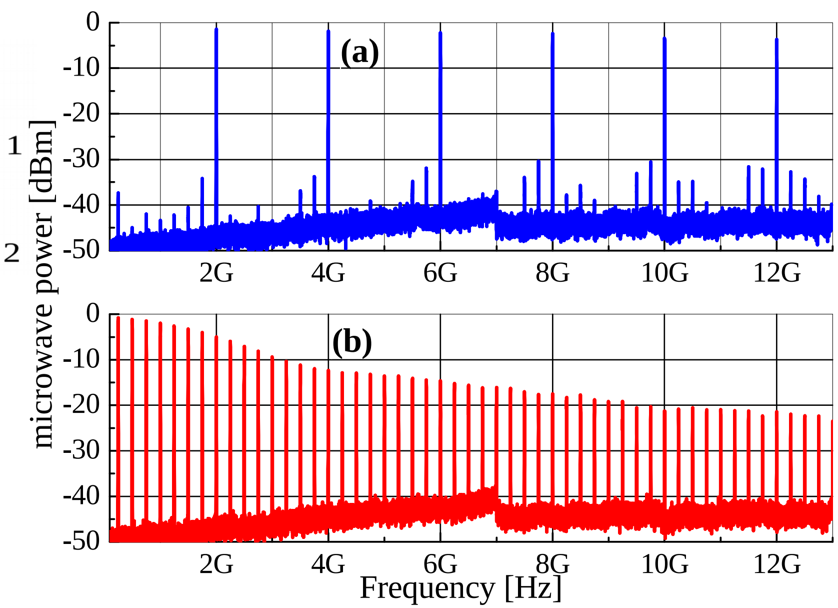
Increase SNR for lower white phase noise floor

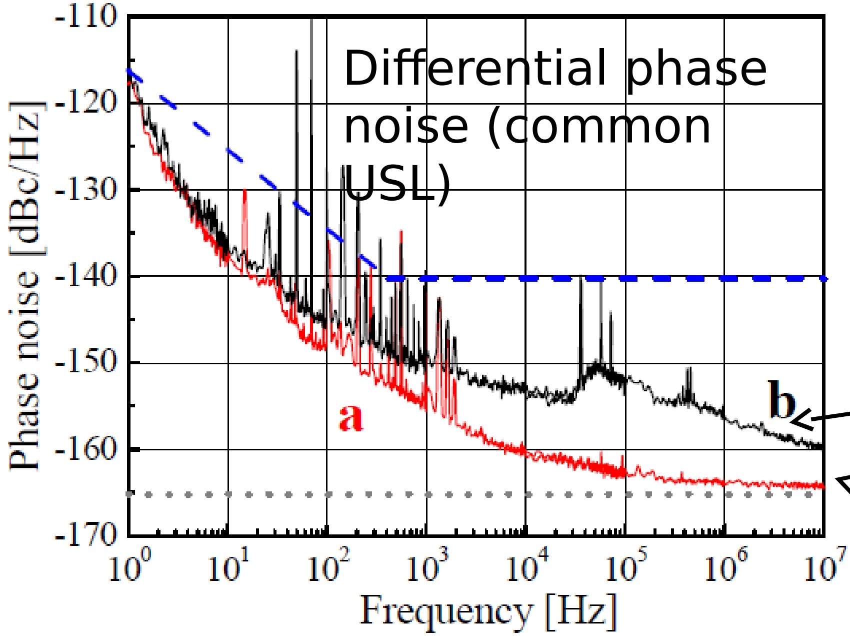
Thermal noise (Johnson-Nyquist) :

A 0 dBm µ-wave signal cannot have a white phase noise limit better than -177dBc/Hz
Solution : increase µ-wave power

- higher optical power+more linear PD
  (in coll. with Discovery semiconductor)
- high rep rate fs laser / external rep rate multiplication
  → less power in undesired harmonics,
  more in the harmonic of interest







2 combs 2 MZM 2 PD
(excess phase noise to investigate)

1 combs 2 MZM 2 PD

Optics Letters 36, 3654 (2011)

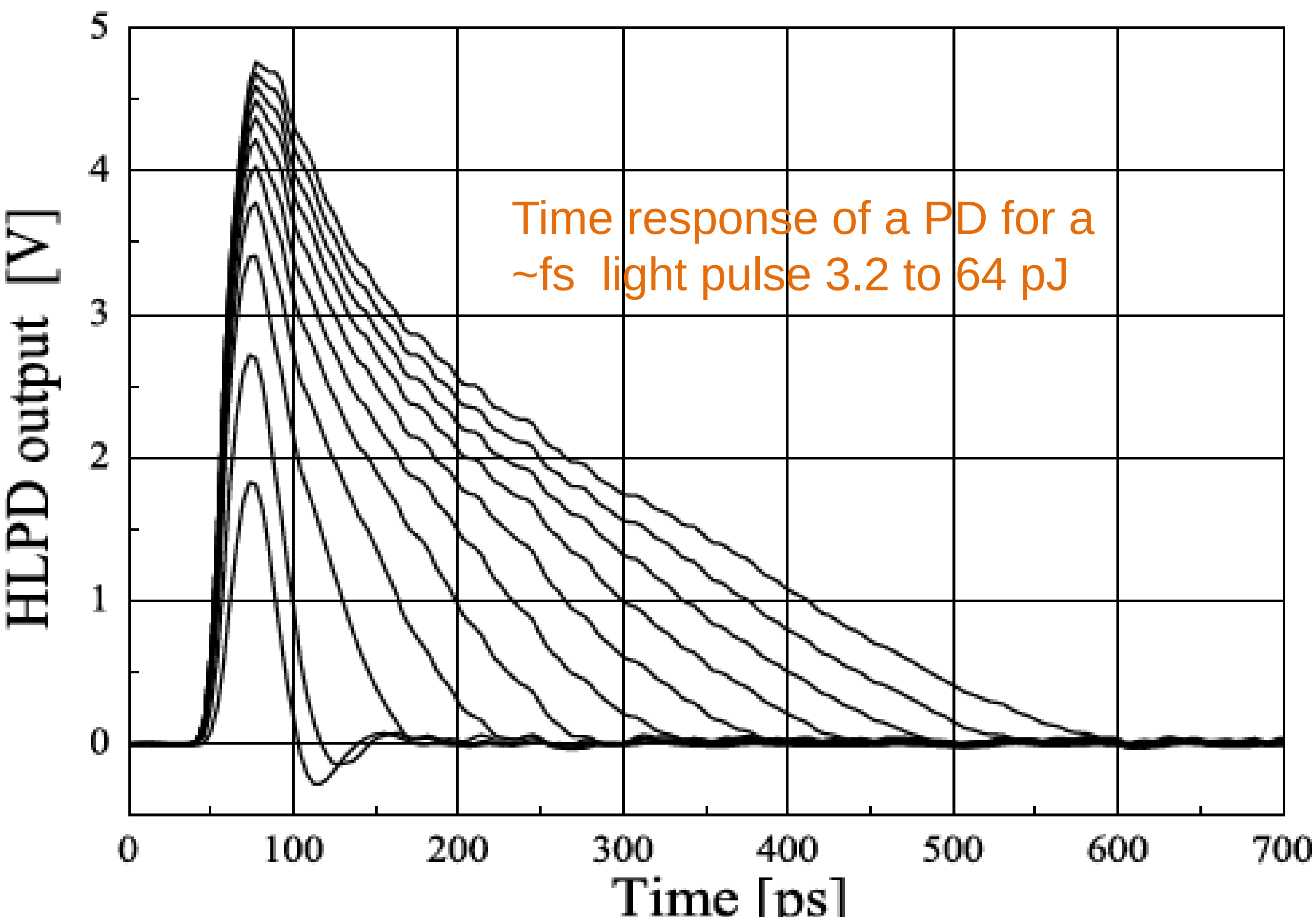
AMPM conv. in f_{rep} and harmonics photodetection

amplitude fluctuations of the fs laser induce fluctuations of phase of f_{rep} (and its harmonics)

→ possible to lock amplitude (but only at low Fourier frequencies)
1.2×10^{-16}@1s generated µwave / 100as synchro

App. Phys. Lett. 96, 211105 (2010)

→ or analyze carefully the physics…

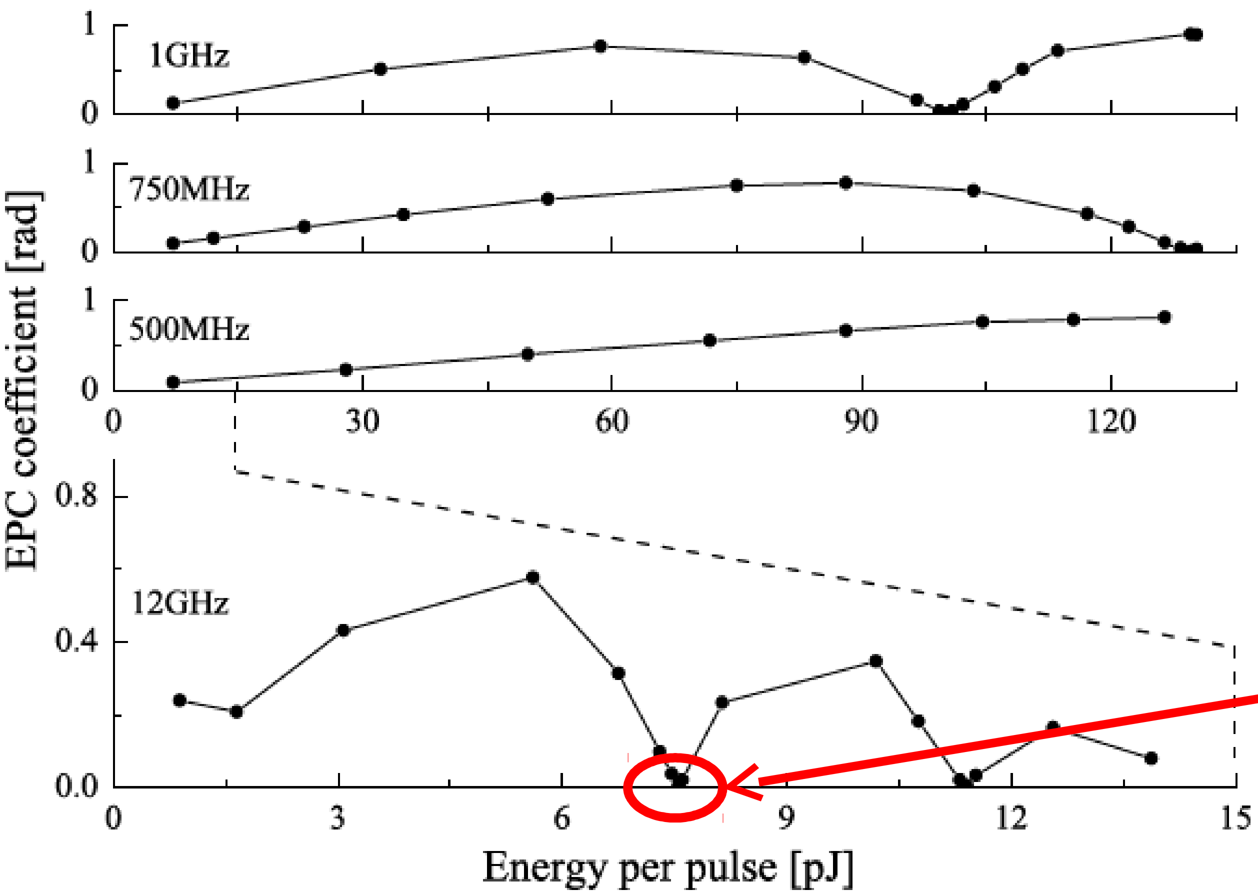


By space-charge screening effect, close to saturation, the PD response is asymmetric

→ AM noise produces PM noise

For harmonic order >1 there are special situations…

Suppression of AMPM conversion



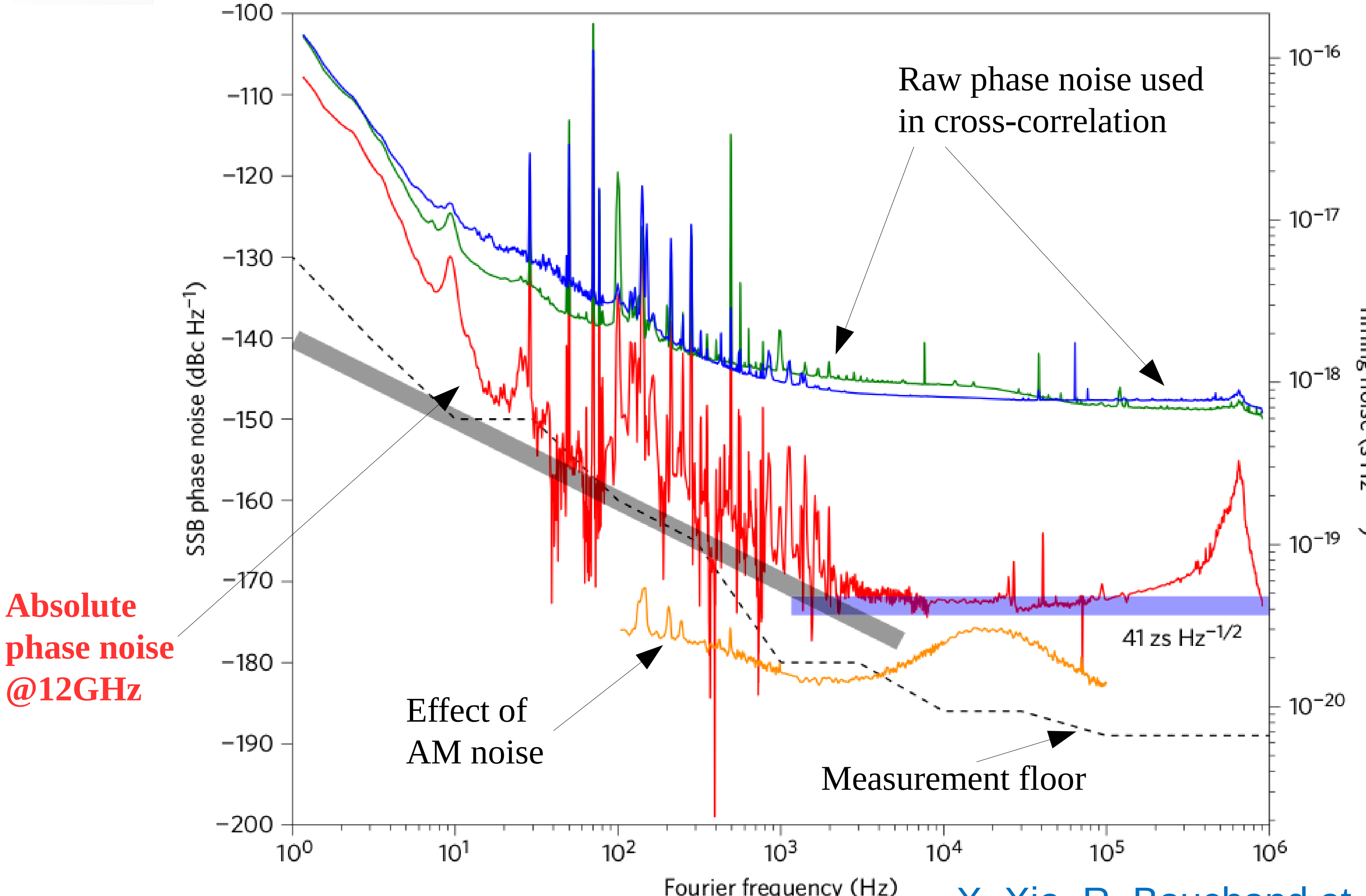
Note : EPC coeff. is really alternatively >0 and <0, only |EPC| is plotted here…

In this area
→ low EPC

Reasonable to passively keep the laser power at ~0.1%
→ From slope of EPC vs. Energy per pulse, EPC < 0.003 rad
→ AMPM-induced excess phase noise = RIN – 50dB !!!

Applied Physics B 106, 301 (2012)
Optics Letters 39, 1204 (2014)

Latest results for low phase-noise µwave



X. Xie, R. Bouchand et al.
Nature Photonics 11, 44 (2017)

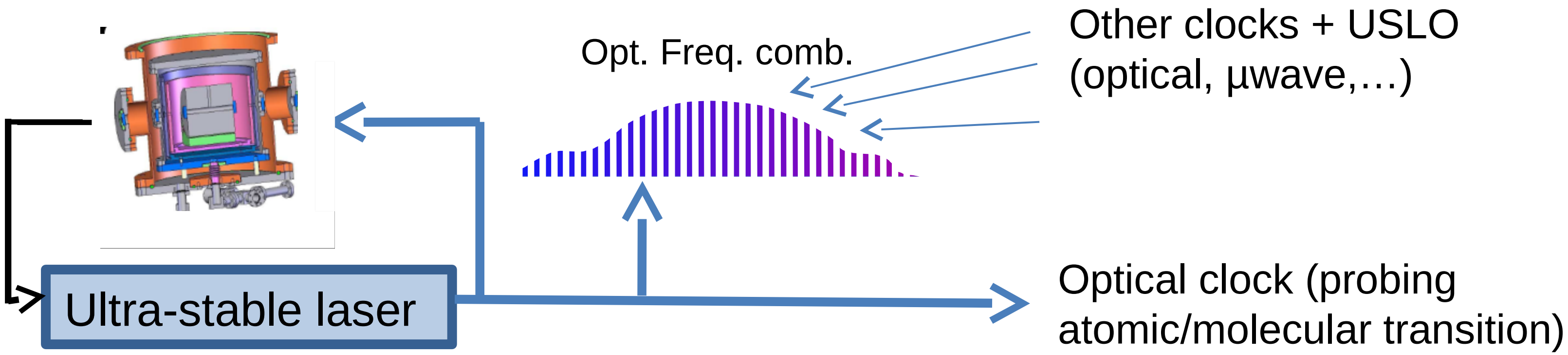
→ This is the lowest phase noise microwave source of all existing technologies and for ~all Fourier frequencies !

Transfer of spectral purity in the optics domain

Motivation

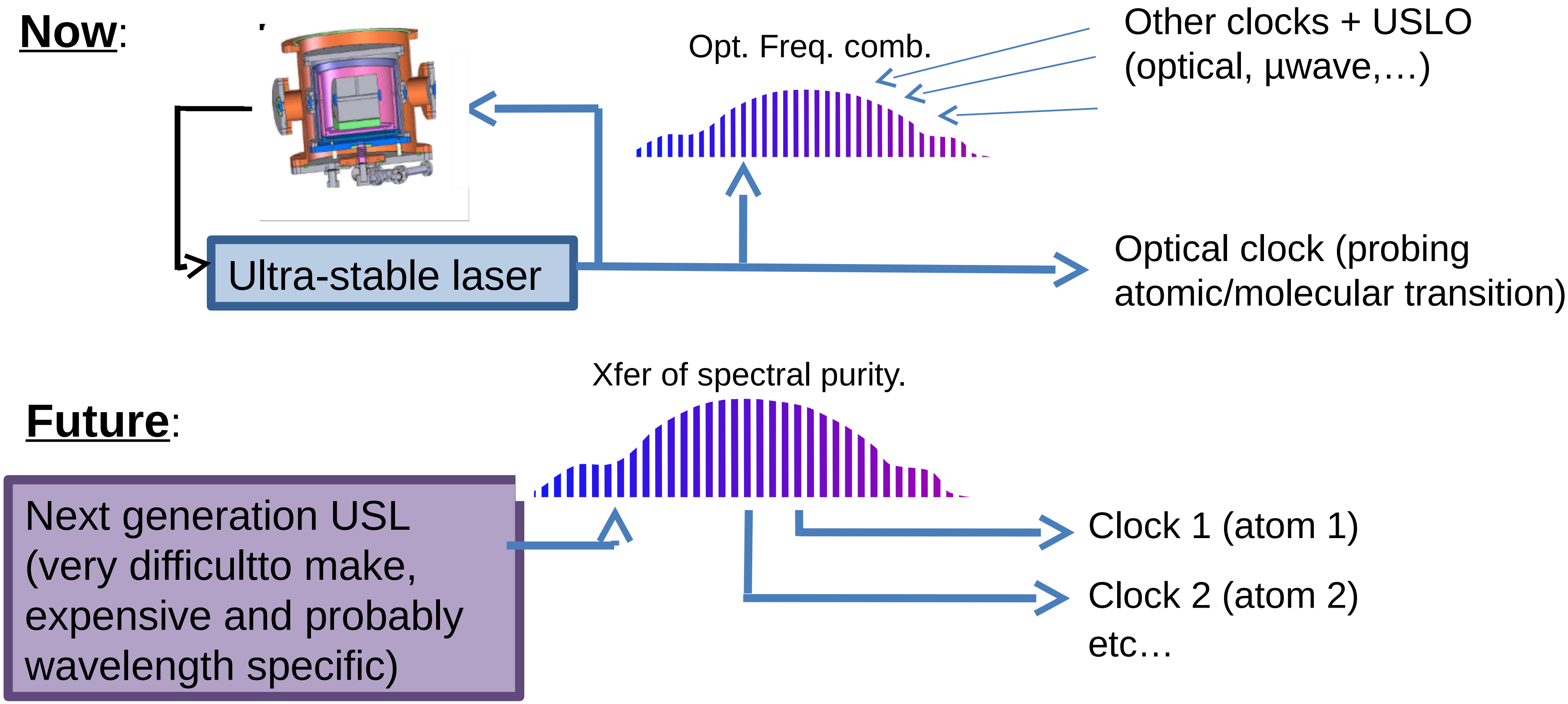
Toward a new paradigm in the optical clock community...

Now:

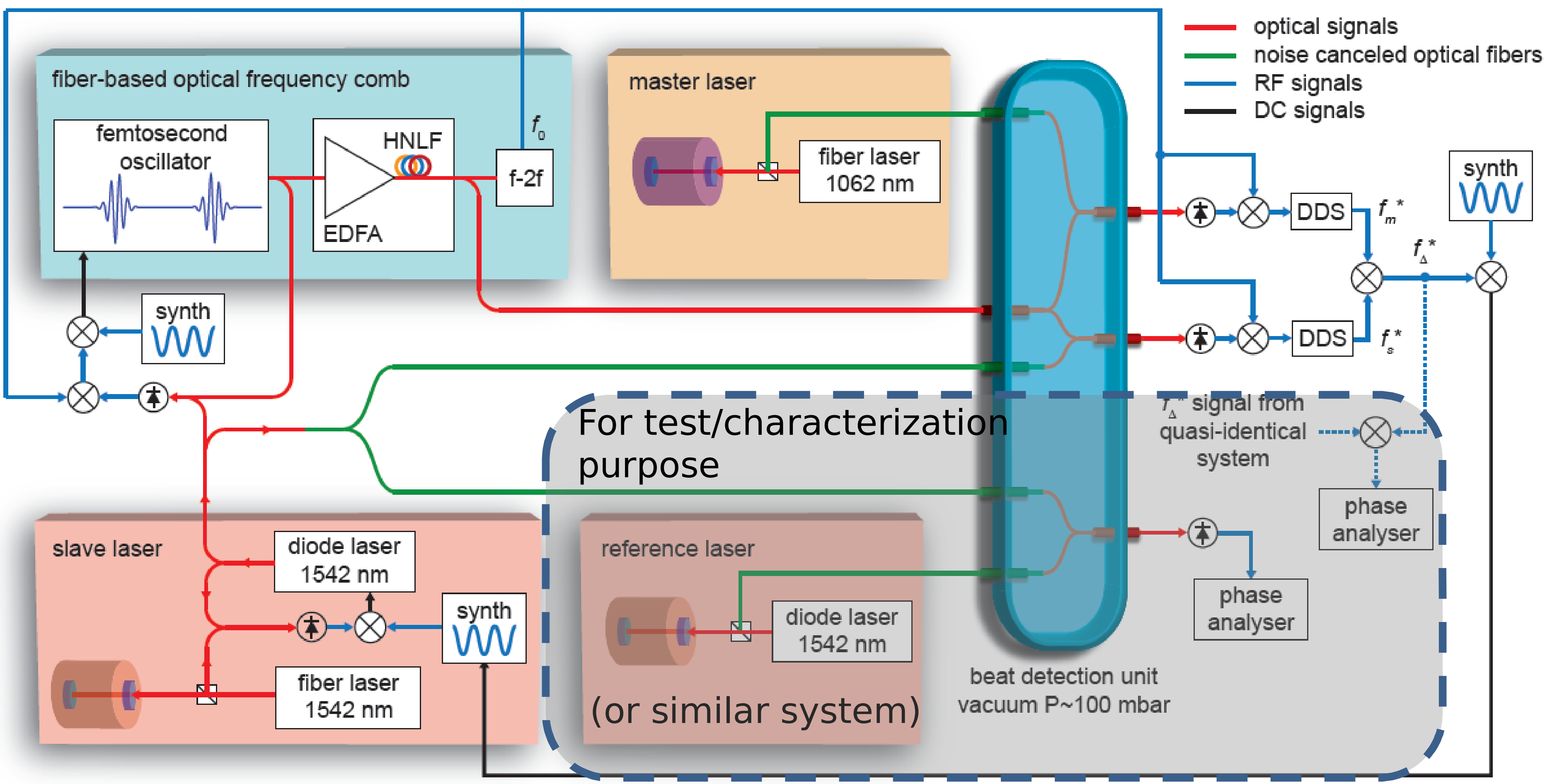


Motivation

Toward a new paradigm in the optical clock community...

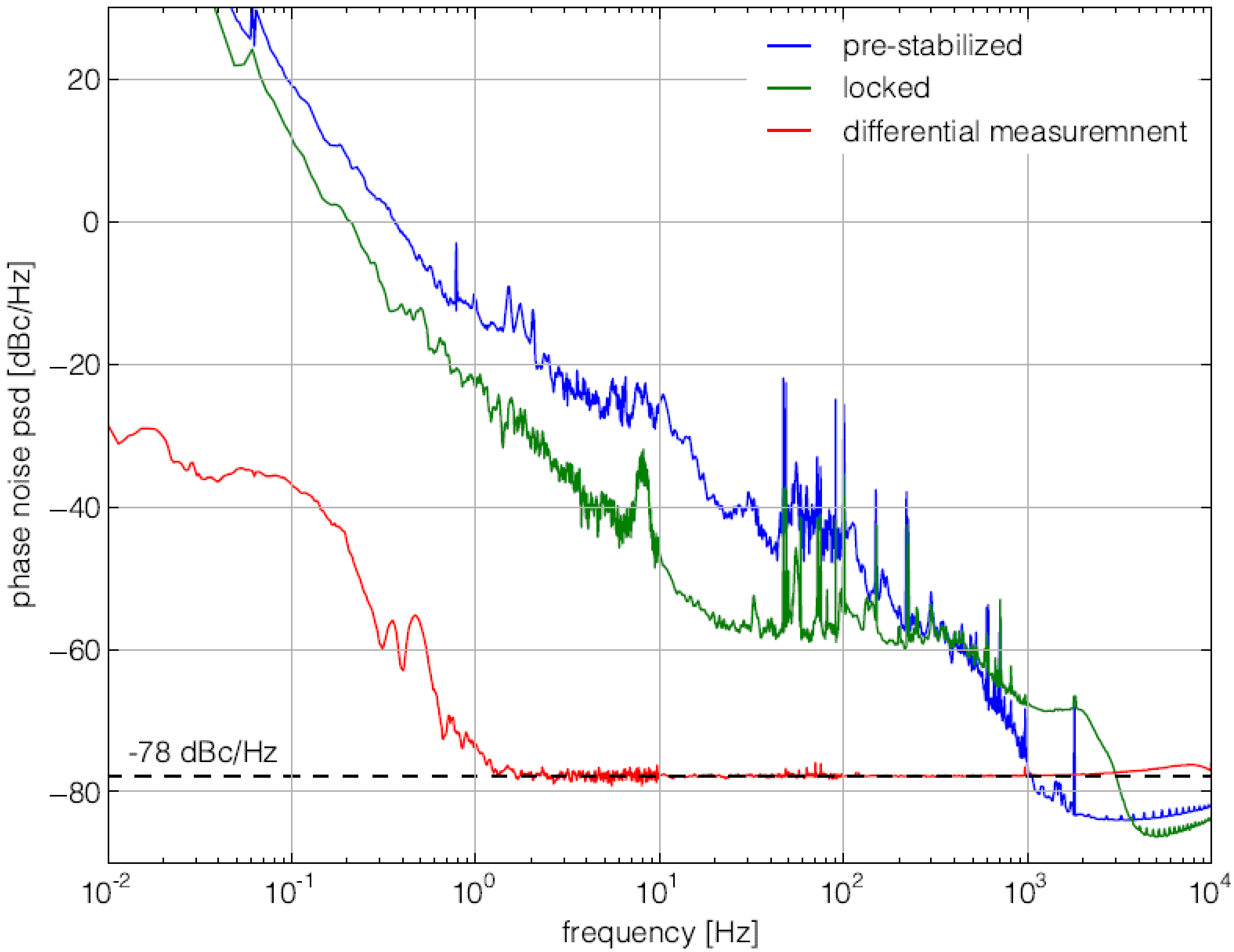


Transfer of spectral purity at the 10^{-18} (@1s) level

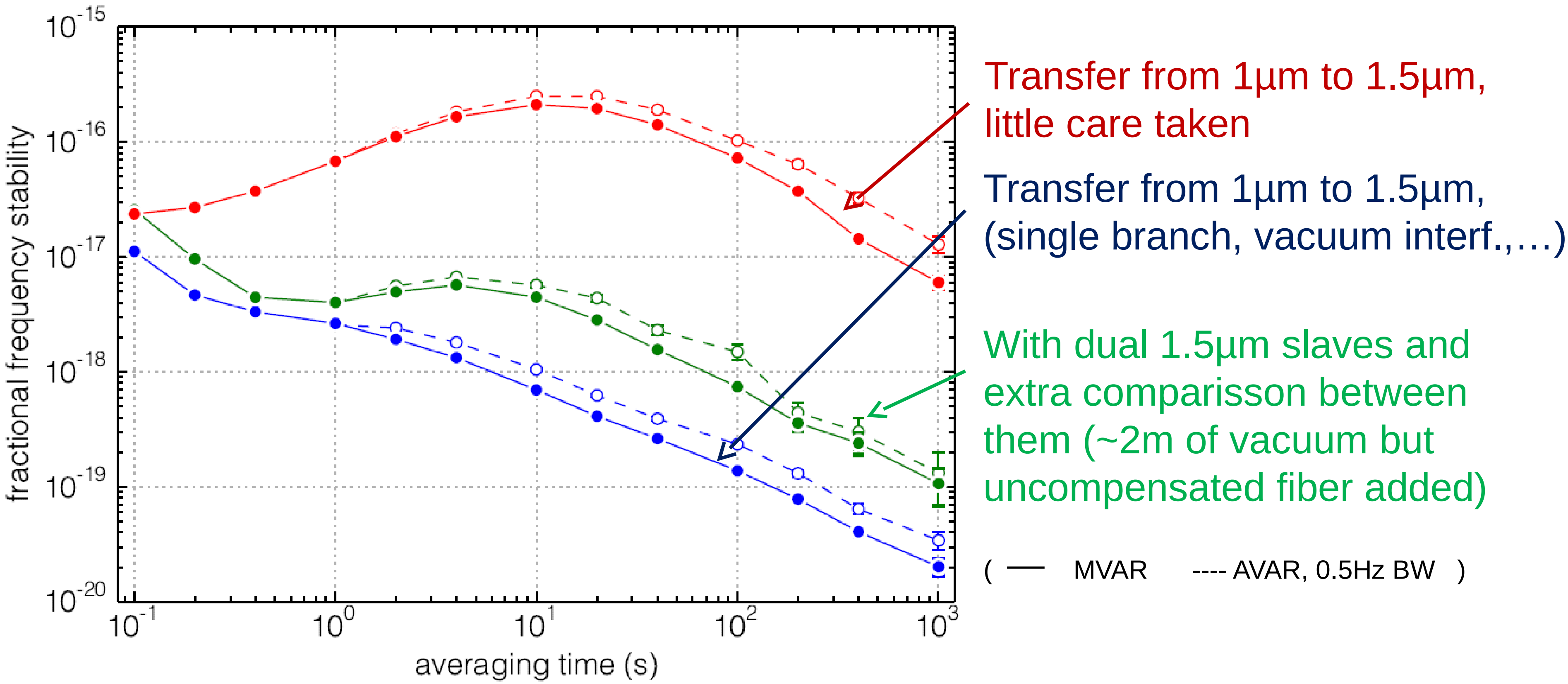


- Don't rely entirely on the lock of the comb to 1.5µm to guess the phase of the combs' teeth wrt to 1.5µm laser (especially those seen after EDFA+HNLF)
- All spectral phase fluctuations of EDFA+HNLF+comb scaling with ν_{optic} up to linear order are common-mode if both 1µm and 1.5µm are measured in the beat detection unit (BUT, low SNR…)

Test of spectral purity transfer



Transfer of spectral purity at the 10^{-18} (@1s) level

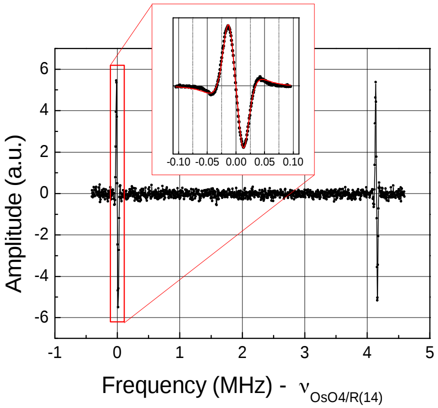


Limitation at a few 10^{-18} at ~1s
(>1 order of magnitude lower than current best demonstrated USL)

D. Nicolodi et al. Nature Photonics 8, 219 (2014)

Transfer of spectral purity in the MIR domain

THz motivation

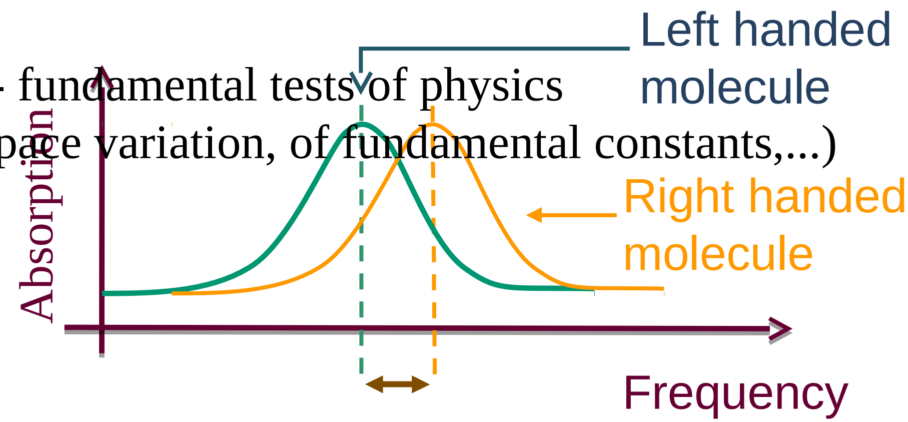


Spectroscopy of molecules at high resolution:

- more complex, richer structure than atoms

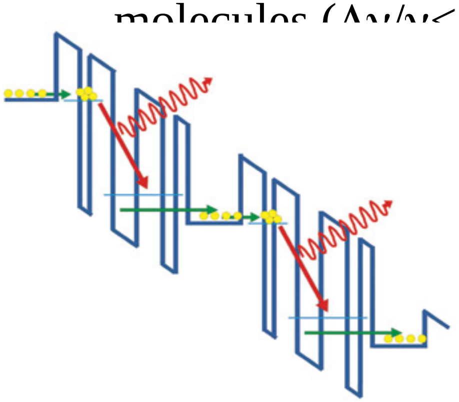
- can be more sensitive to certain effects

- fundamental tests of physics (EDM, time-space variation, of fundamental constants,...)



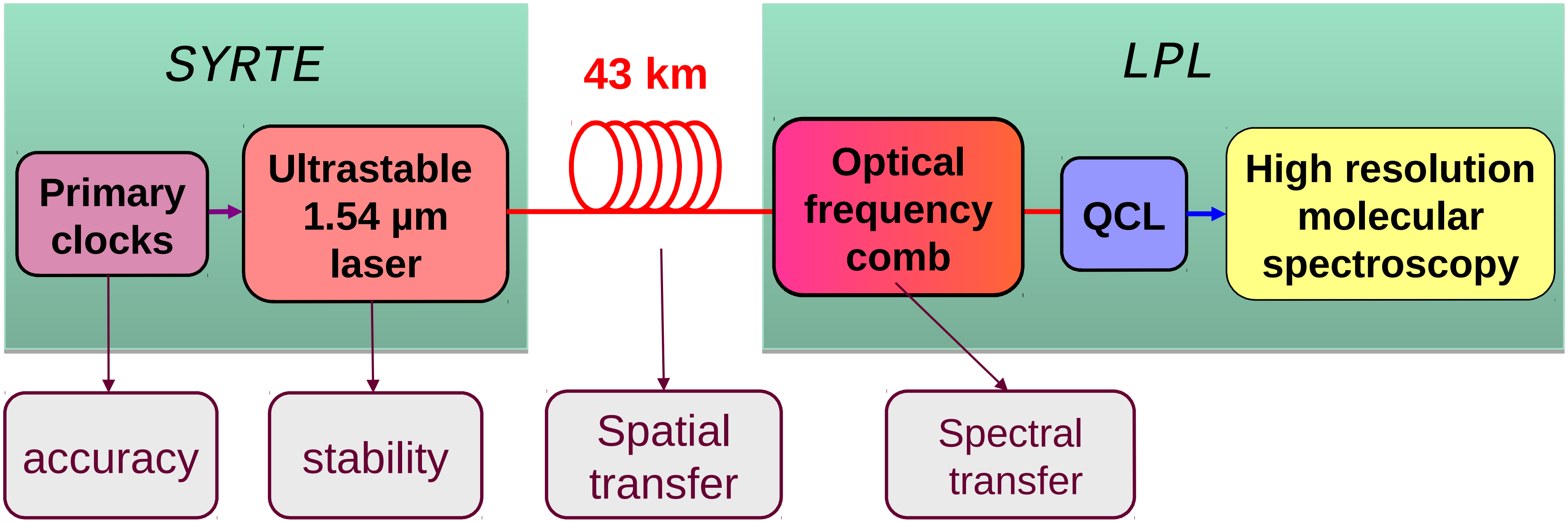
Exemple : test of parity violation in molecules
- due to weak interaction

- seen in atoms and nuclear physics
- never seen in molecule because too weak
- ultra-high resolution spectroscopy in chiral molecules ($\Delta\nu/\nu < 10^{-13}$)



Mid-IR QCL ideal tool for ro-vibrational lines (tunable, relatively easy to operate,...)
BUT large free-running linewidths (~1MHz)
→ necessary to servo its frequency

QCL stabilization onto a NIR frequency reference



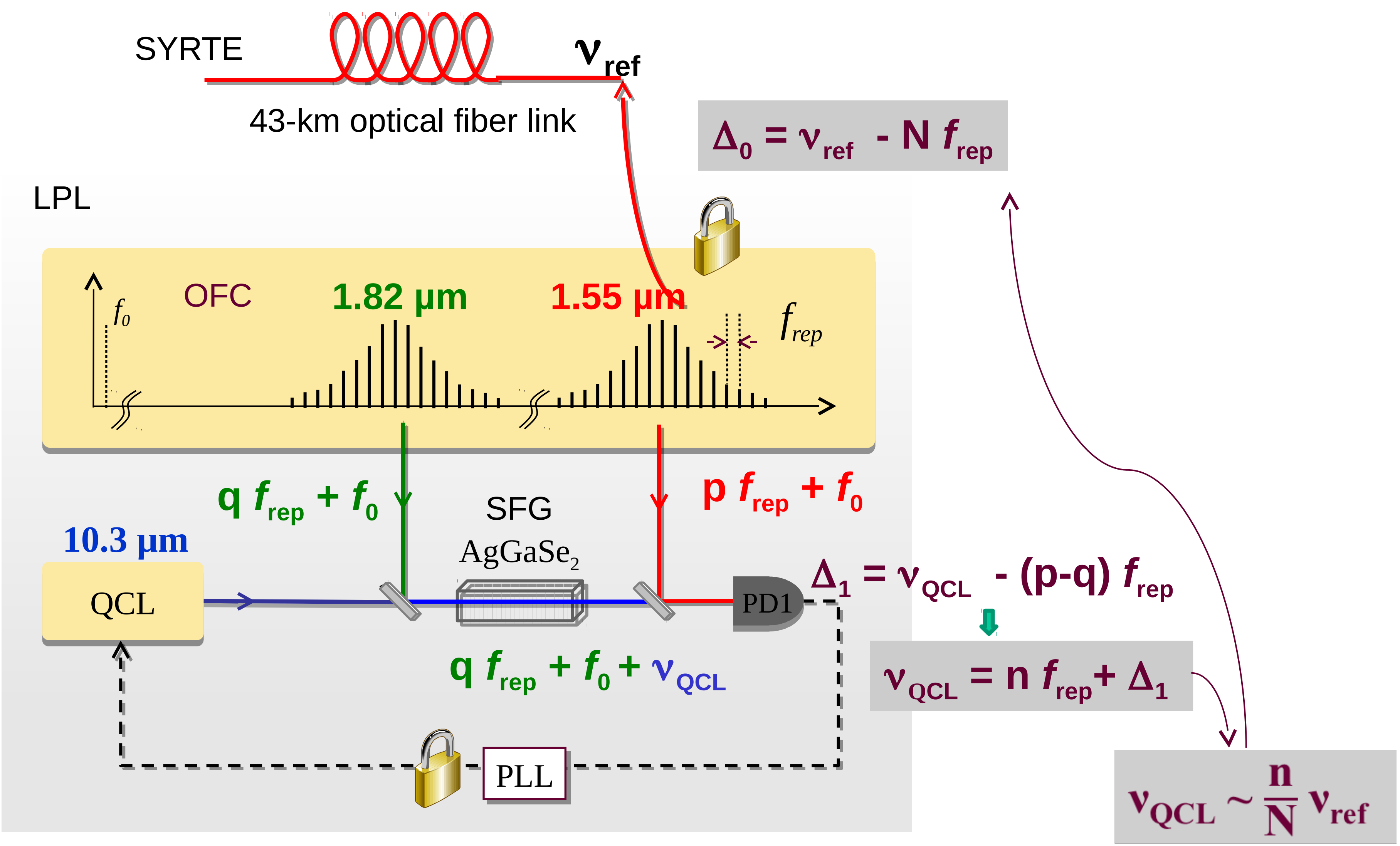
Near-IR frequency reference

- 1s-stability ~10^{-15}
- Accuracy ~10^{-14} (with primary clocks: 3×10^{-16})

Optical fiber link

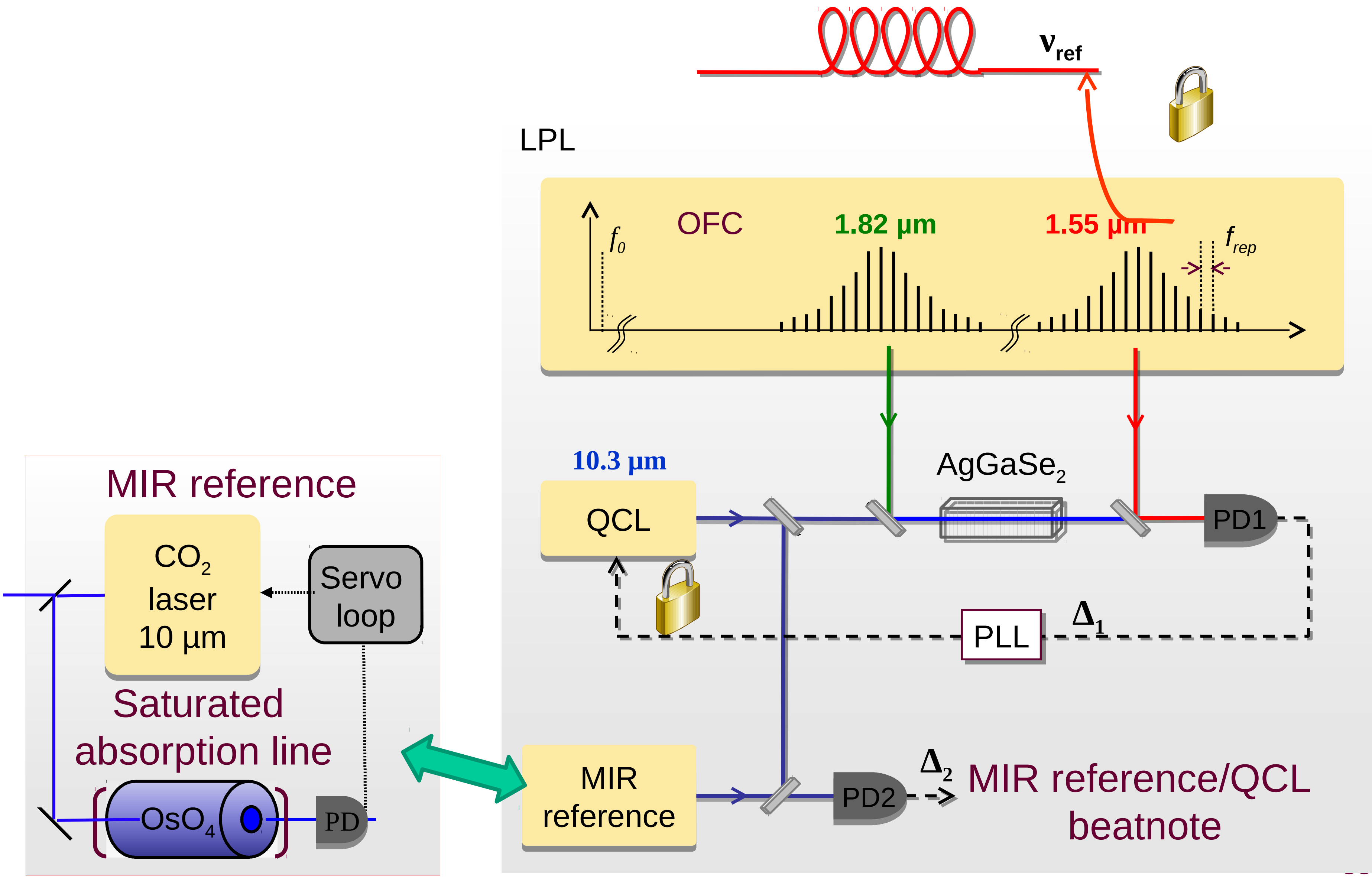
- Free running stability ~≤ 10^{-14}
- Stabilized: $10^{-15}\,\tau^{-1}$ from 1 to 10^4 s

QCL frequency stabilization

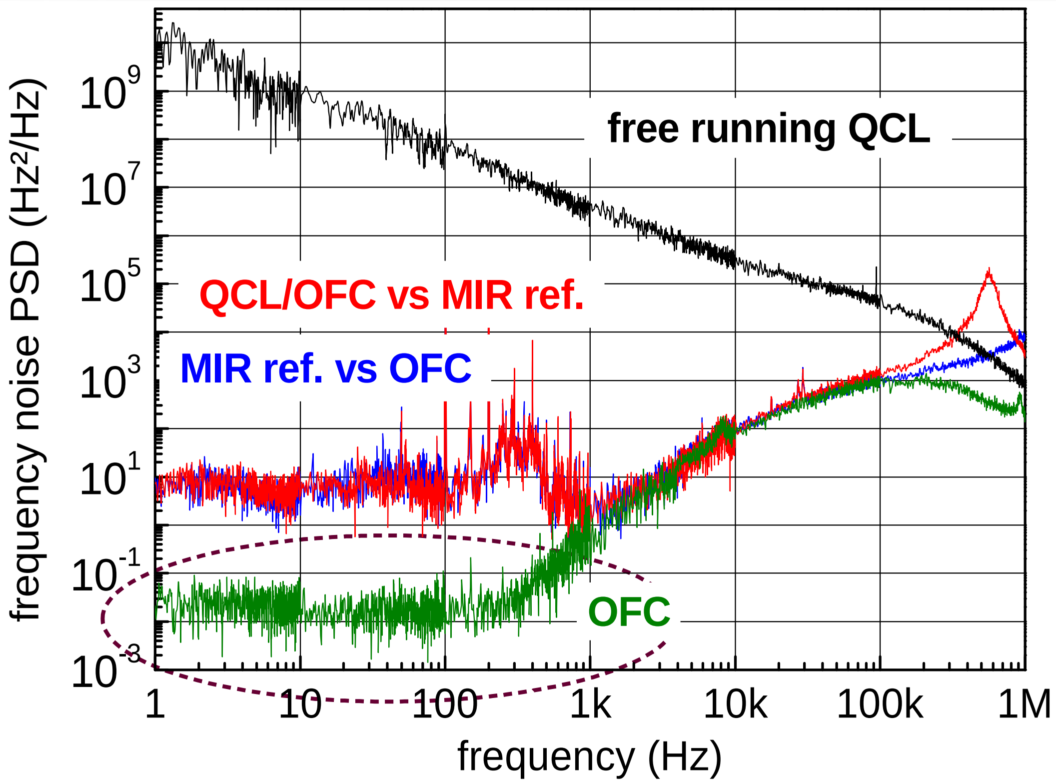


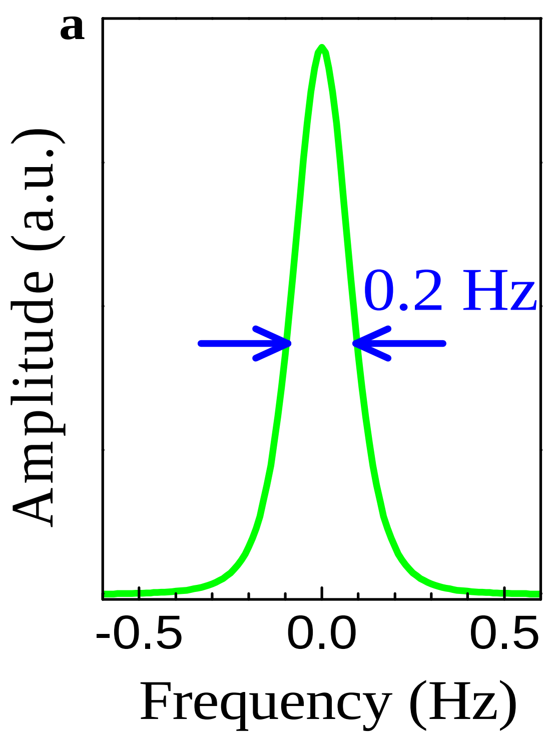
Sum-frequency generation with 9-11 µm tunability

QCL performance evaluation



Locked QCL performance





⇒ QCL frequency noise ~10^{-2} Hz^2/Hz between 1 and 100 Hz

⇒ QCL linewidth ~ 0,2 Hz ($7x10^{-15}$)

⇒ QCL stability ~ $1.5x10^{-15}$ (1s-100s)

B. Argence et al. Nature Photonics 9, 456 (2015)