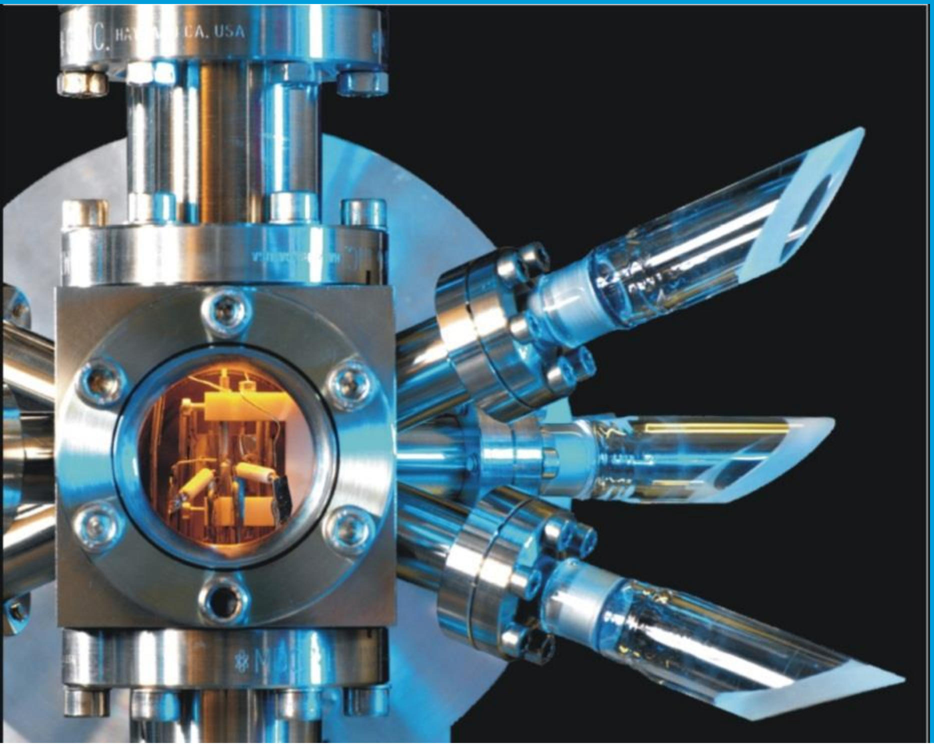
Trapped Ion Optical Clocks



Rachel Godun

"Optical Clock School 2018", Gressoney la Trinité, 10th – 14th Sep 2018

Optical clocks with 10^{-18} uncertainty

NPL National Physical Laboratory

European consortium working to reach 10^{-18} uncertainties by 2019

EURAMET OC18

Outline

- Atomic clocks based on caesium
- The role of optical clocks
- Single-ion optical frequency standards
- Minimising systematic frequency shifts
  - Motion
  - Electric fields
  - Magnetic fields
  - Gravity
- Optical frequency metrology
  - Stability, absolute frequencies and ratios
- Summary and future perspectives

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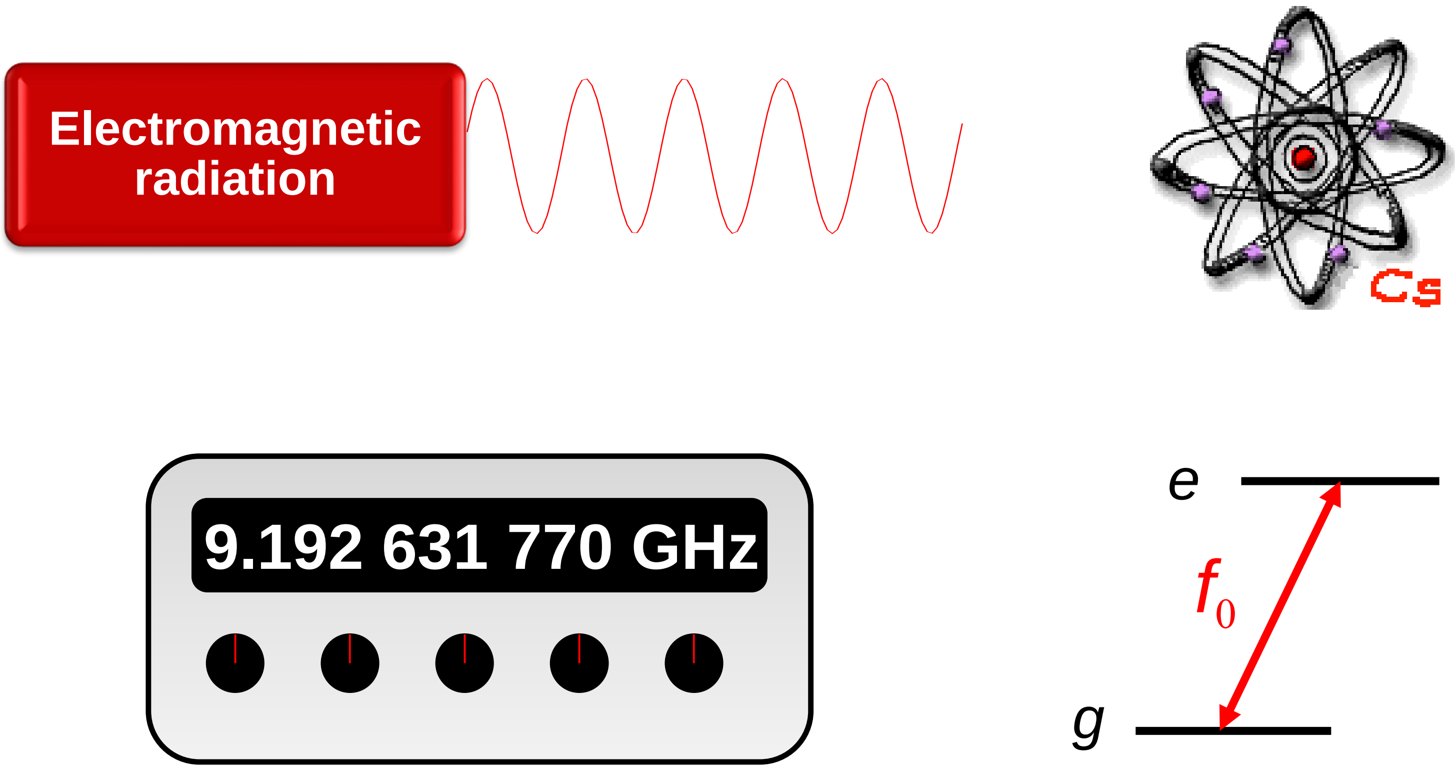
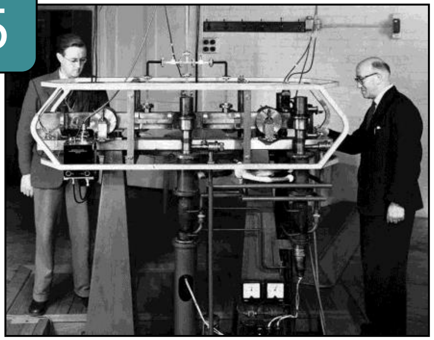
Atomic ~~clock~~ frequency standard basics



- Tune the frequency of the radiation to drive an atomic transition

Introduction of atomic time

1955

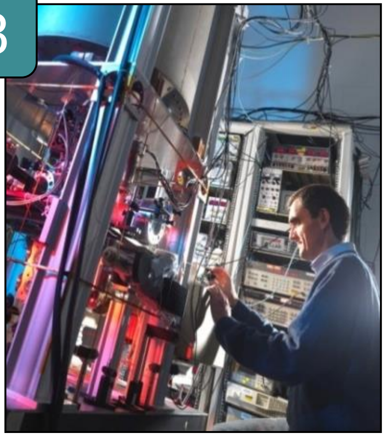


First caesium atomic frequency standard developed at NPL by Essen & Parry, accurate to 1 part in 10^{10}

1967

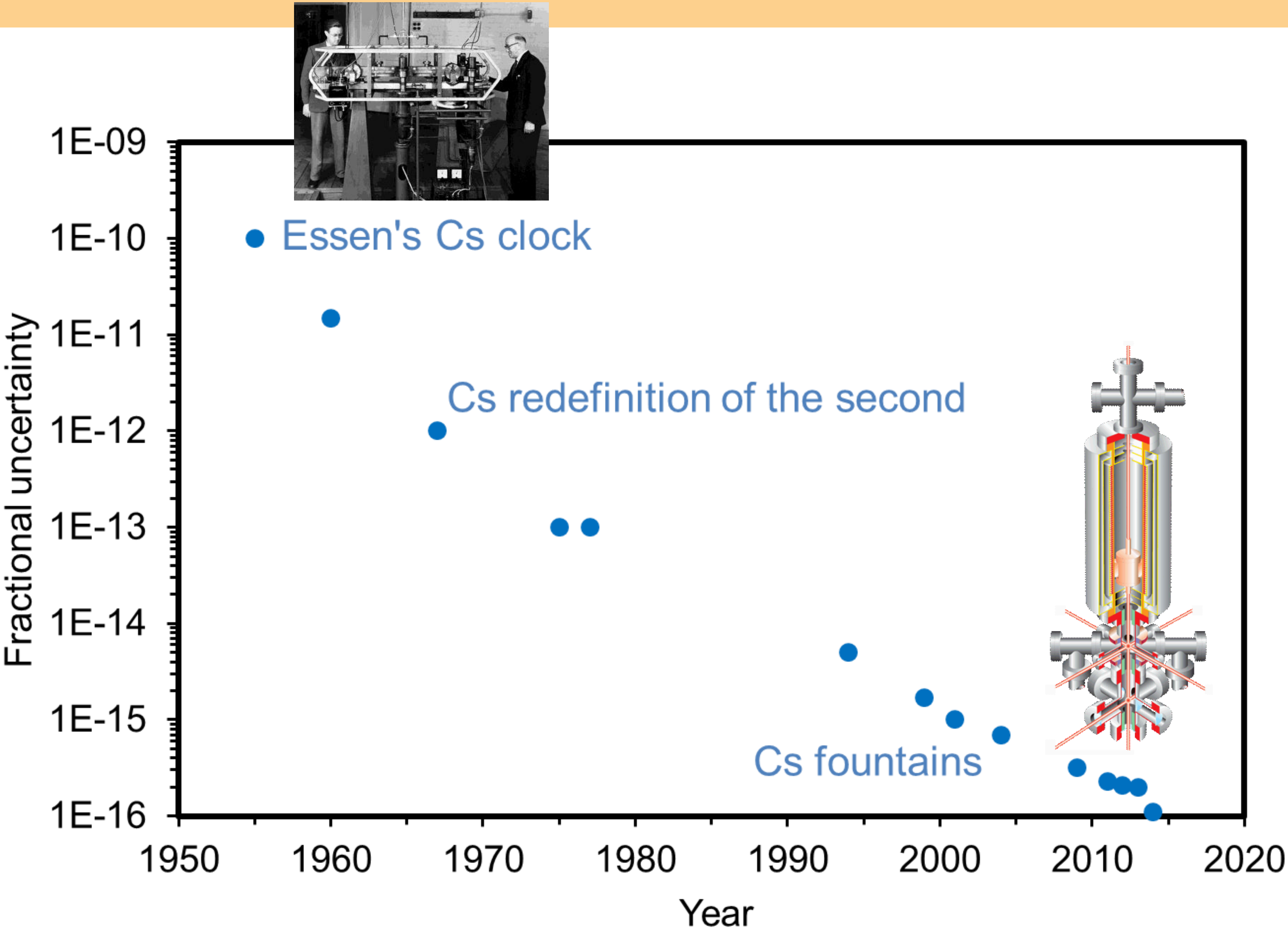
The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.

2018

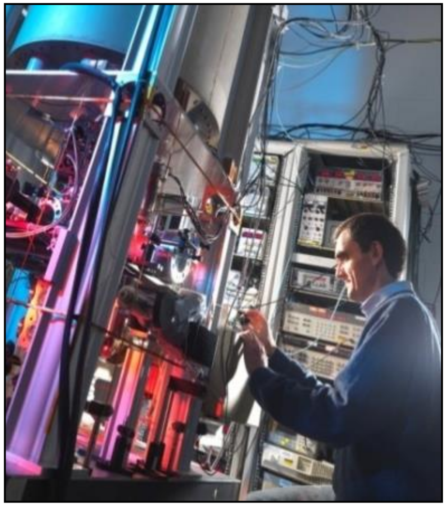


The best caesium fountain primary frequency standards now have accuracies of 1 – 2 × 10^{-16}

Improvements in caesium atomic clocks

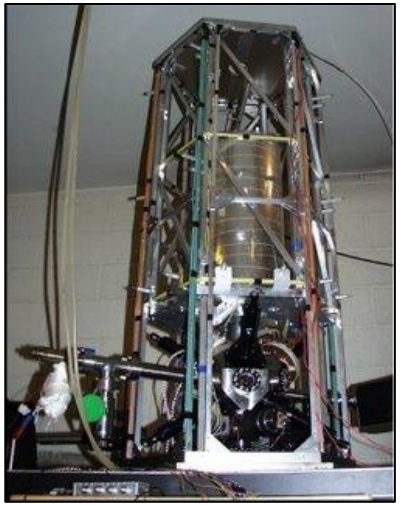


Cs fountain primary frequency standards



NPL-CsF2

INRIM ITCsF2



LNE-SYRTE FO2-Cs

NIST-F2

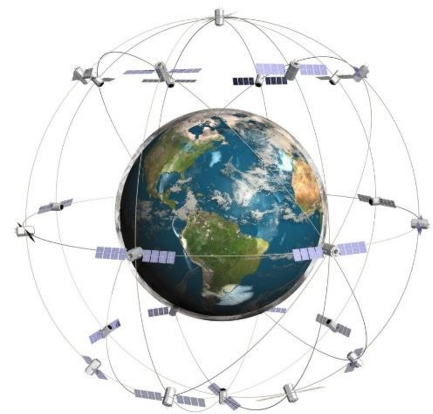
PTB-CSF2

NPL Commercial

Cs clocks are widely used

International time scale

Patrizia Tavella



Navigation

Network synchronisation

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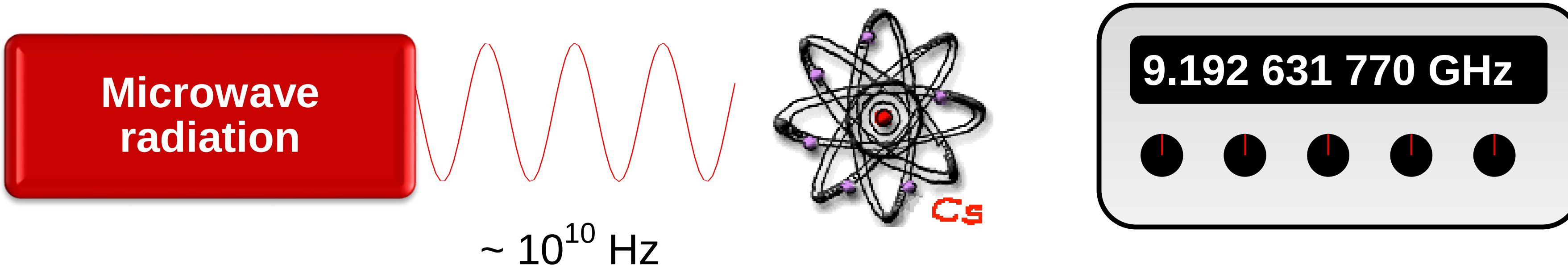
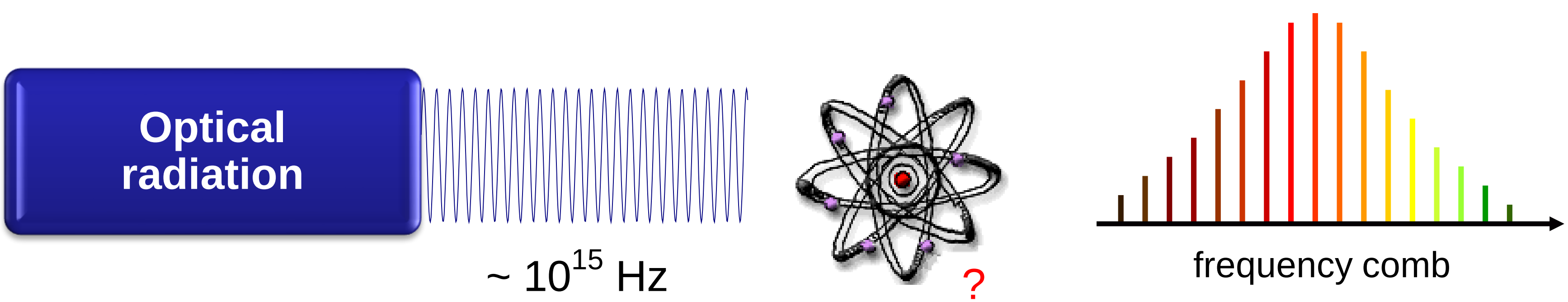
Optical clock basics

- Caesium clock

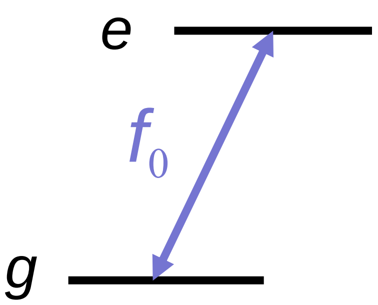


- Optical clock



Yann Le Coq

Performance of a frequency standard

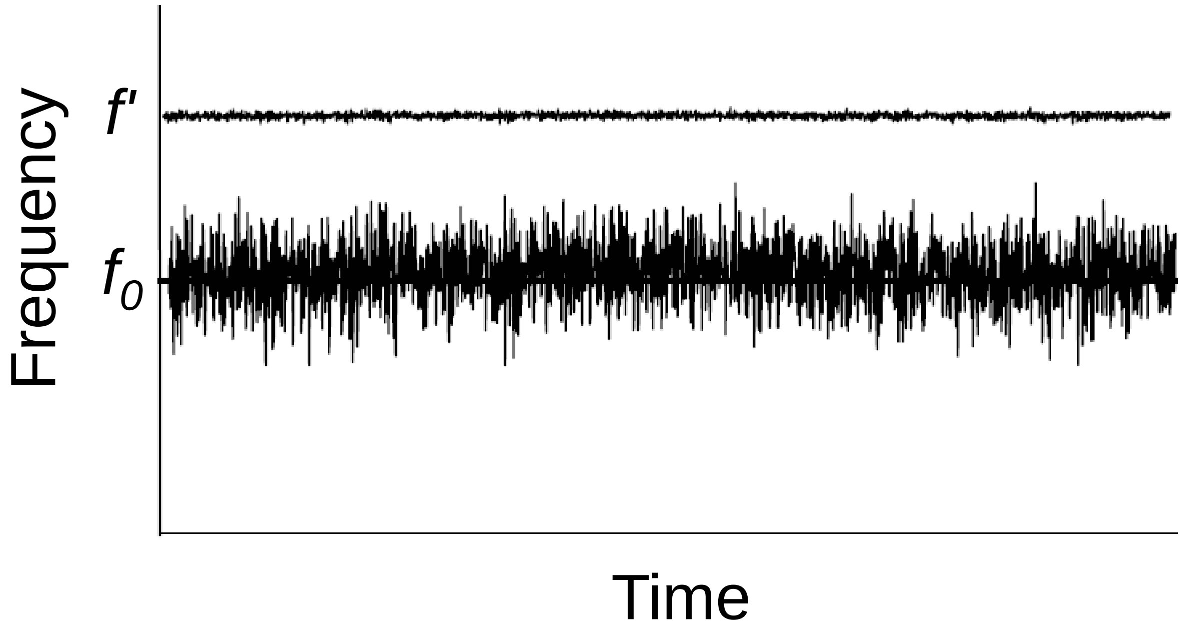


Stability

Level of frequency fluctuations over time (statistical uncertainties)

Accuracy

Level of offset from correct frequency (systematic uncertainties)



Advantage of optical frequency standards

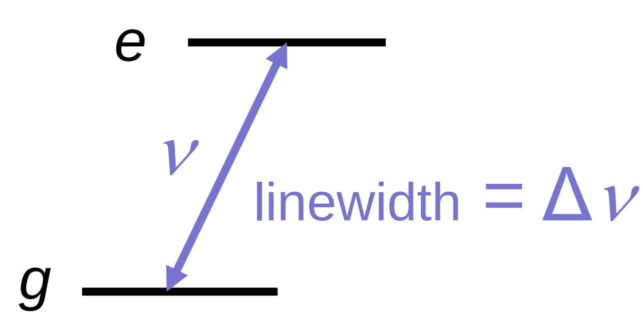
Stability:

Fractional instability

$$\sigma(\tau) = \frac{\Delta\nu}{\nu}\frac{\eta}{(S/N)}\sqrt{\frac{T}{\tau}}$$

	Microwave	Optical
$\Delta\nu$	~ 1 Hz	~ 1 Hz
ν	~ 10^{10} Hz	~ 10^{15} Hz
reach 10^{-15}	~ 1 day	~ seconds

$\Delta\nu$ = linewidth
ν = optical frequency
(S/N) = signal-to-noise ratio
T = probe time
τ = total averaging time
η ~ 1 depends on probing technique and shape of resonance



Accuracy:

Microwave	Optical
~ 10^{-15} - 10^{-16}	~ 10^{-17} - 10^{-18}

Improvements in optical clocks

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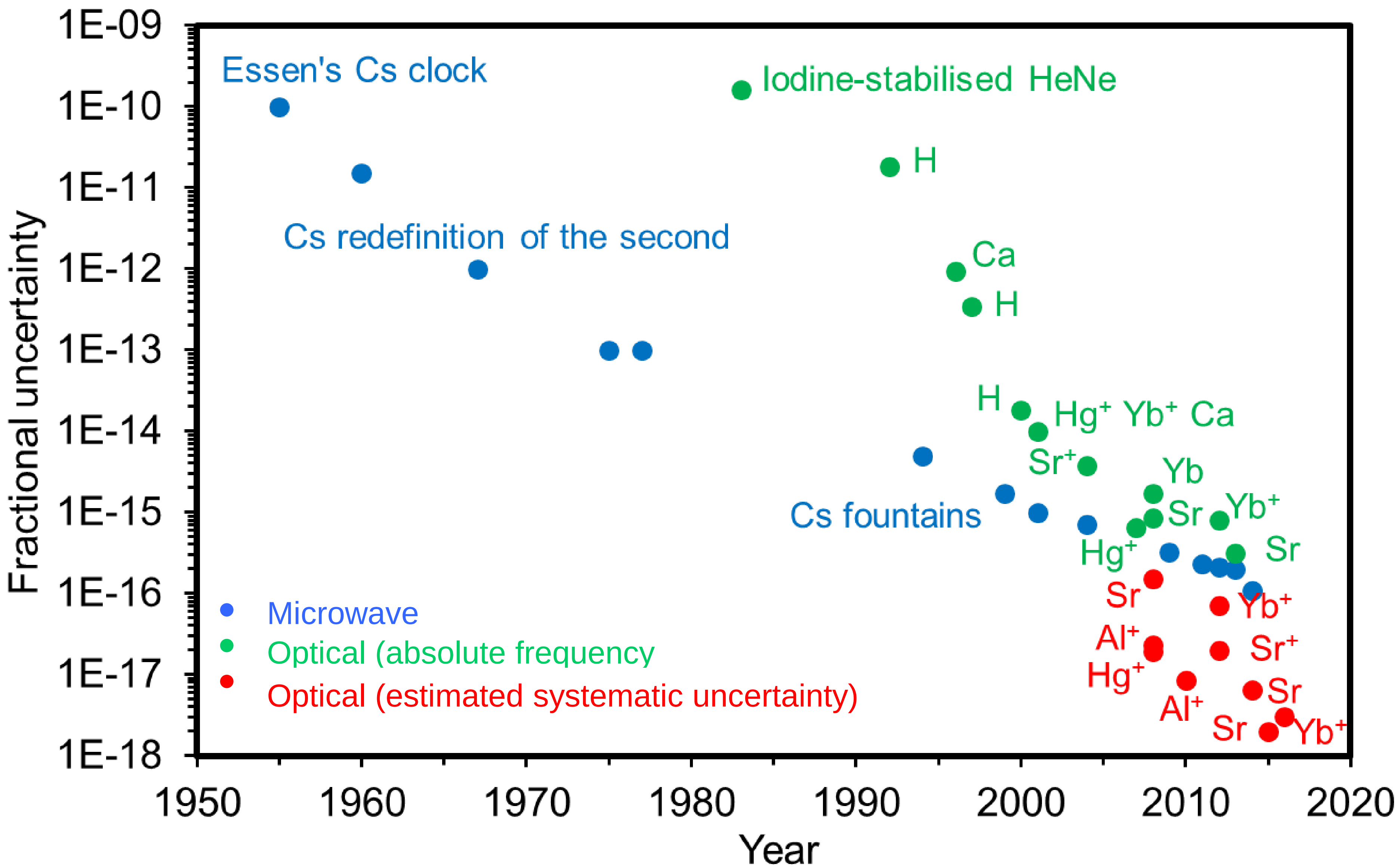
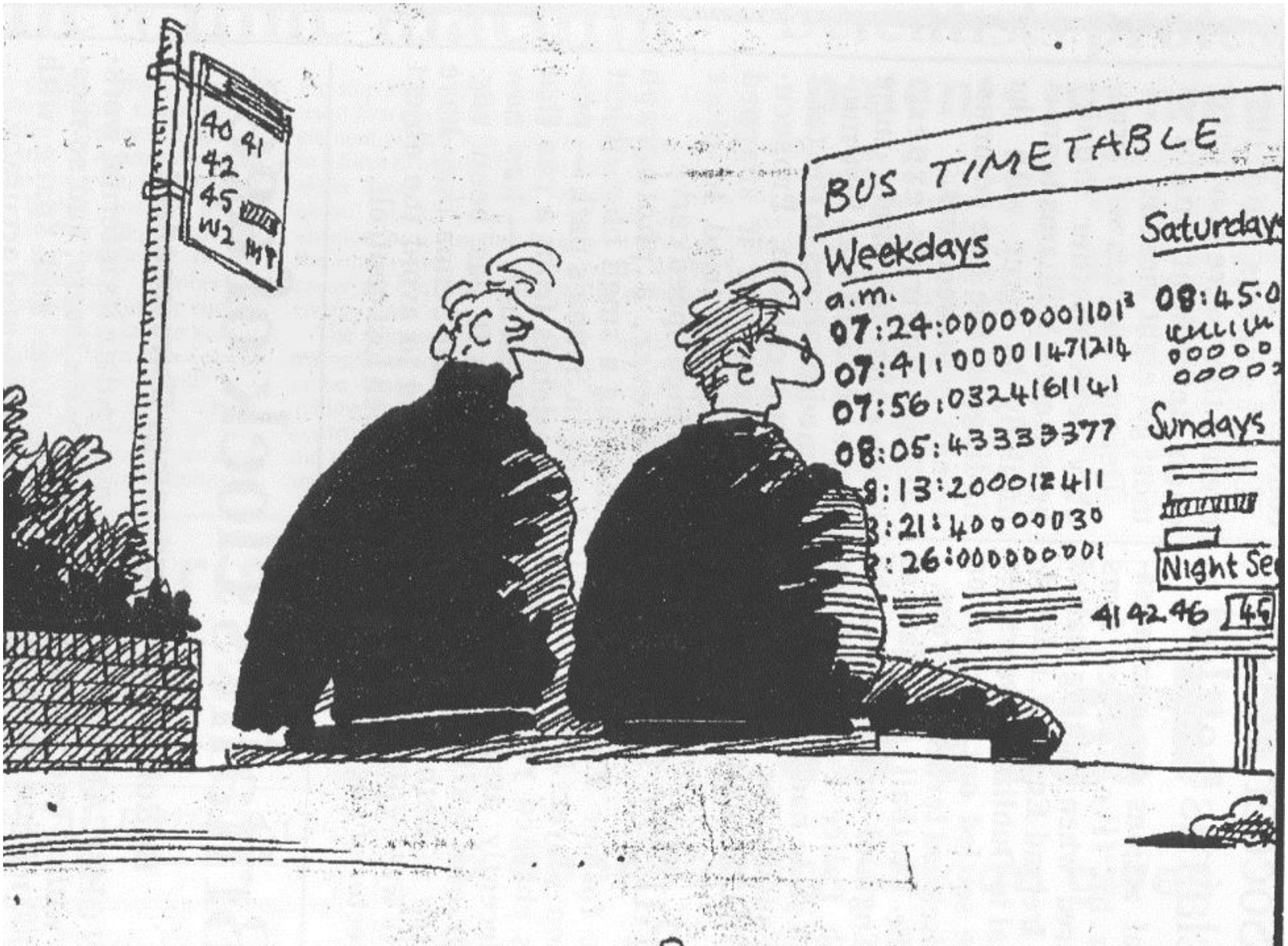


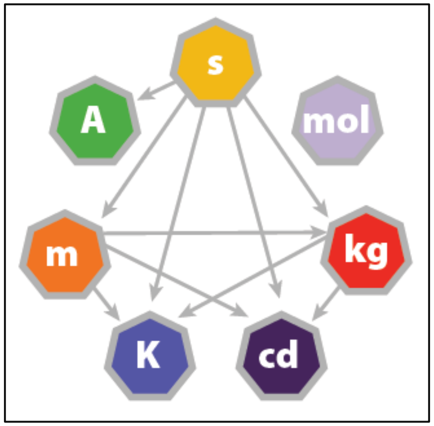
What difference does this make?



Impact of optical clocks

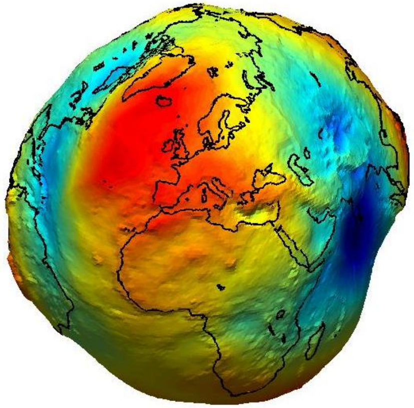
- Greater accuracy and stability opens up yet more opportunities for atomic clocks

Top-level SI



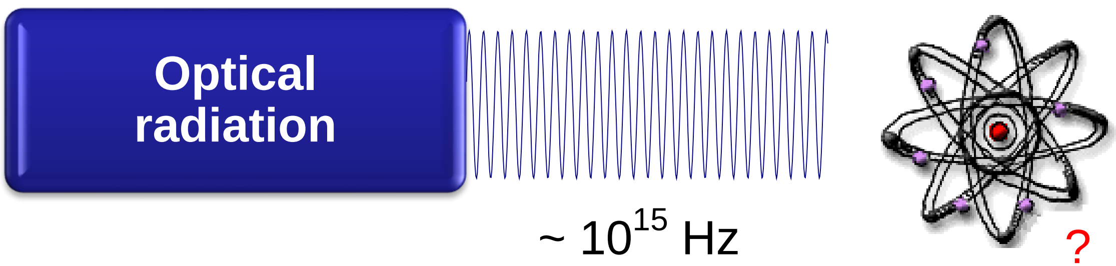
Fundamental physics

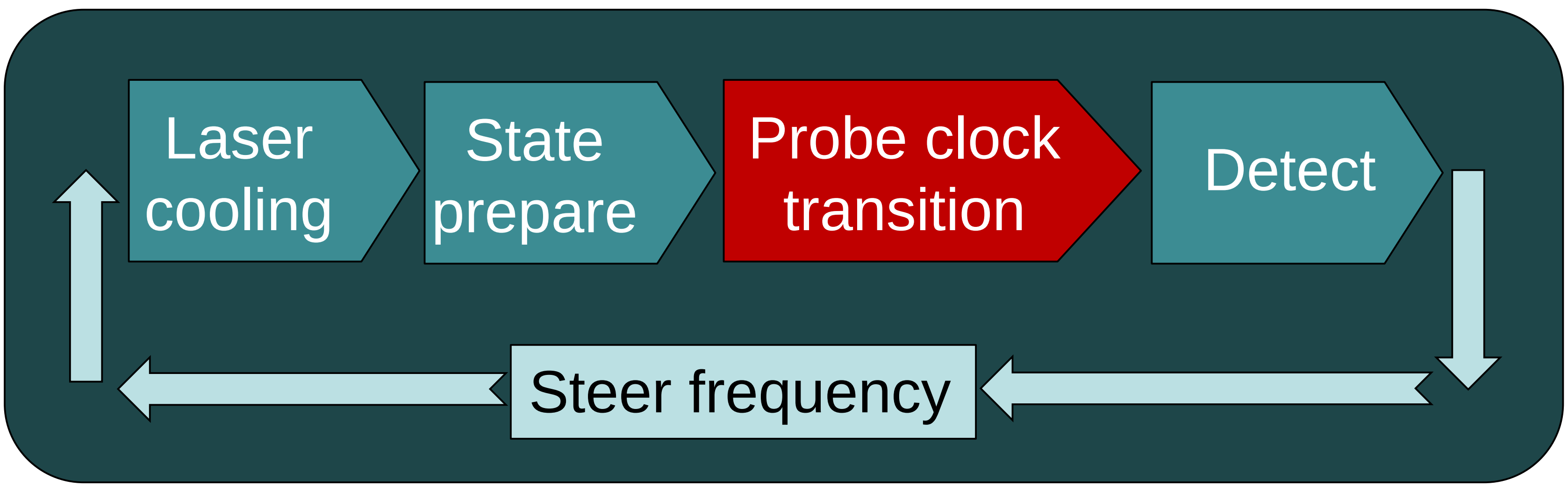
Geodesy



Astronomy, GNSS

Stabilising frequency to an atomic reference





What could possibly go wrong?

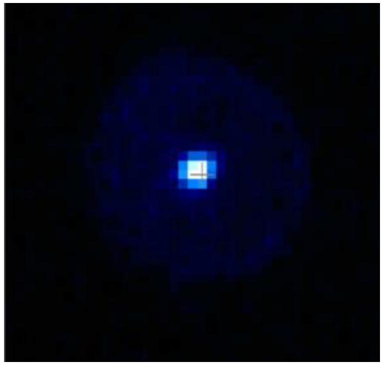
- Whether neutral atom or ion, need to think about:

  Fundamental: motion, E-fields, B-fields, gravitational effects

- Each setup may also have other issues to avoid

  Technical: shutter leakage / collisions / servo offsets / frequency chirps …

- Single ions are good because almost an isolated atom at rest



- Atoms good because large number gives better S/N

Fractional instability $\sigma(\tau) = \frac{\Delta\nu}{\nu}\frac{\eta}{(S/N)}\sqrt{\frac{T}{\tau}}$

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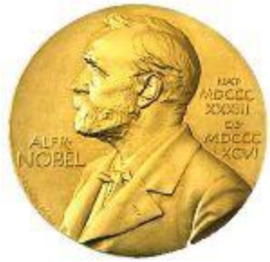
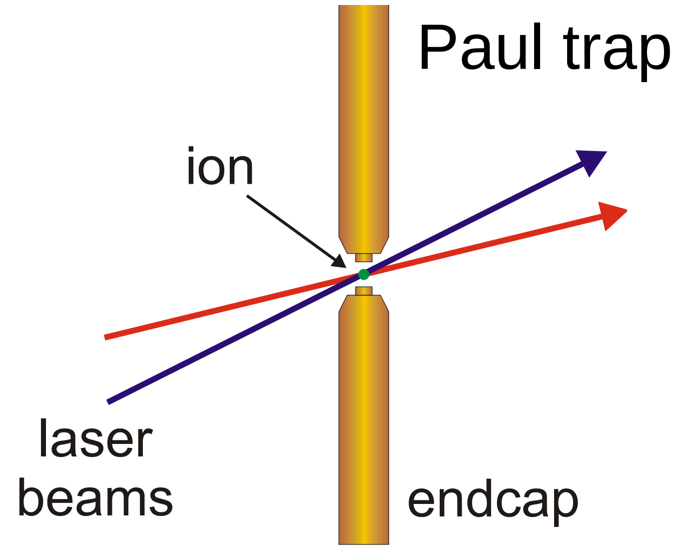
Ion clocks

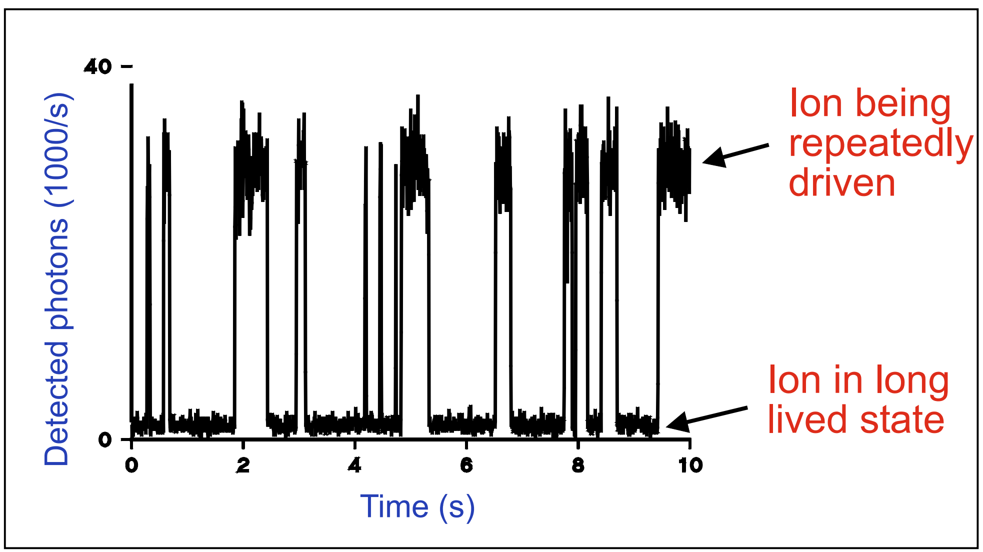
- Nobel Prize 1989

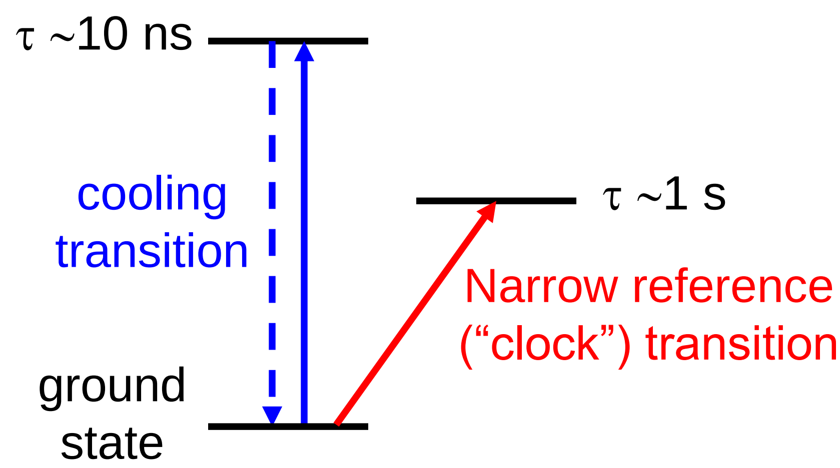
  Dehmelt, Paul, Ramsey





- Dehmelt proposed using ions for optical frequency standards and ‘electron shelving’ for detection





[H. Dehmelt, IEEE Trans. Instrum. Meas. **IM-31**(2), 83 (1982)]

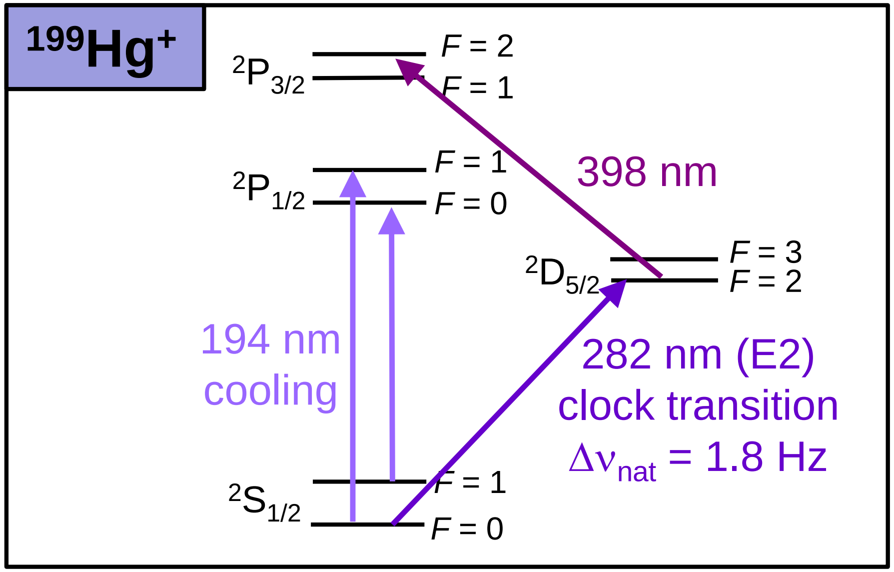
Candidate systems (1)

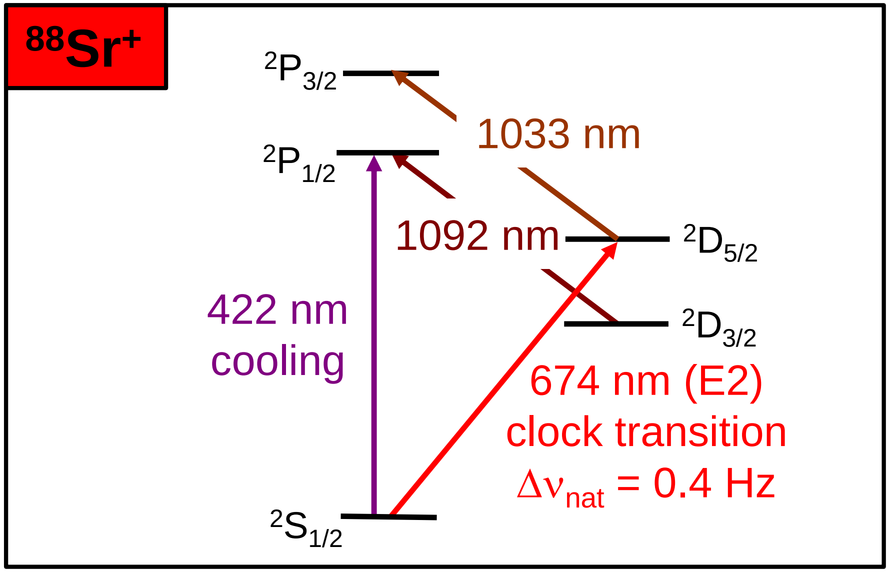
Ions with alkali-like or quasi-alkali-like atomic structure

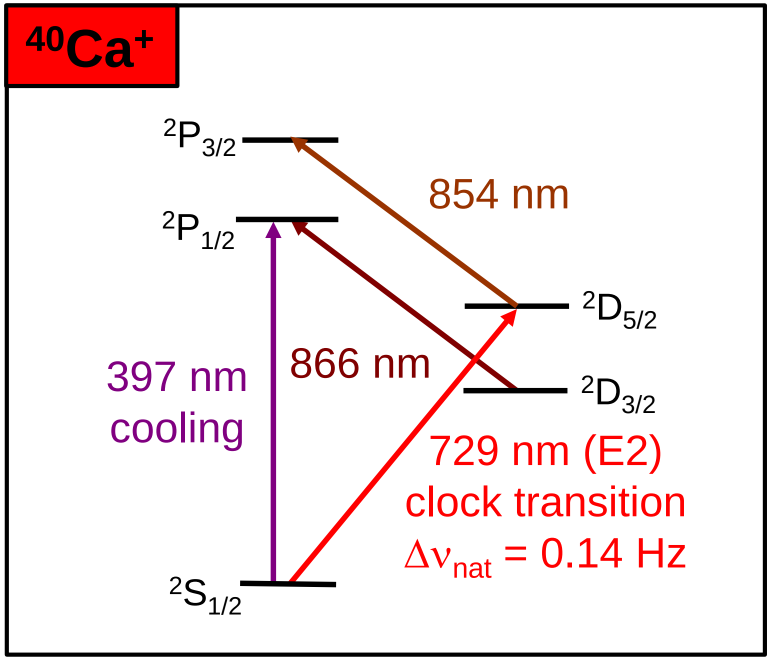
1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac	104 Unq	105 Unp	106 Unh	107 Uns	108 Uno	109 Une	110 Unn								

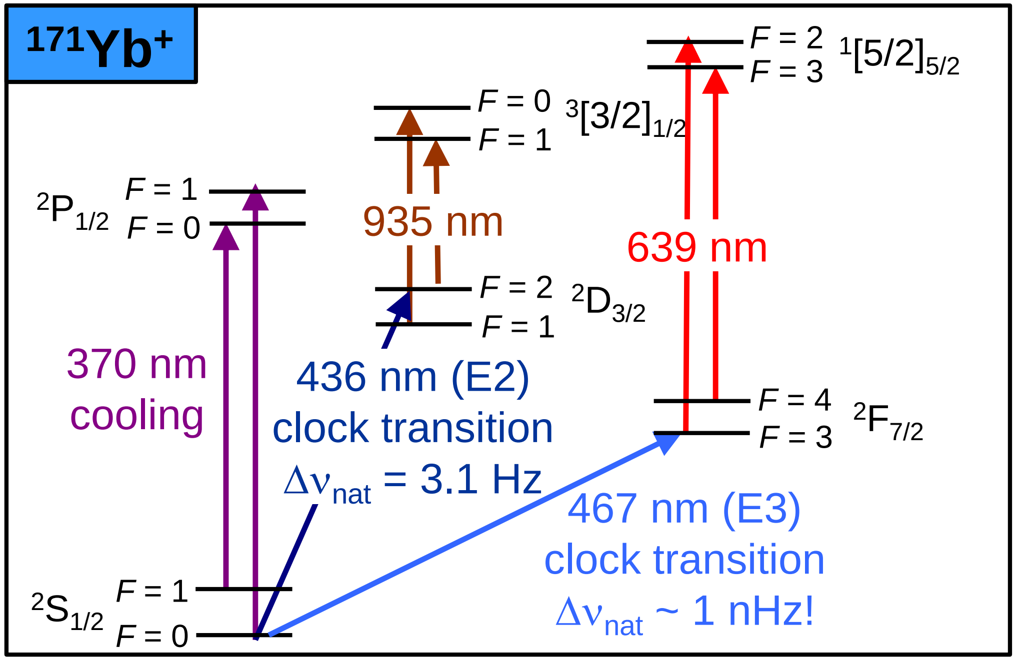
58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

Alkali-like systems









Candidate systems (2)

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Ions with atomic structure similar to alkaline earth elements

1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac	104 Unq	105 Unp	106 Unh	107 Uns	108 Uno	109 Une	110 Unn								

58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

Alkaline-earth-like systems

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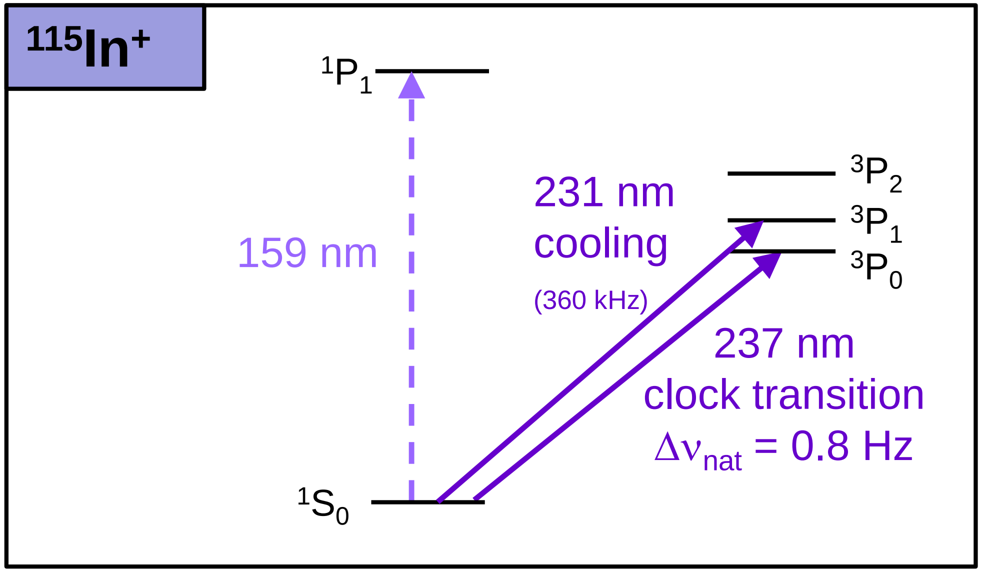
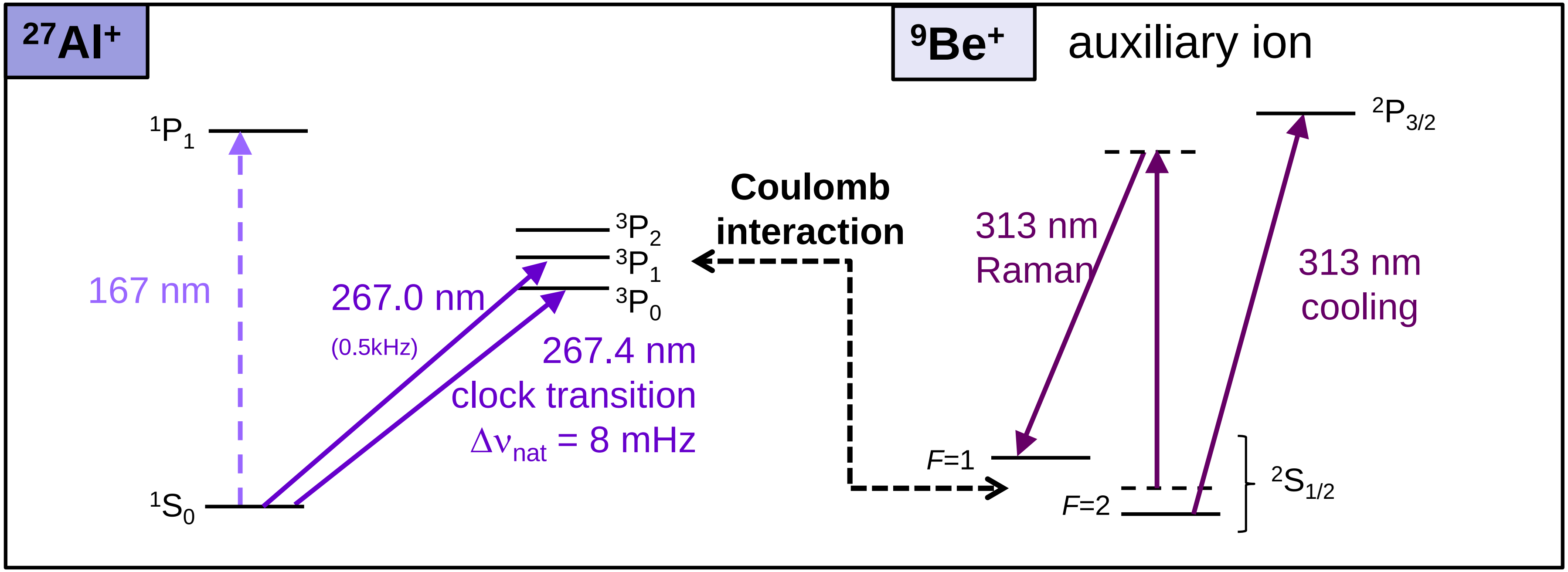
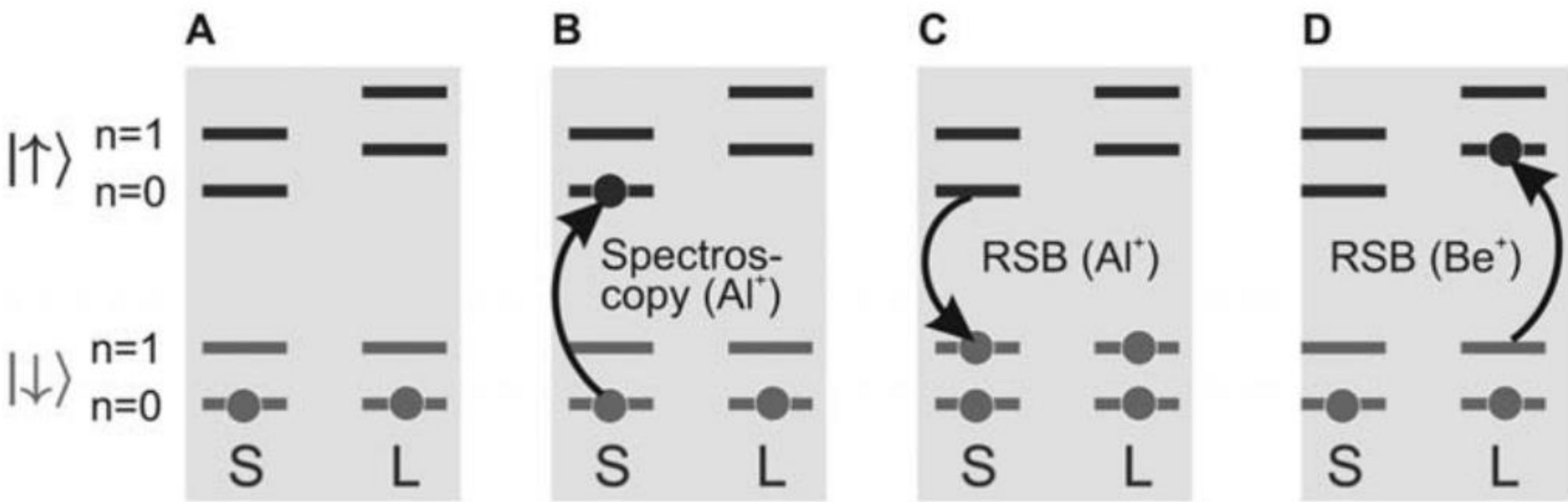




Aluminium ion clock

- Quantum logic spectroscopy



Initialisation to ground states	Clock spectroscopy on Al^+	Transfer Al^+ internal superposition state to motional superposition state	Transfer motional superposition state to internal superposition state in Be^+

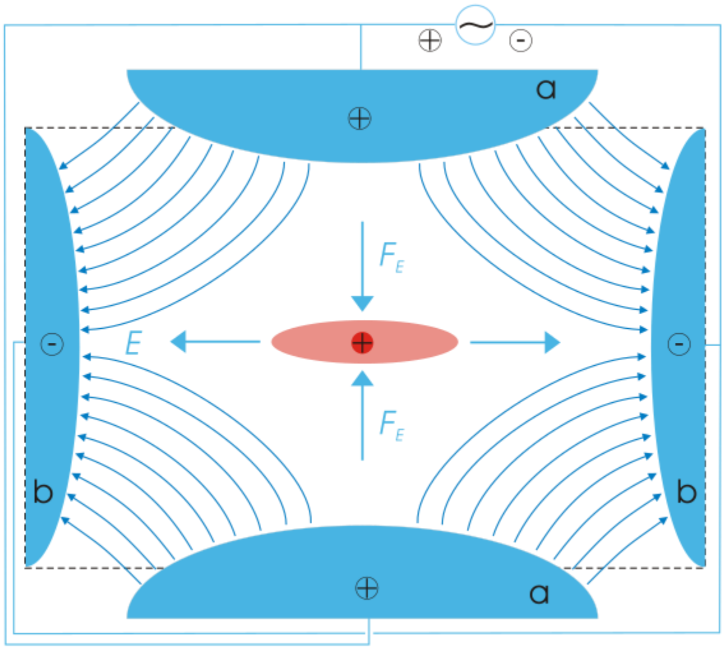
[P.O. Schmidt *et al.*, Science **309**, 749 (2005)]

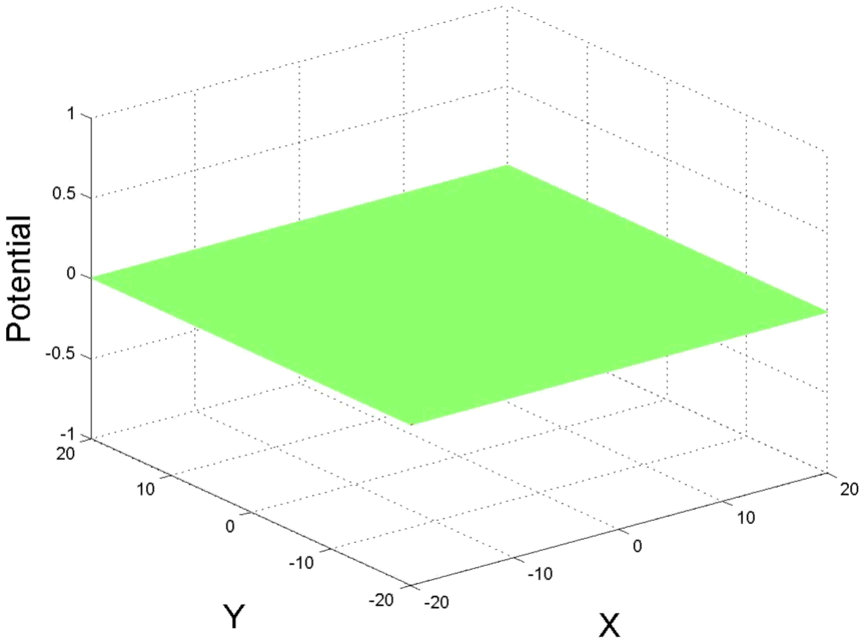
Principles of ion trapping

Quadrupole potential:

$$V(t) = A(t)\,(r^2 - 2z^2)$$

$$V(t) = \varepsilon(V_{dc} + V_{ac}\cos\Omega t)\,\frac{(r^2 - 2z^2)}{2r_o^2}$$



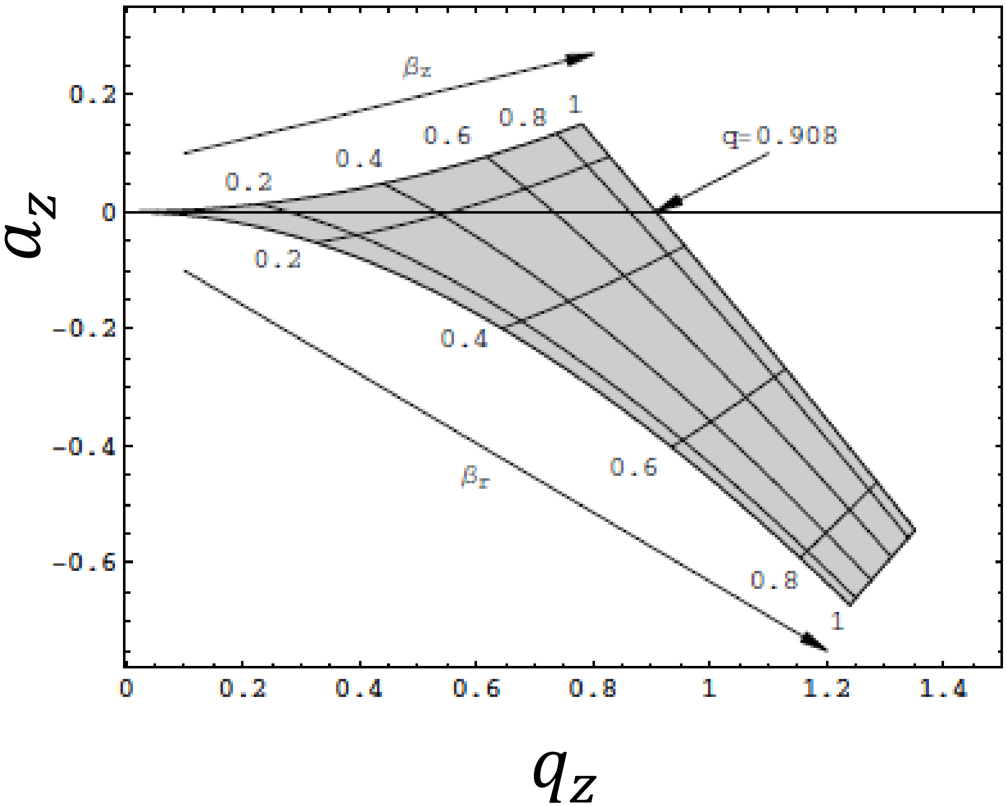


Radiofrequency voltage applied to top and bottom electrodes

→ ion trapped in time-averaged pseudo-potential minimum

Stable solutions

- Pseudo-potential gives stable trapping for parameters in the shaded region



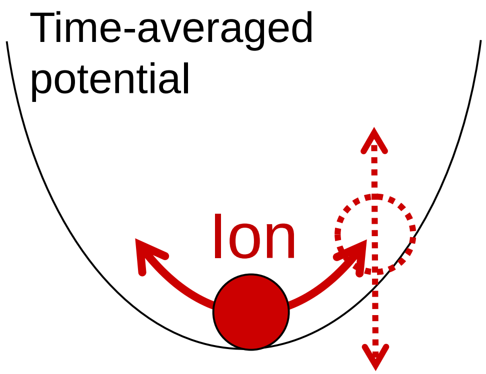
where $a_z = \frac{-8e\varepsilon V_{dc}}{m r_o^2 \Omega^2}$ and $q_z = \frac{4e\varepsilon V_{ac}}{m r_o^2 \Omega^2}$

Motion of the trapped ion

Stable ion motion can be separated into two parts:

Secular motion Thermal motion associated with time-averaged confining potential (characteristic frequencies ω_r and ω_z)

Micromotion Driven oscillatory motion at frequency Ω (vanishes at trap centre)

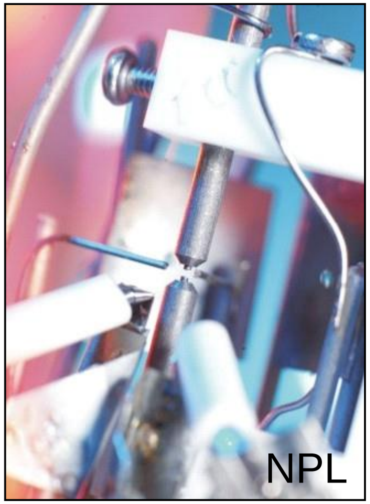


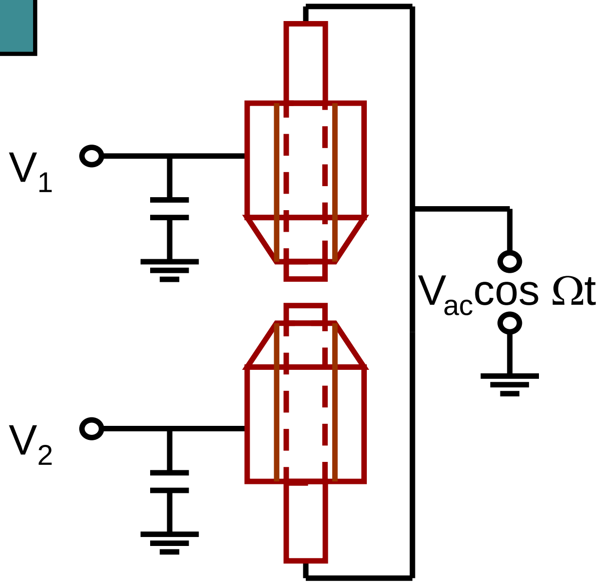
Mathematical solution for ion's position:

$$z(t) = \bar{z} + z_a \cos(\omega_z t) - z_a \frac{q_z}{2} \cos(\omega_z t) \cos(\Omega t) - \bar{z} \frac{q_z}{2} \cos(\Omega t)$$

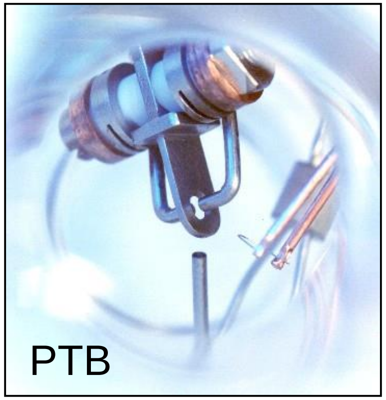
Ion traps for optical frequency standards

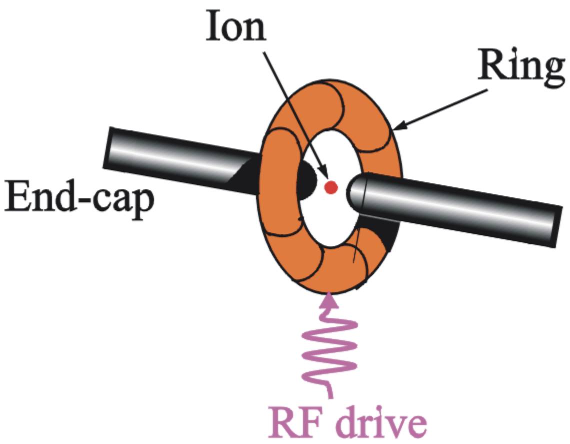
Endcap traps



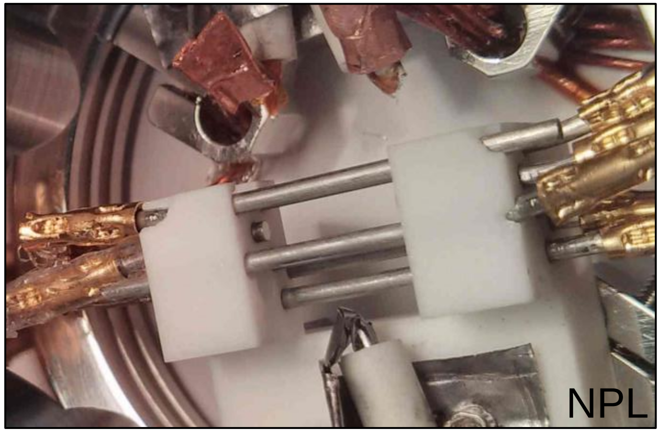


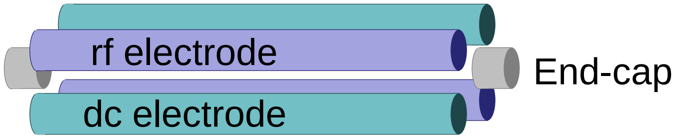
Ring traps

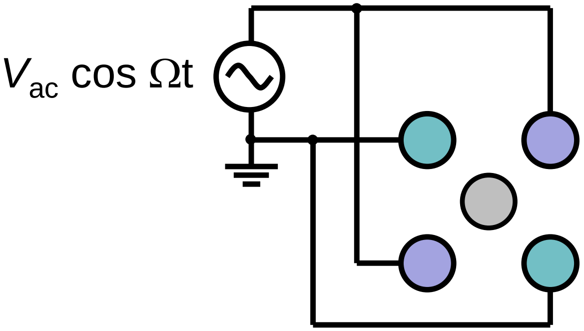




Linear traps







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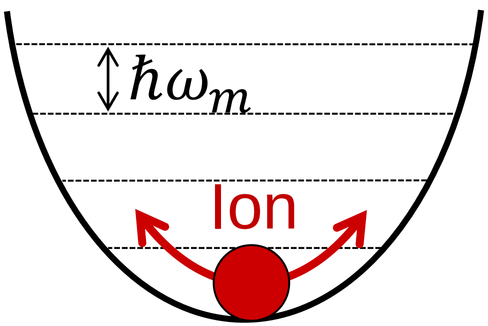
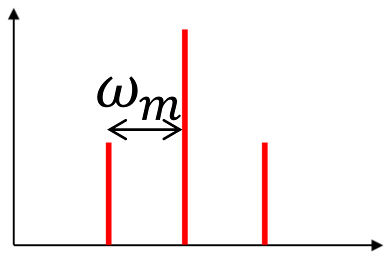
Ion motion causes Doppler shifts

- Motion leads to Doppler shift through $\frac{\Delta f}{f_0} = \frac{v}{c}$
- For **unconfined** ion at 1mK, Doppler broadening gives FWHM of optical probe ~ 1MHz
- **Confined** ion in harmonic potential





- Sidebands negligible if weak modulation, i.e. small enough amplitude of motion

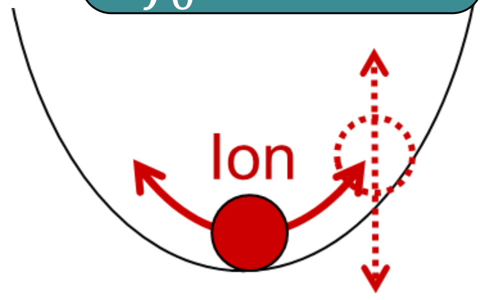
$$d < \frac{\lambda}{\pi}$$

Lamb-Dicke criterion
- eliminates 1st order Doppler broadening

Ion motion causes Doppler shifts

- 2nd order Doppler shifts (relativistic time dilation) still present so need to measure residual motion

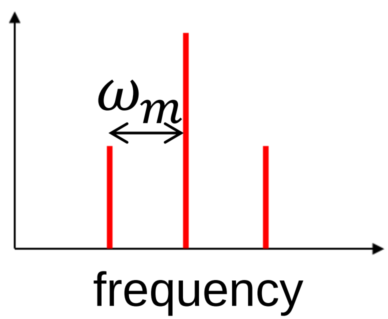
$$\frac{\Delta f}{f_0} = -\frac{\langle v^2 \rangle}{2c^2}$$



Secular motion

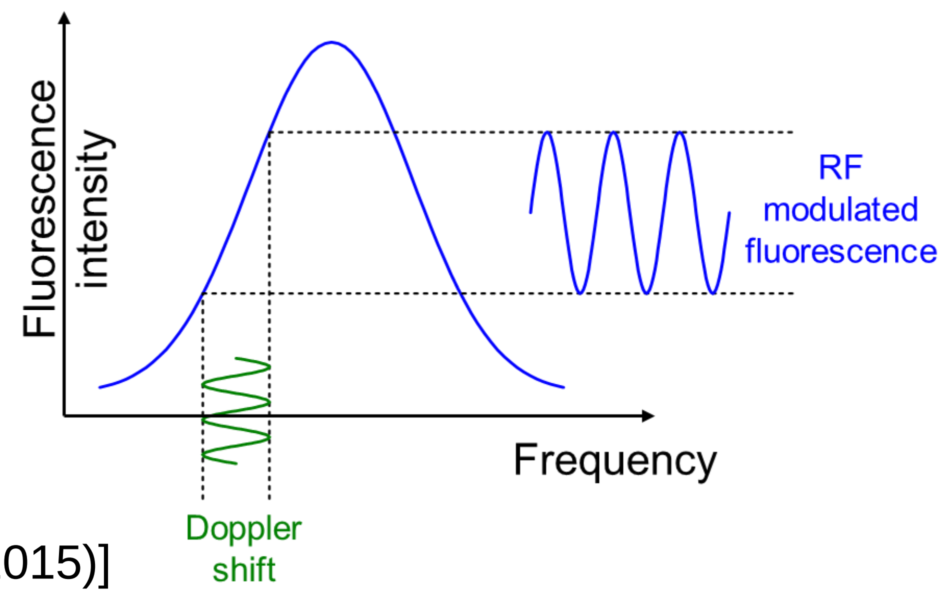
Micromotion

Sideband-to-carrier ratio



Micromotion

RF-photon correlation technique:



[J. Keller *et al.*, J. Appl. Phys. **118**, 04501 (2015)]

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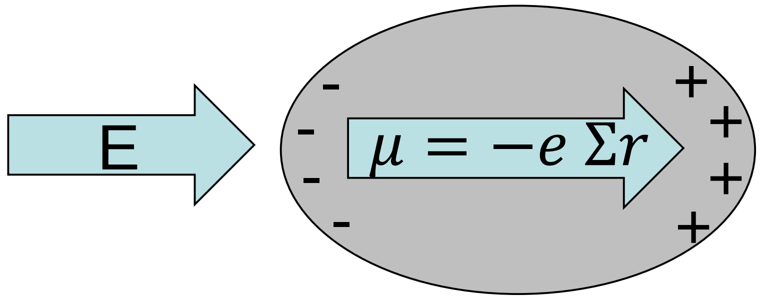
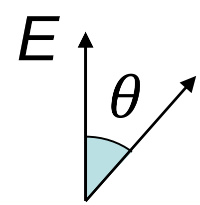
Electric fields cause Stark shifts

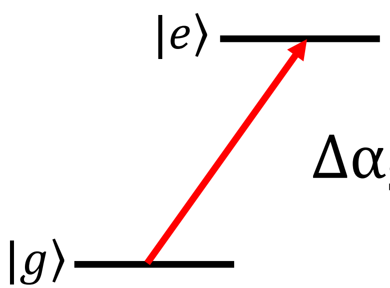
- External electric fields induce a dipole moment in the ion,



then interact with that induced dipole as a 2nd order effect (shifts proportional to E^2)

$$h\Delta f = -\frac{1}{2}\Delta\alpha_{sc}E^2 - \frac{1}{4}\alpha_{ten}\frac{[3M_F^2 - F(F+1)]}{F(2F-1)}(3\cos^2\theta - 1)E^2$$



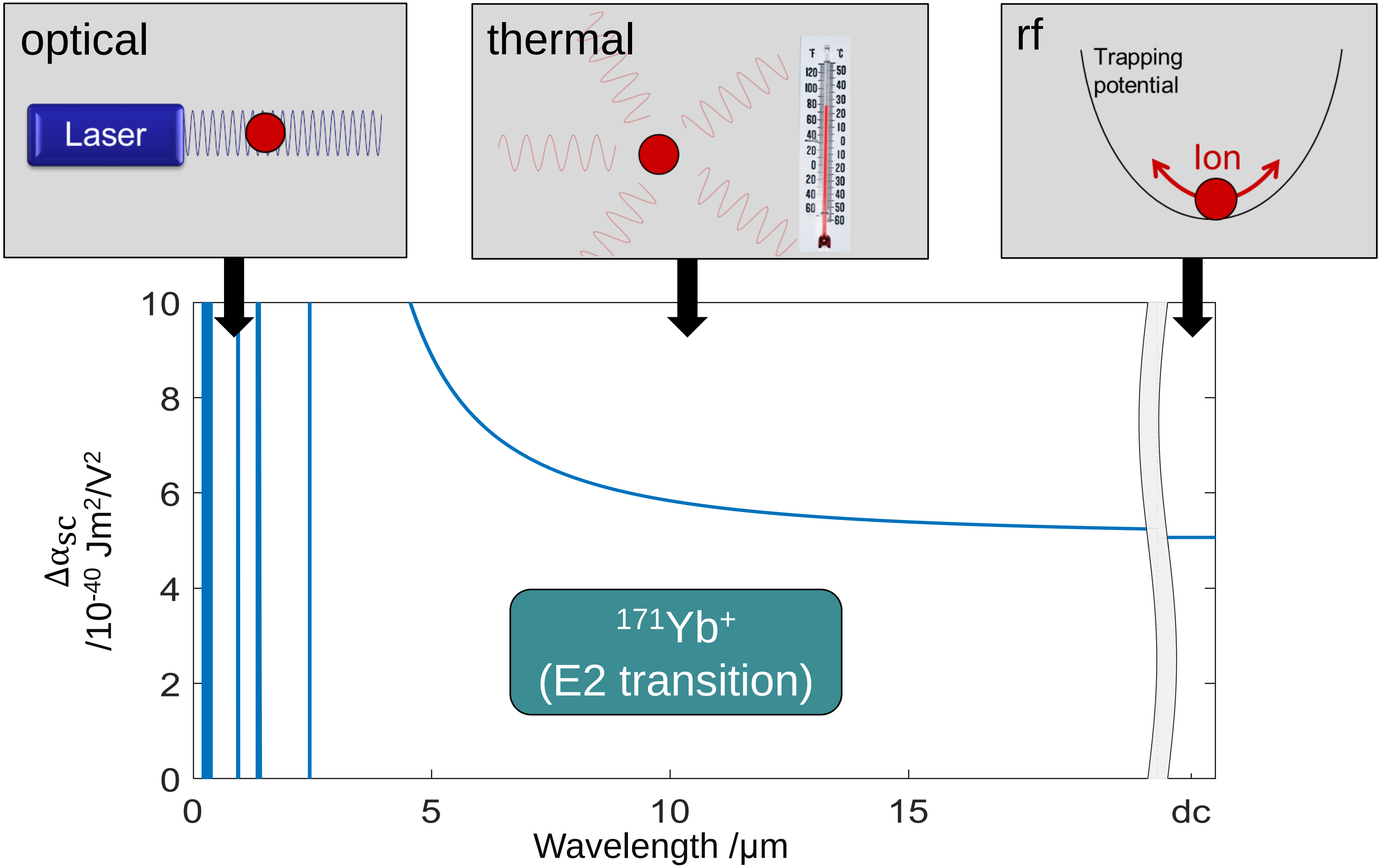


$$\Delta\alpha_{sc} = \alpha_{sc}^{e} - \alpha_{sc}^{g}$$

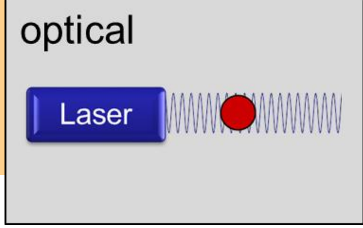
[J. R. P. Angel and P. G. H. Sandars, Proc. Roy. Soc. A **305**, 125 (1968)]

Electric fields cause Stark shifts

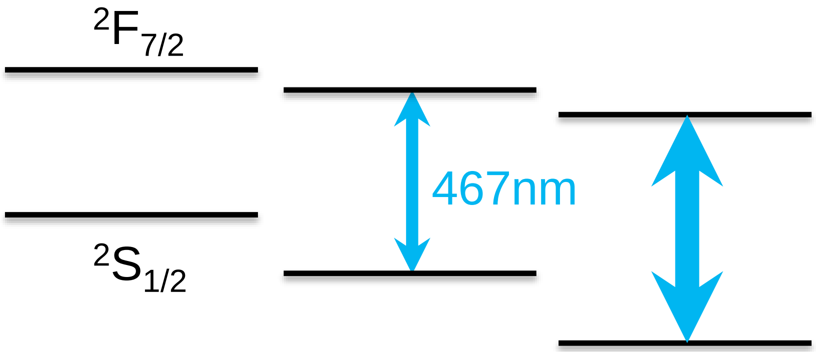
- Different frequency electric fields, so different polarisabilities



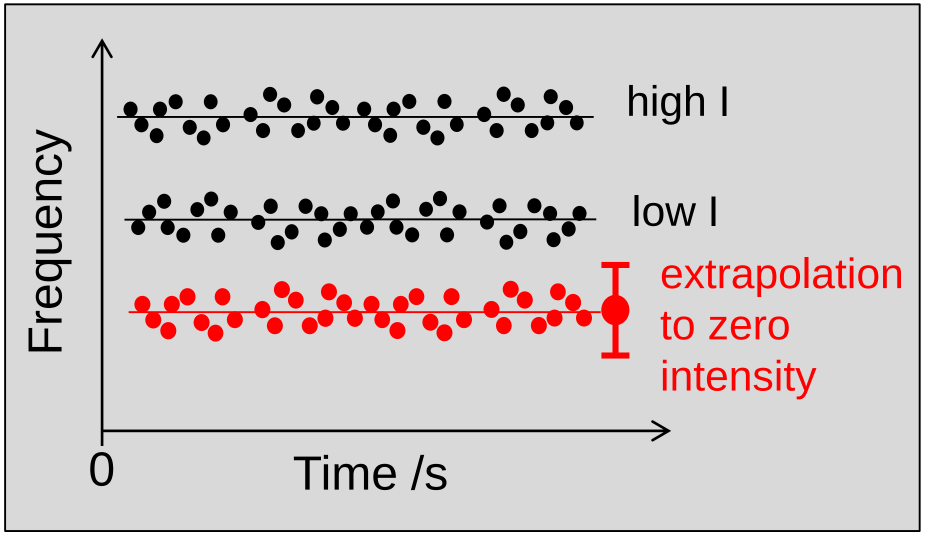
Stark shift – optical



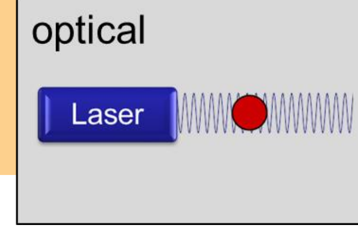
- Strong extinction of cooling and repumper beams is vital
- Negligible shift due to probe laser at typical intensities (exception is 467 nm E3 transition in $^{171}Yb^{+}$)

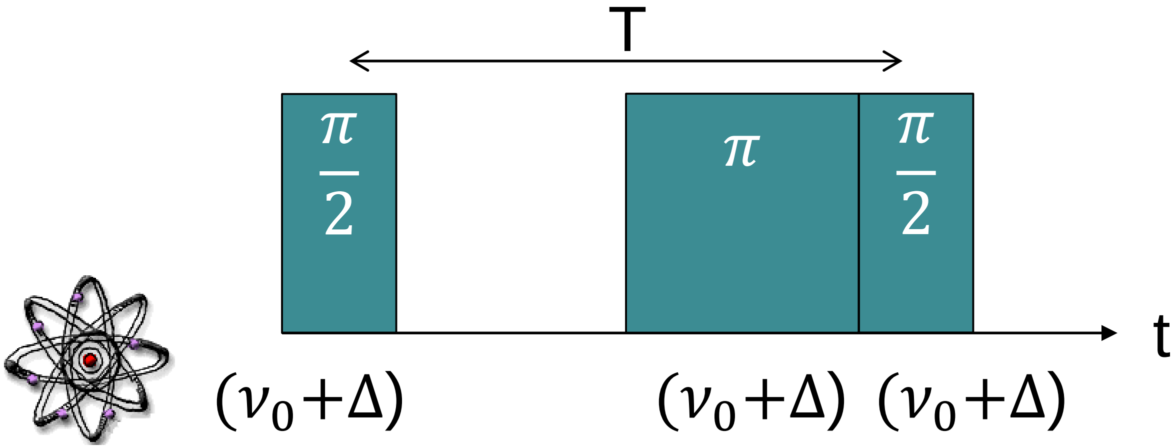


Can interleave high/low power probes and extrapolate, or use a modified Ramsey scheme



Modified Ramsey schemes





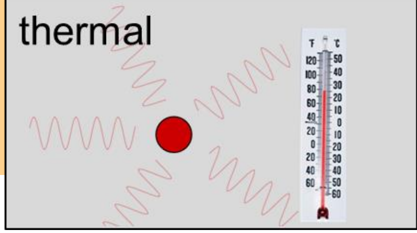
Ekkehard Peik

- Problem: need to step frequency by exactly Δ to avoid shift
- Or else:

- Insert π pulse to reduce sensitivity of any error in the step
- Invert phase of π pulse to reduce sensitivity to pulse area
- Interleave sequences with 90 deg phase shifts on $\pi/2$ pulses
- Interleave sequences with different T – auto balanced Ramsey

PRL **109**, 213002 (2012); Phys. Rev. A **93**, 010501 (2016); PRL **120**, 053602 (2018)

Stark shift – thermal



- One of the hardest shifts to determine
- Blackbody radiation:

  $< E^2 > = 8.55 \times 10^{-5}\, T^4$ (V²/m²)

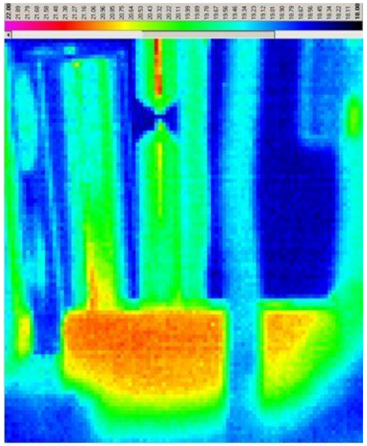
  $h\Delta f = -\frac{1}{2}\Delta\alpha_{sc}E^2$

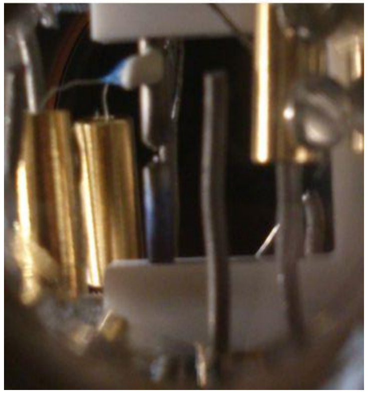
- Difficult to characterise E-field when it's not a blackbody distribution

- Ion mostly 'sees' trap structure. Need to characterise temperature rise and emissivities of all components, or use thermal imaging

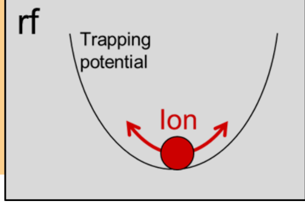
- $\Delta\alpha_{sc}$ (and hence BBR shift & uncertainty) generally smaller for ions than neutral atoms

Thermal image of trapping region (18-22°C)

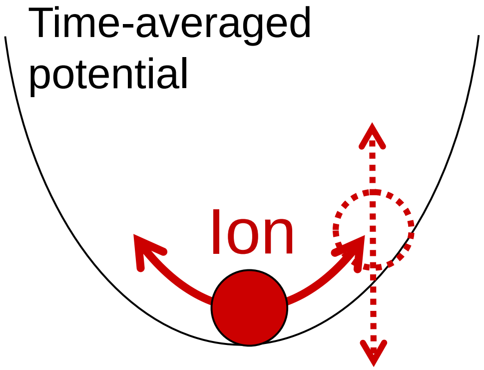




Stark shift – rf trapping



- Motion in the trap leads not only to Doppler shifts, but also to the ion experiencing a time-averaged non-zero electric field



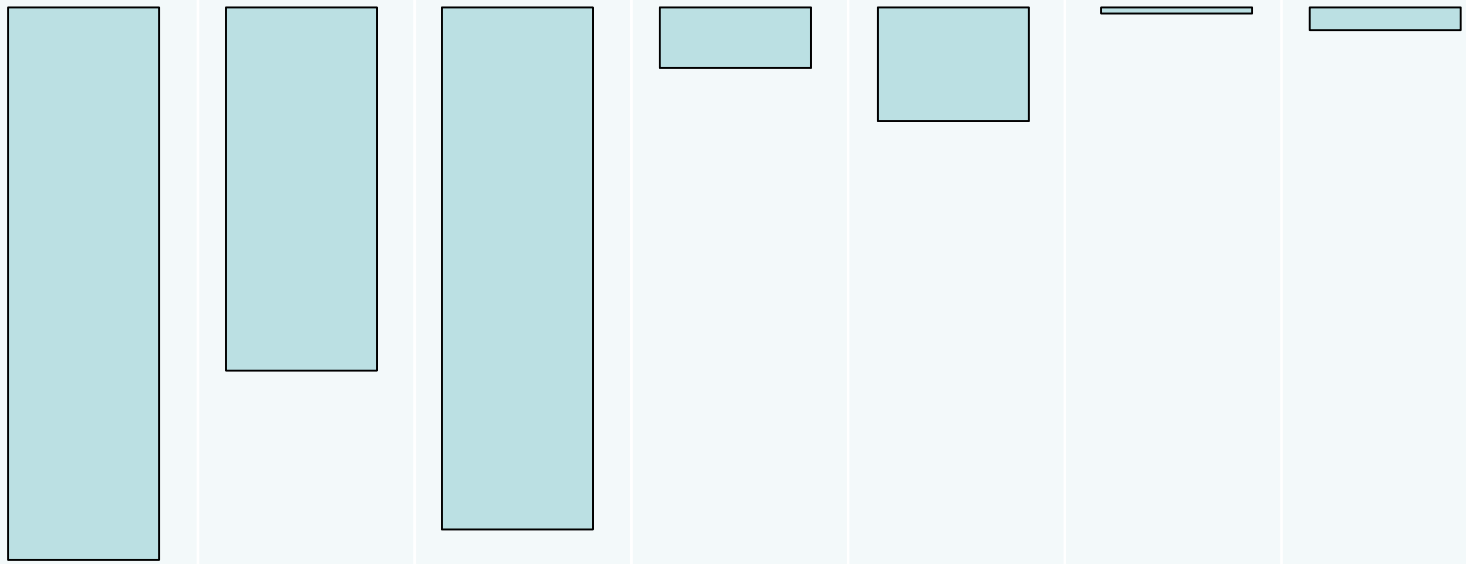
Secular motion

Micromotion

- Measure the motion to deduce:

Velocity → Amplitude of motion → $\langle E^2 \rangle$ → Stark shift

dc polarisabilities for different ions

	Ca^+	Sr^+	Yb^+ E2	Yb^+ E3	Hg^+	Al^+	In^+
$\Delta\alpha_{sc}$ 10^{-41} Jm^2/V^2	-73.0	-47.938	+69	+8.88	+15	+0.82	+3.3
$\Delta\alpha_{sc}$ relative magnitudes							
$\Delta\alpha_{ten}$ 10^{-41} Jm^2/V^2	-24.51	-78.6	-136	+1	-3	0	0

- Note that some polarisabilities are negative and some are positive

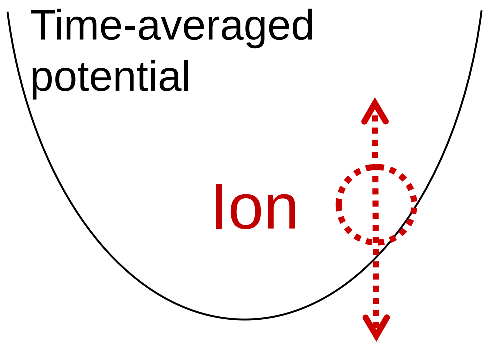
[A. Ludlow *et al.* Rev. Mod. Phys., **87**, No. 2 (2015)]

Magic trap drive frequency for Sr^+ and Ca^+

- Micromotion will cause both a Doppler and a Stark shift

$$\frac{\Delta\nu_\mu}{\nu_0} = -\left(\frac{\Omega}{\omega_0}\right)^2 \left[1 + \frac{\Delta\alpha_0}{\hbar\,\omega_0}\left(\frac{m\,\Omega\,c}{e}\right)^2 \right] \sum_{x,y,z} R_i$$

Doppler Stark



- Sr^+ and Ca^+ have negative $\Delta\alpha$, so Doppler & Stark shifts cancel at the 'magic trap drive frequency'

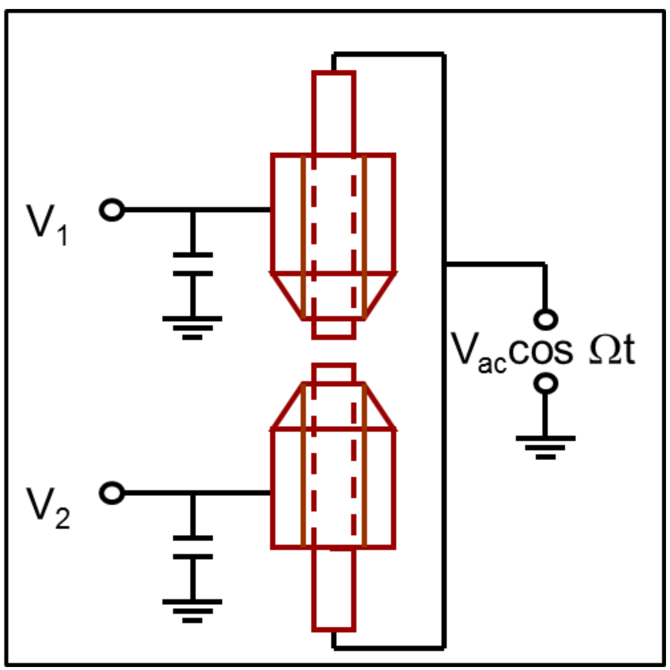
$$\Omega_0 = \frac{e}{m\,c}\sqrt{-\frac{\hbar\,\omega_0}{\Delta\alpha_0}}$$

- Sr^+: 14.4 MHz
- Ca^+: 24.6 MHz

[P. Dubé *et al.*, Phys. Rev. Lett. **112**, 173002 (2014)]

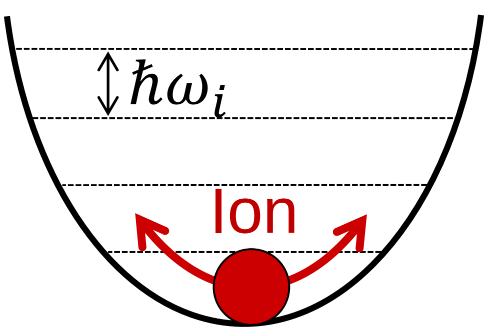
Designing traps to minimise shifts

- Motional effects → want low ion motional heating rate
  - Avoid noise on electrical signals and surface contaminants
  - Use large separation, d between rf electrodes (ion heating = $1/d^{\alpha}$, where $2 < \alpha < 4$)
  - Operate with large Ω (and hence V_{ac}) to achieve large ω_i



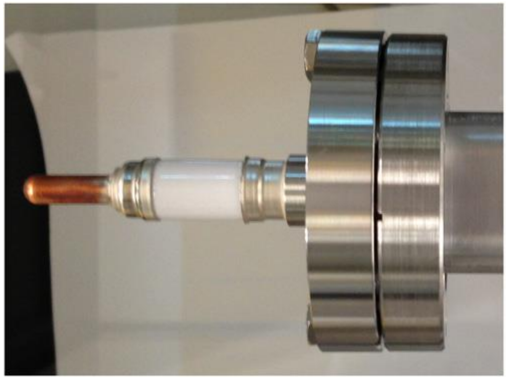
$$\omega_i \approx \frac{q_i \Omega}{2\sqrt{2}}$$

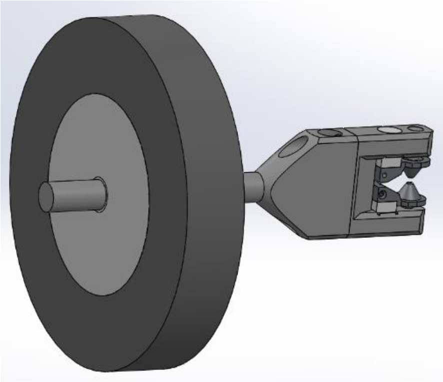
$$q_z = \frac{4e\varepsilon V_{ac}}{m r_o^2 \Omega^2}$$



Designing traps to minimise shifts

- Thermal E-fields → want low temperature rise
  - Operate with low V_{ac} to minimise dielectric heating, and increase separation between V_{ac} and ground



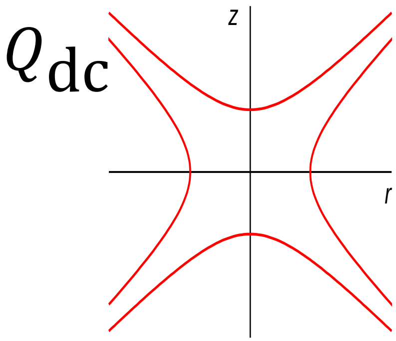


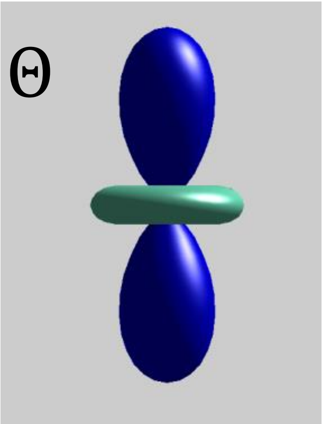
  - Careful choice of materials
  - Polish electrode surfaces to increase emissivity
- rf E-fields → want no residual fields at trap centre
  - Symmetric connections to rf electrodes
  - Compensation electrodes in pairs

Short break

Electric field gradients cause quadrupole shifts

- Electric field gradients interact with electric quadrupole moments





- States with ang. mom. $J \geq 1$ possess a quad. Mom., eg. $^2D_{5/2}$, $^3P_1$.
- Lattice clock transitions $^1S_0 - {}^3P_0$ are free from this effect

$$\Delta f \propto \; Q_{\mathrm{dc}} \cdot \Theta \cdot \left(3m_F^2 - F(F+1)\right)\left(3\cos^2\beta - 1\right)$$

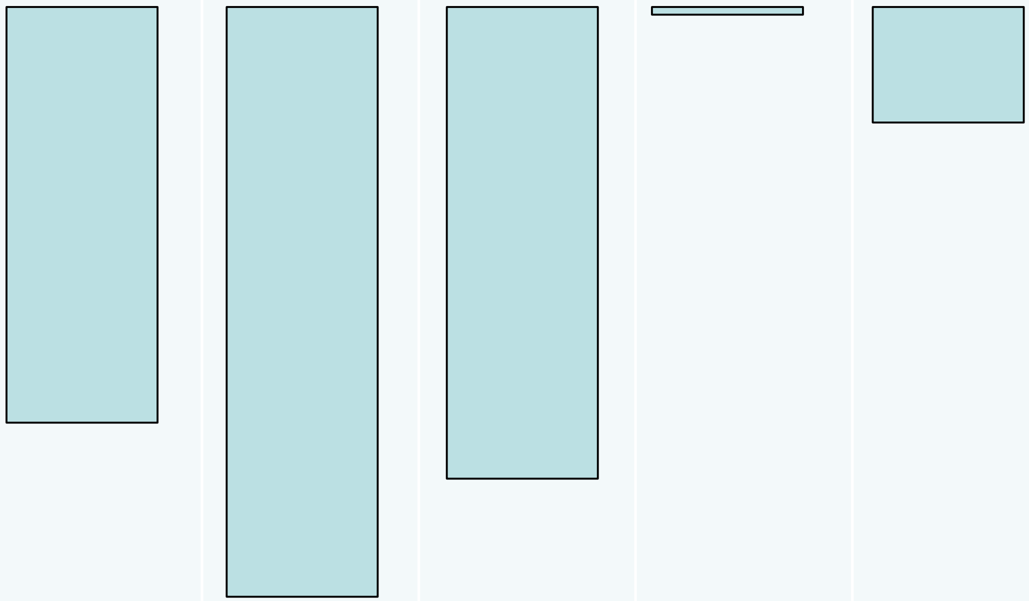
- Q_{dc}: quadrupole field gradient
- Θ: quadrupole moment of clock state
- m_F: magnetic qu. number of clock state
- β: angle between quadrupole field axis & quadrupole moment

Quadrupole moments for various systems

	Ca^+	Sr^+	Yb^+ E2	Yb^+ E3	Hg^+	Al^+	In^+
$\Theta\ (ea_0^2)$	1.83	2.6	2.08	-0.041	-0.510	0	0

Θ

relative magnitudes



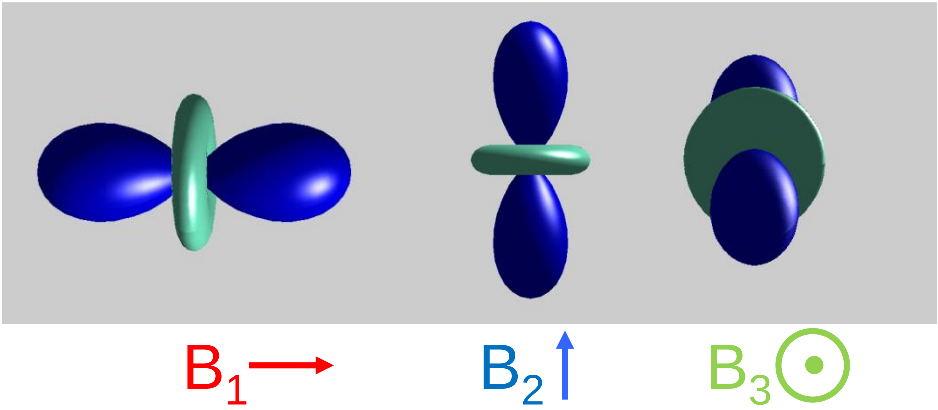
- No quadrupole moment in $^{115}In^+$ or $^{27}Al^+$ because $^1S_0 - {}^3P_0$
- For other systems, shift may be several Hz or more, so must be nulled.

[A. Ludlow *et al.* Rev. Mod. Phys., **87**, No. 2 (2015)]

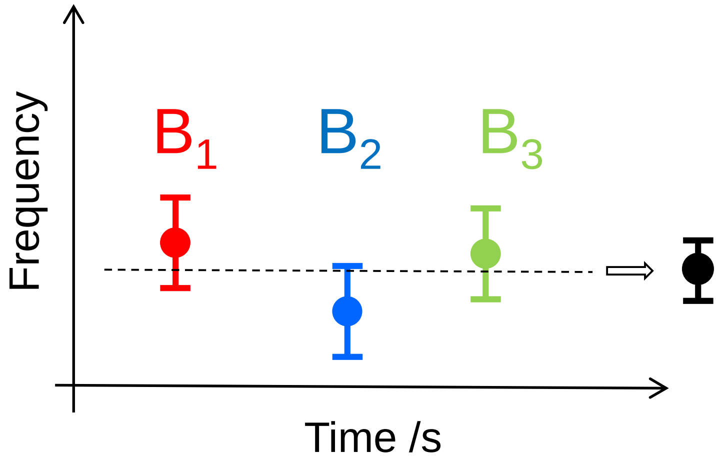
Nulling the quadrupole shift

$$\Delta f \propto (3\cos^2\beta - 1)$$

β = angle between B-field and E-field gradient



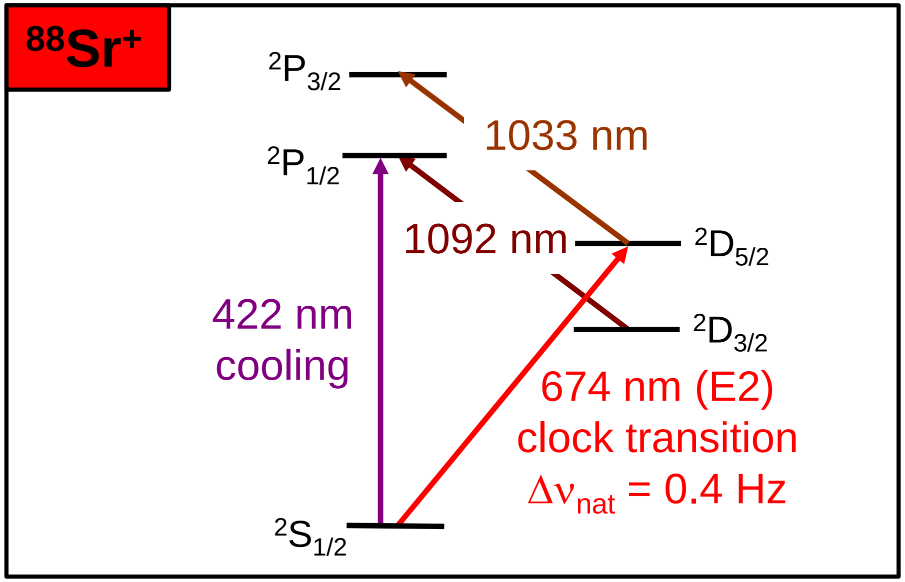
- Measure in three orthogonal B-field directions
- (Alternatively, measure 3 different pairs of Zeeman states)
- $(3\cos^2\beta - 1)$ averages to zero

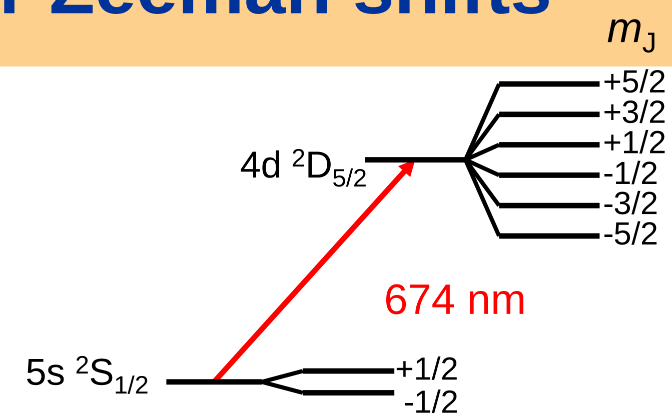


Outline

- Atomic clocks based on caesium
- The role of optical clocks
- Single-ion optical frequency standards
- **Minimising systematic frequency shifts**
  - Motion
  - Electric fields
  - **Magnetic fields**
  - Gravity
- Optical frequency metrology
  - Stability, absolute frequencies and ratios
- Summary and future perspectives

Magnetic fields – 1^{st} order Zeeman shifts

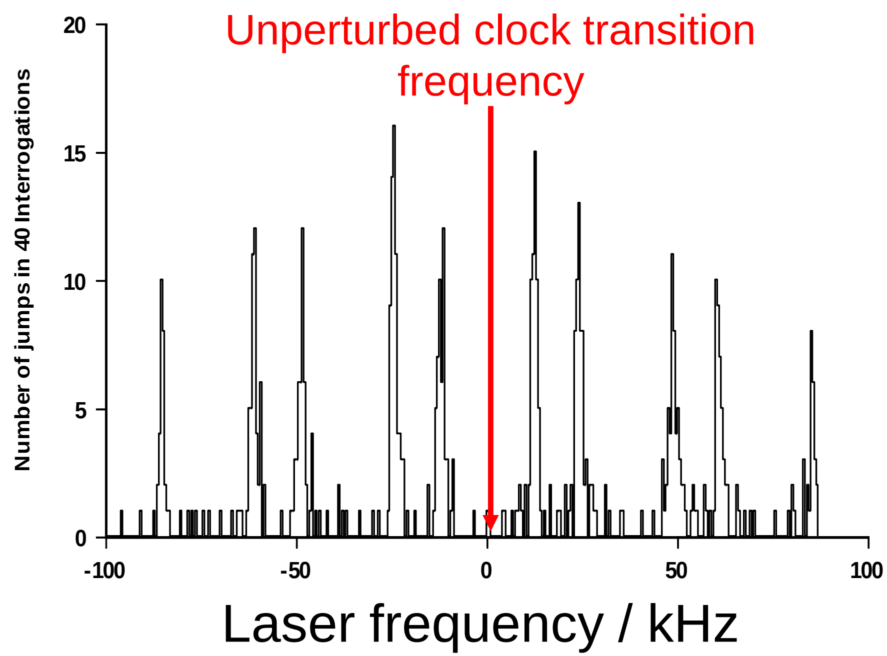




All components exhibit a linear Zeeman shift

~ 10kHz in 1μT for $^{88}Sr^{+}$

Eliminated by probing two Zeeman components symmetrically placed about line centre, and taking the average frequency.



Magnetic fields – 2nd order Zeeman shifts

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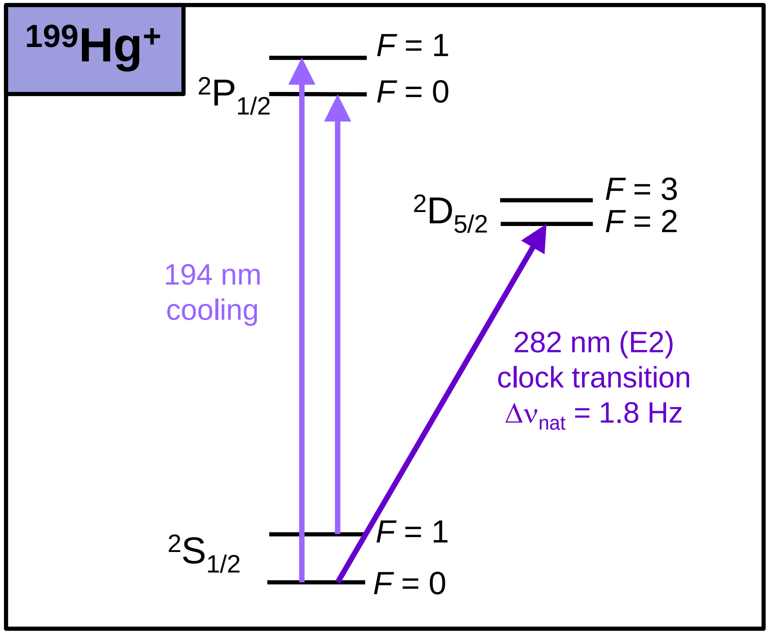


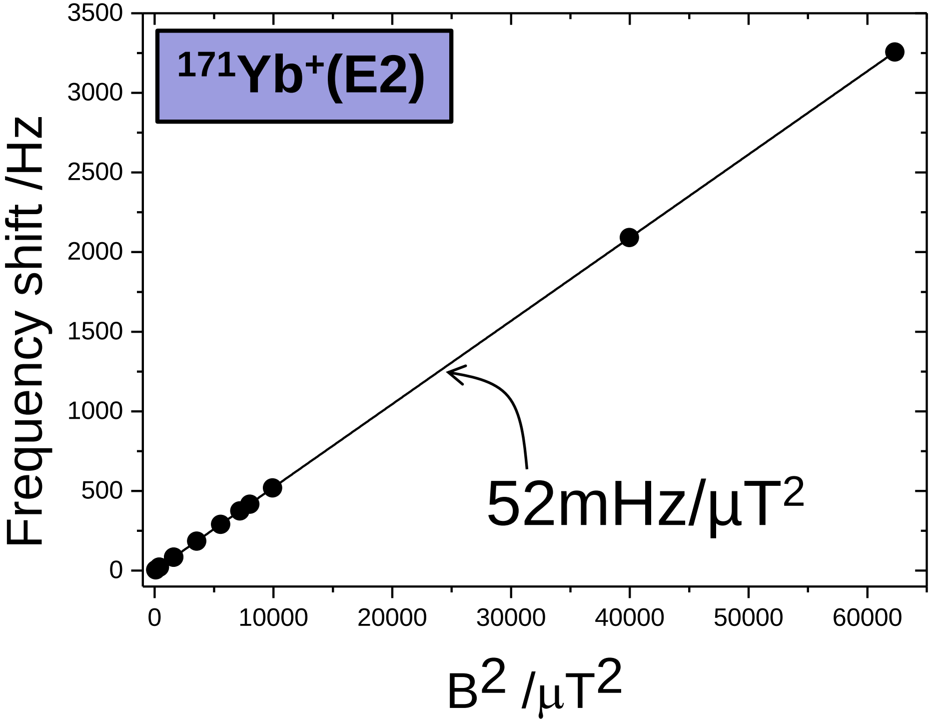
For odd isotope ions, with half-integer nuclear spin, can probe

$m_F = 0 – m_F = 0$,

so there is no 1st order Zeeman shift.

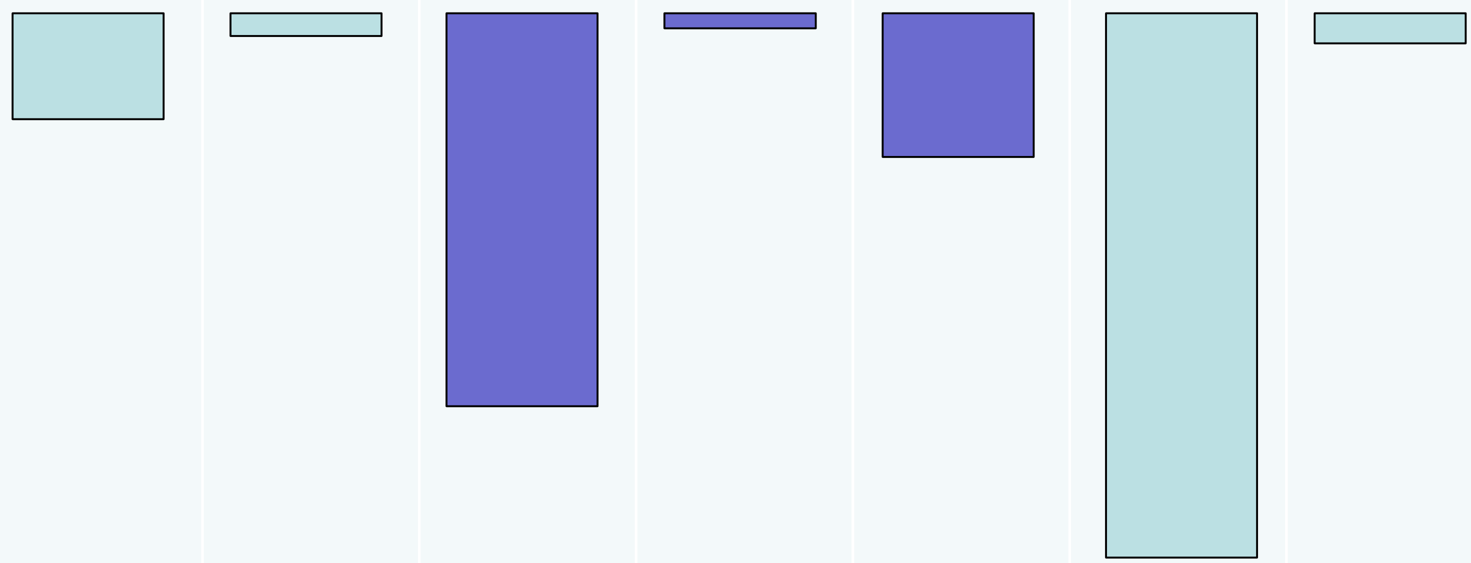
2nd order Zeeman shift,

$\Delta\nu = \kappa B^2$



2^{nd} order Zeeman shift for different ions

	Ca^+	Sr^+	Yb^+ E2	Yb^+ E3	Hg^+	Al^+	In^+
2^{nd} order coefficient Hz/mT^2	+14.355	+3.1223	+52,096	-2,030	-18,900	-71.988	+4.09
2^{nd} order Zeeman relative magnitudes							



- The purple bars are 1,000$\times$ larger

[A. Ludlow *et al.* Rev. Mod. Phys., **87**, No. 2 (2015)]

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Gravity potential affects clock frequency

- General relativity: time in different gravity potentials will run at different rates

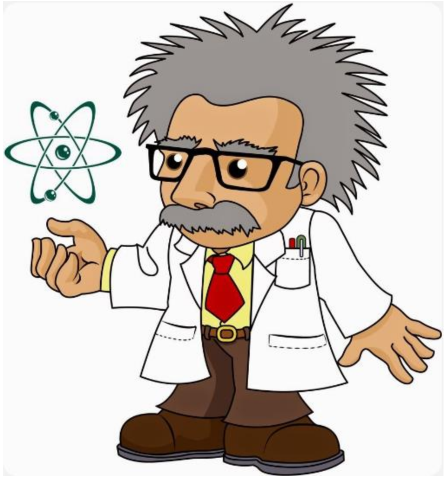
- Frequency shift $$\frac{\Delta f}{f_0} = \frac{g\Delta h}{c^2}$$

g = local acceleration due to gravity
Δh = height above reference level
c = speed of light

- Fractional change = 10^{-16} for every 1 m change in height at Earth's surface

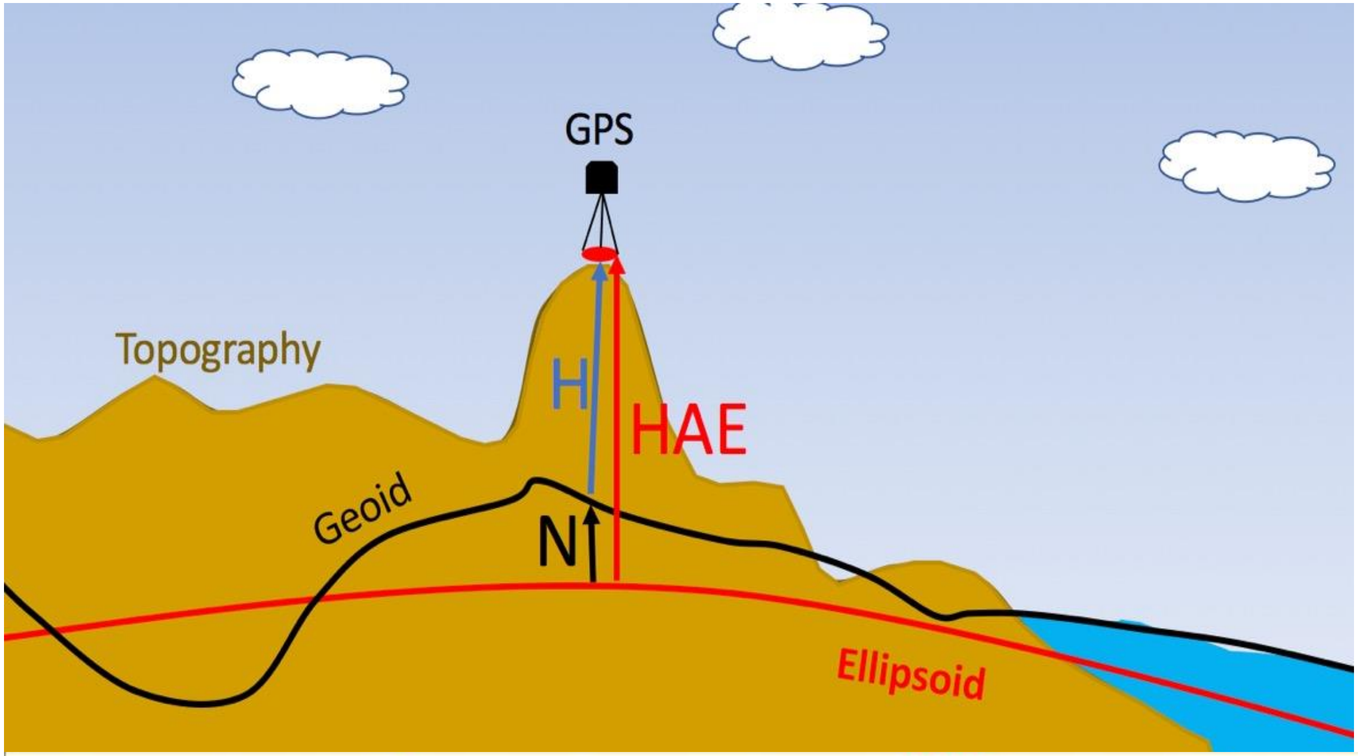
- 10^{-18} clocks sensitive to 1 cm height changes

Fact:
Your head ages faster than your feet by a couple of ns per year!



Potential above reference height

- Geoid - equipotential reference surface, local fluctuations
- GNSS gives height relative to ellipsoid
- Local modelling for difference between ellipsoid and geoid



Orthometric ht. (H) = Ht. Above Ellipsoid (HAE) – Geoid ht. (N)

Measuring gravity potentials at NMIs

- Surveys in 2013 – 2014 at INRIM, NPL, PTB, SYRTE
- 1 – 3 absolute gravity measurements at each site
- 35 – 99 relative gravity measurements in surrounding area
- Accuracy of potential above geoid is about 0.25 m^2/s^2 (equivalent to 2.5 cm)

PTB

Summary of fundamental frequency shifts

NPL National Physical Laboratory

Doppler shifts	– due to motion of ion in trap
Stark shifts	– due to external electric fields (optical, thermal, rf trap)
Electric quadrupole shift	– due to electric field gradients
Zeeman shifts	– due to external magnetic fields
Gravitational redshift	– due to location of ion

Uncertainty budget

- PTB Yb^+ E3 – single-ion clock with best accuracy

Effect	$\delta\nu/\nu_0$ (10^{-18})	u/ν_0 (10^{-18})
Second-order Doppler shift	−3.7	2.1
Blackbody radiation shift	−70.5	1.8
Probe light related shift	0	1.1
Second-order Zeeman shift	−40.4	0.6
Quadratic dc Stark shift	−1.2	0.6
Background gas collisions	0	0.5
Servo error	0	0.5
Quadrupole shift	0	0.3
Total	−115.8	3.2

[N. Huntemann *et al.*, PRL **116**, 063001 (2016)]

Which ions make the best clocks?

	Ca^+	Sr^+	Yb^+ E2	Yb^+ E3	Hg^+	Al^+	In^+
Linewidth, $\Delta\nu$	✗	✗	✗	✓	✗	✓	✗
Stark shift, $\Delta\alpha_{SC}$	✗	✗	✗	(✓)	✓	✓	✓
Quadrupole shift	✗	✗	✗	✓	✗	✓	✓
2nd order Zeeman	✓	✓	✗	✗	✗	✓	✓
Simple lasers	✓	✓	✓	✓	(✓)	✗	✗

- Compact / simple systems – Ca^+, Sr^+
- Low systematic frequency shifts – In^+, Al^+
- High stability – Yb^+(E3), Al^+

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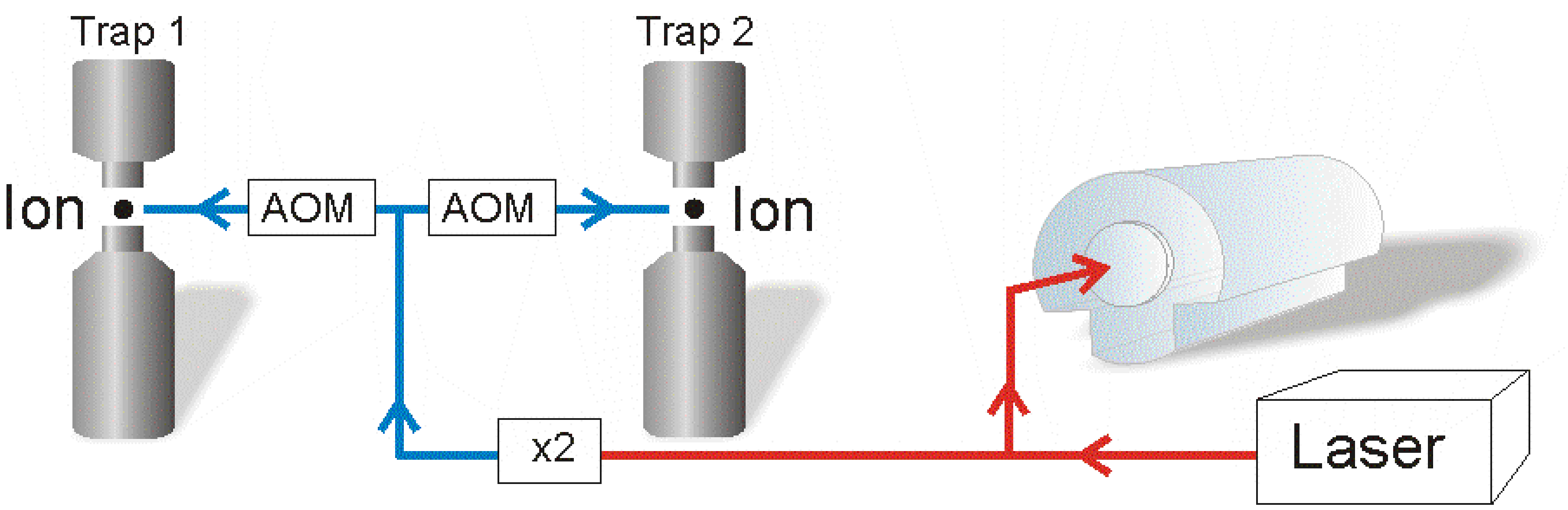
Measuring reproducibility and stability

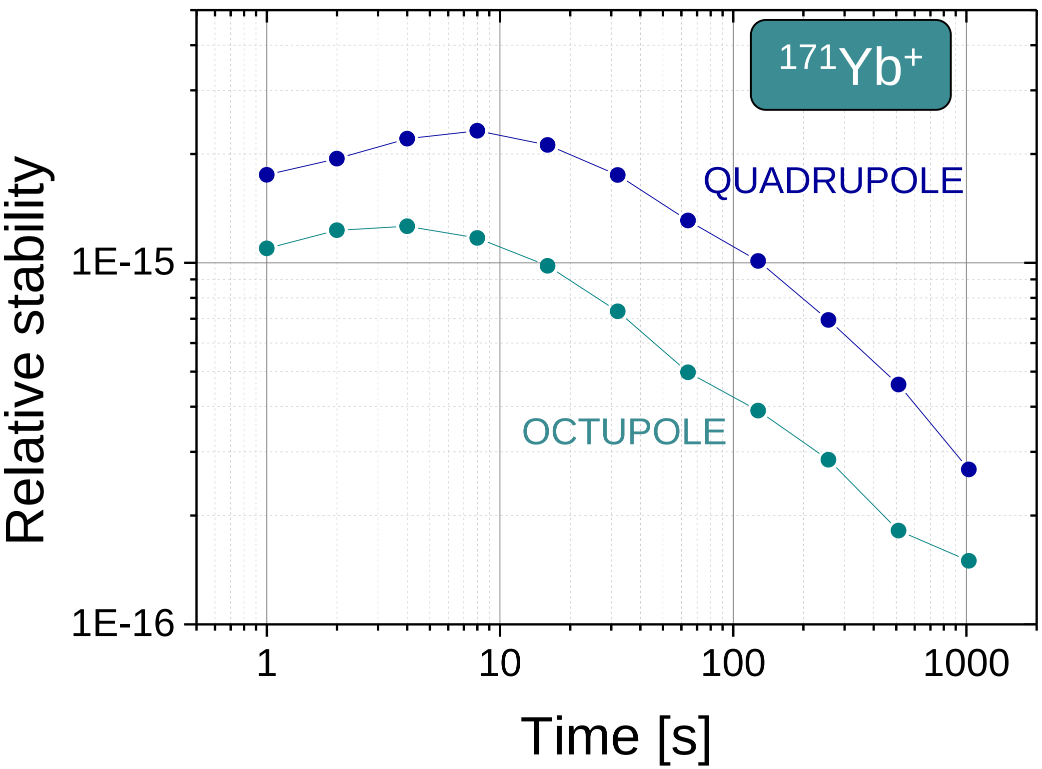
Fractional instability $$\sigma(\tau) = \frac{\Delta\nu}{\nu}\frac{\eta}{(S/N)}\sqrt{\frac{T}{\tau}}$$

- Compare two independent optical frequency standards
- Measure $(f_1 - f_2)$ for a period of time, repeatedly.



Stability depends on linewidths

Fractional instability $$\sigma(\tau) = \frac{\Delta\nu}{\nu} \frac{\eta}{(S/N)} \sqrt{\frac{T}{\tau}}$$

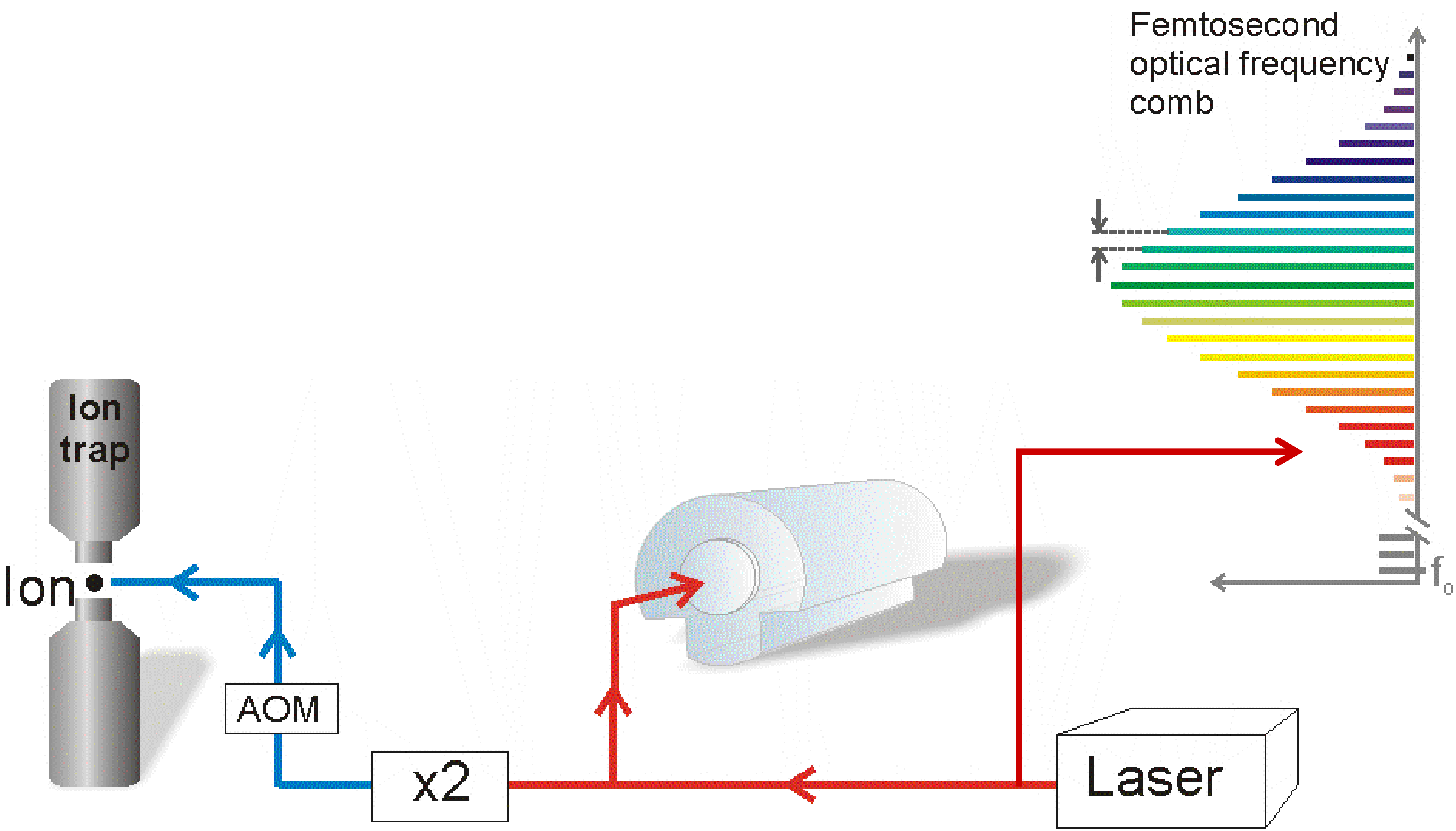


- Octupole stability limited only by laser linewidth as Yb^{+}(E3) natural linewidth ~ 1 nHz

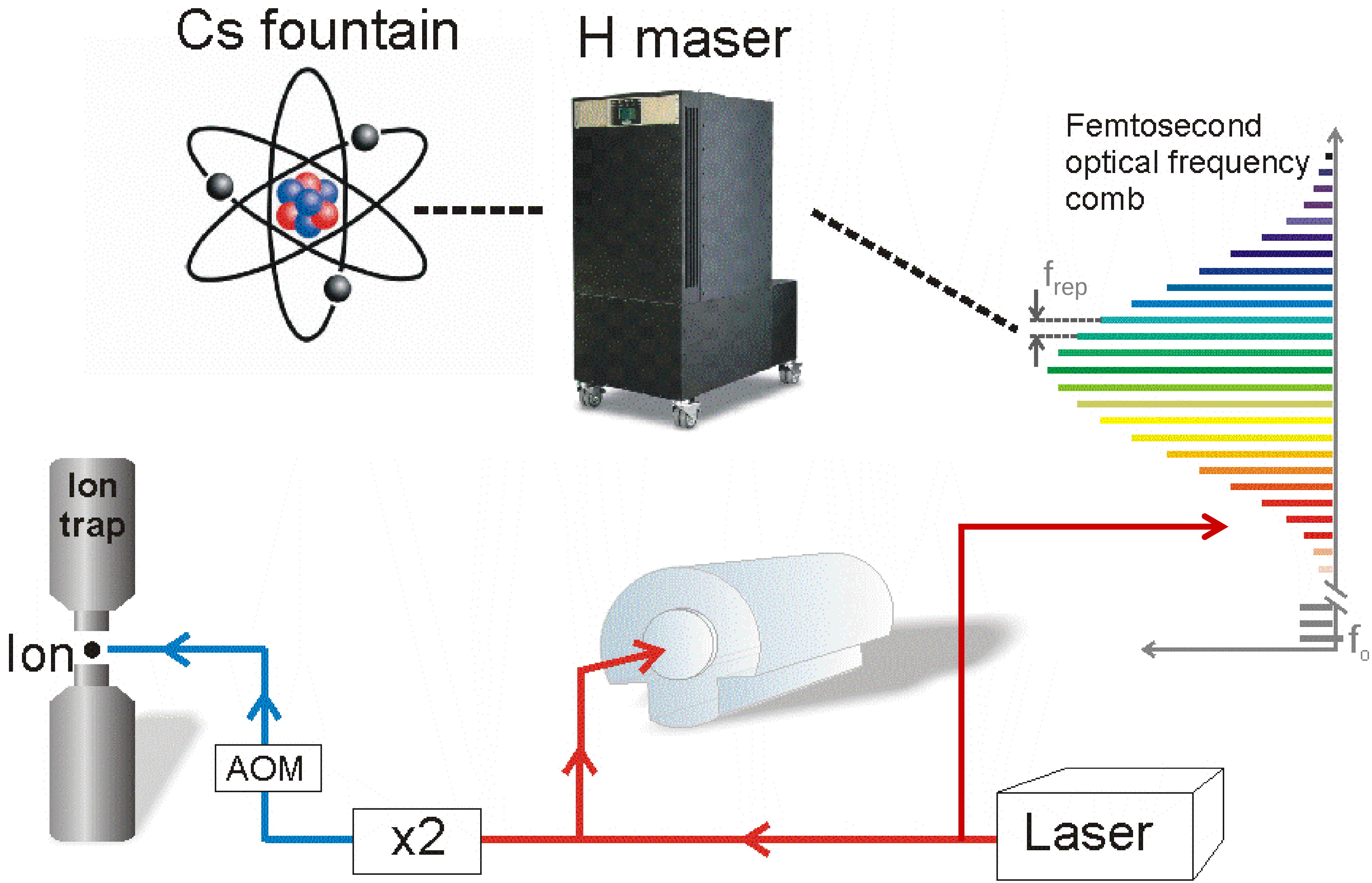
Octupole:
$\sigma(\tau) = 5 \times 10^{-15} / \sqrt{\tau}$
100ms probe pulses
10Hz linewidths

Quadrupole:
$\sigma(\tau) = 1 \times 10^{-14} / \sqrt{\tau}$
30ms probe pulses
30Hz linewidths

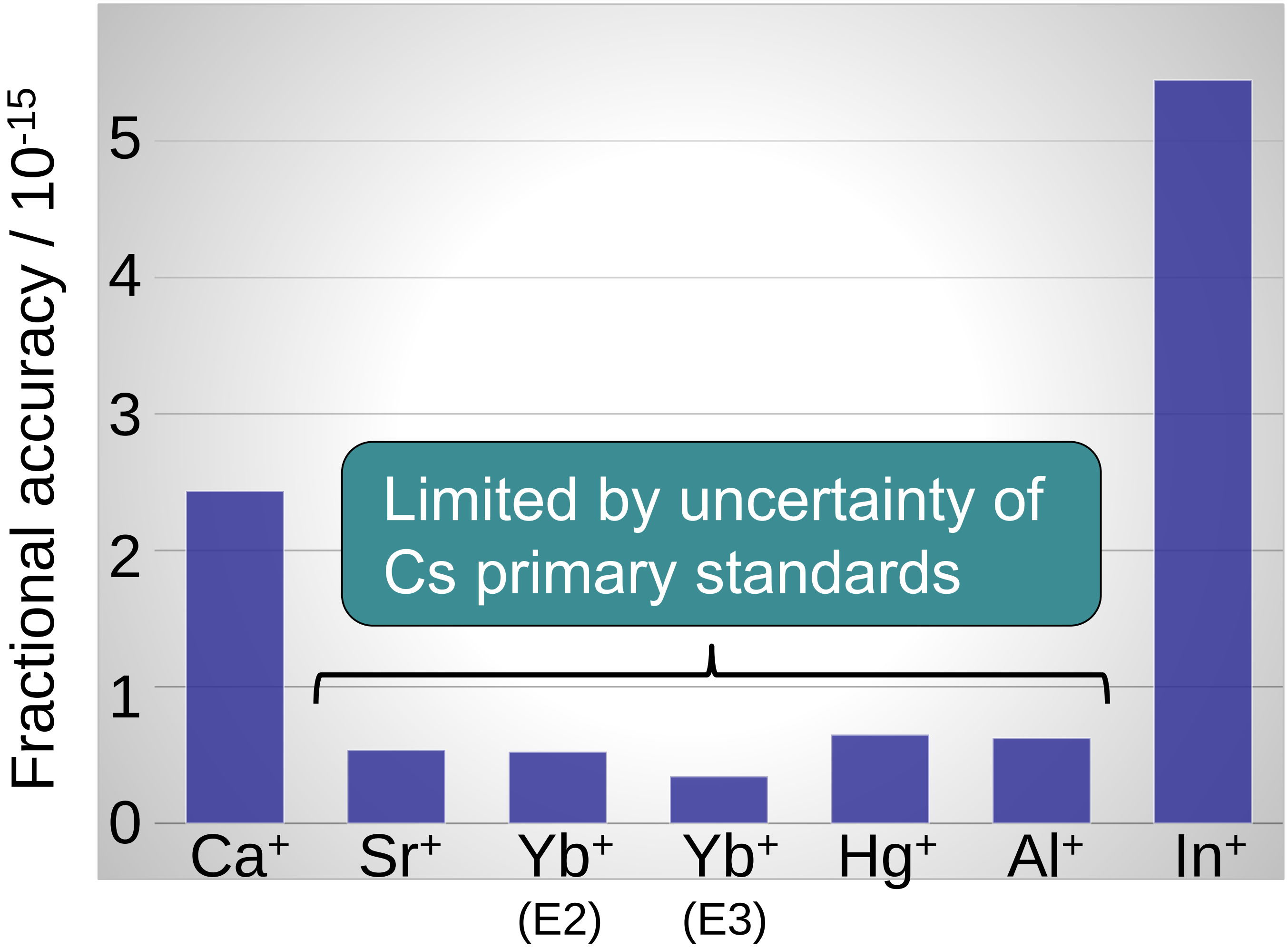
Measuring the absolute frequency



Measuring the absolute frequency



Absolute frequency measurements



[Ca^+: PRL **102**, 023002 (2009); Sr^+:PRA **89**, 050501 (2014); Yb^+(E2): PRA **89**, 023820 (2014); Yb^+(E3): EFTF2018 proceedings; Hg^+: Appl. Phys. B, **89**, 167 (2007); Al^+: Science **319**, 1808 (2008); In^+: Opt. Exp. **25**, 11725 (2017).]

Secondary representations of the second

- Optical frequency standards can be used to realise the SI second (although uncertainty cannot be better than Cs primary standard)
- List of secondary representations of the second now includes eight optical frequency standards

Atom or ion	Transition	Wavelength	Recommended fractional uncertainty
^{199}Hg	$^1S_0 - {}^3P_0$	266 nm	5×10^{-16}
$^{27}Al^+$	$^1S_0 - {}^3P_0$	267 nm	19×10^{-16}
$^{199}Hg^+$	$^2S_{1/2} - {}^2D_{5/2}$	282 nm	19×10^{-16}
$^{171}Yb^+$	$^2S_{1/2} - {}^2D_{3/2}$	436 nm	6×10^{-16}
$^{171}Yb^+$	$^2S_{1/2} - {}^2F_{7/2}$	467 nm	6×10^{-16}
^{171}Yb	$^1S_0 - {}^3P_0$	578 nm	5×10^{-16}
$^{88}Sr^+$	$^2S_{1/2} - {}^2D_{5/2}$	674 nm	15×10^{-16}
^{87}Sr	$^1S_0 - {}^3P_0$	698 nm	4×10^{-16}

Measuring optical frequency ratios

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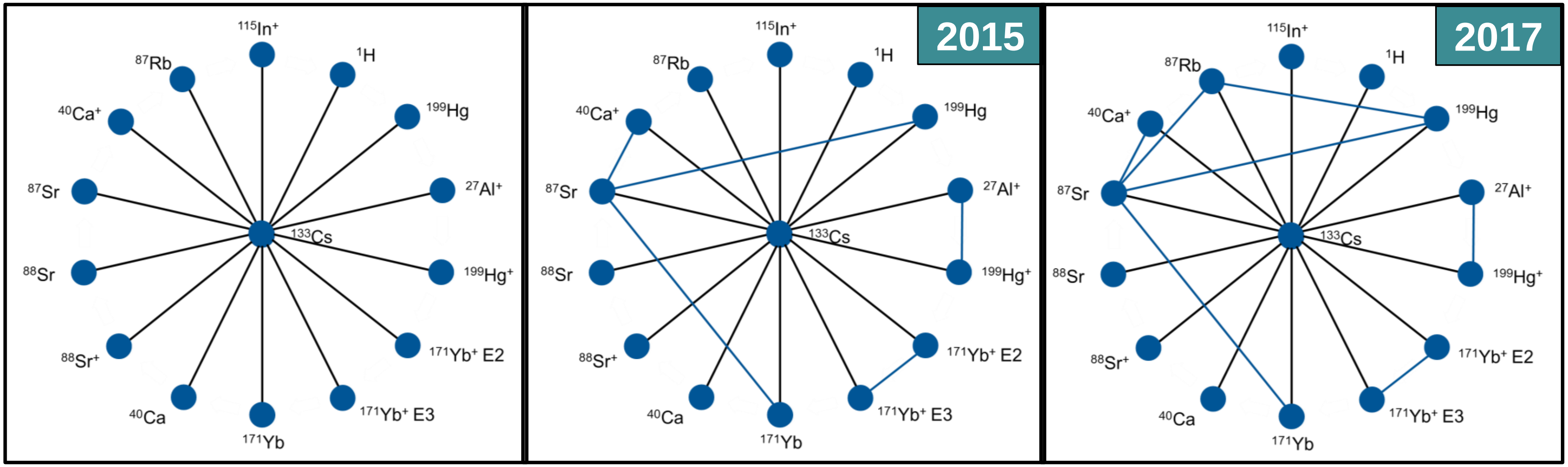
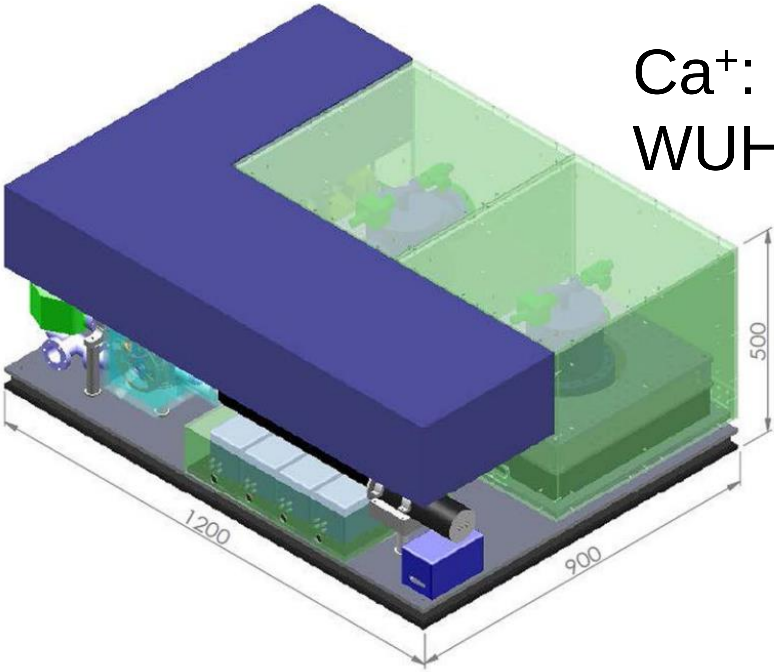


Optical frequency ratios between labs

- Local comparisons
- Transportable clocks
- Satellite links
  (Two-way and GPS)
- Optical fibre links



Ca^+: 7.8×10^{-17}
WUHAN

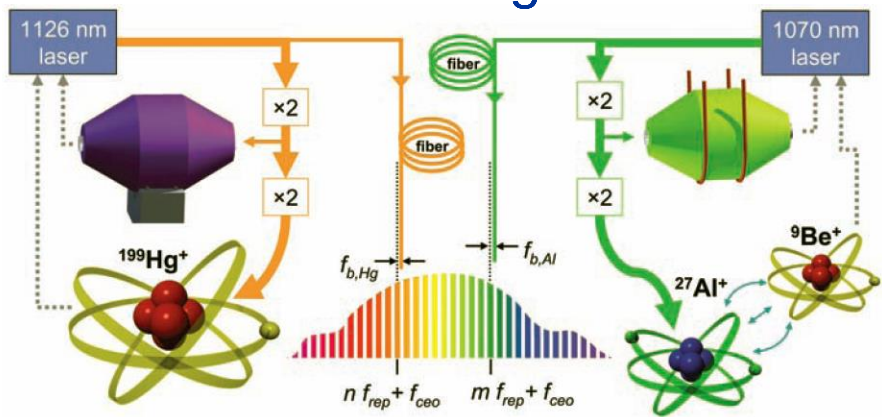
[J. Cao et al., Appl. Phys. B 123,112 (2017)]

Ion optical ratios – variation of constants

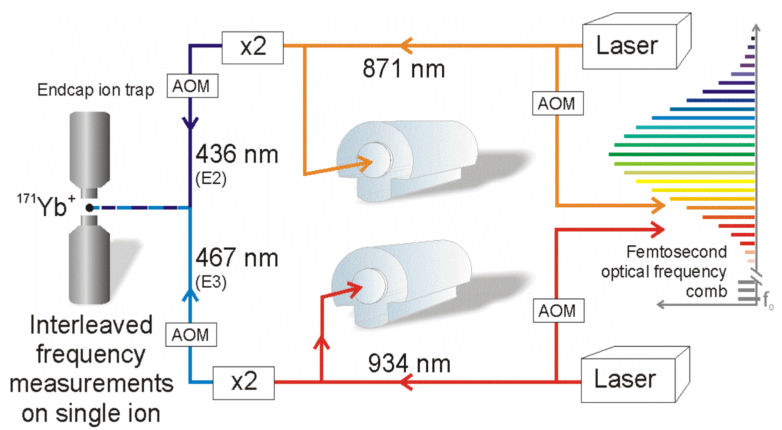
- Sensitivity to variation of fine structure constant

Ion	Relative sensitivity	*A*
Sr^+		0.43
Yb^+(E2)		1.00
Yb^+(E3)		-5.95
Hg^+		-2.94
Al^+		0.008
In^+		0.18

NIST - $^{199}Hg^+$: $^{27}Al^+$



NPL, PTB - $^{171}Yb^+$(E3):(E2)



[Rosenband *et al*., Science **319**, 1808 (2008);
Godun et al., PRL 113, 210801 (2014); Huntemann et al., PRL 113, 210802 (2014)]

Outline

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- Atomic clocks based on caesium
- The role of optical clocks
- Single-ion optical frequency standards
- Minimising systematic frequency shifts
  - Motion
  - Electric fields
  - Magnetic fields
  - Gravity
- Optical frequency metrology
  - Stability, absolute frequencies and ratios
- **Summary and future perspectives**

Summary of ion optical clocks

- ✓ Systematic frequency shifts are generally low
- ✓ Laser systems generally simple
- ✓ Traps can be compact
- ✓ Sensitive tools for fundamental physics tests
- ✗ Single ions give poor frequency instability

Fractional instability $$\sigma(\tau) = \frac{\Delta\nu}{\nu}\frac{\eta}{(S/N)}\sqrt{\frac{T}{\tau}}$$

- Need to reduce instability:

(a) Increase probe time to reduce $\Delta\nu\sqrt{T}$ – no limit for Yb^{+}(E3)

(b) Increase ν by choosing an ion with UV transitions

(c) Increase S/N by increasing number of ions

Ions with UV transitions increase ν_0

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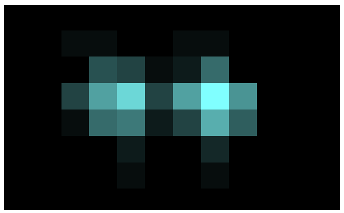
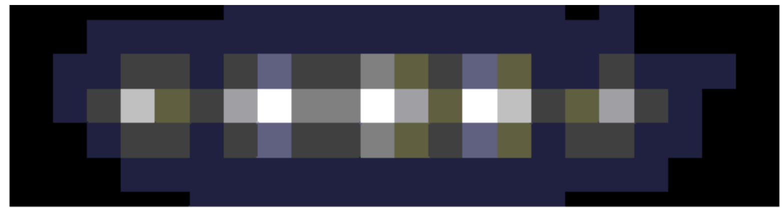
	Highly charged ions	Thorium
Example	Ir^{17+}, sympathetically cooled	$^{229}Th^{2+}$, $^{229}Th^{3+}$
UV transition	Electrons more tightly bound	Nuclear transition
Low systematics	Transitions shift more towards the UV, hence smaller dc and rf electric polarisabilities	Nucleus is highly isolated from environment due to electron cloud
Highly relativistic	Larger relativistic shifts provide enhanced sensitivity to variation of constants	Sensitivity to variation of constants speculated to be enhanced by as much as 5–6 orders of magnitude

Multi-ion clocks improve SNR by $1/\sqrt{N}$

- Challenge: to retain homogeneous frequency shifts across whole string of ions





- Linear traps - dc electric field gradients along axis, so need ions with low quadrupole moment (eg Al^+, In^+, Yb^+(E3))
- Entanglement schemes give further $1/\sqrt{N}$ improvement; still many forms of decoherence present a challenging obstacle

	Stability	**Time to reach 1×10^{-18}**
Single ion	$1 \times 10^{-15} / \sqrt{\tau}$	10^6 s = 12 days
10 uncorrelated ions ($1/\sqrt{N}$)	$3 \times 10^{-16} / \sqrt{\tau}$	10^5 s = 1.2 days

Conclusion

Ions are still very promising candidates for optical clocks

Further reading

National Physical Laboratory

Text books

- F. Riehle, “Frequency Standards”, Wiley-VCH
- C. J. Foot, “Atomic Physics”, Oxford University Press

Publications

- A. D. Ludlow, M. M. Boyd, Jun Ye, E. Peik, P. O. Schmidt, “Optical Atomic Clocks”, Rev. Mod. Phys. **87**, 637 (2015)

- OC18 specifications document ... coming soon to www.oc18.eu