Trapped Ion Optical Clocks

Rachel Godun

“Optical Clock School 2018”, Gressoney la Trinité, 10th – 14th Sep 2018
Optical clocks with $10^{-18}$ uncertainty

European consortium working to reach $10^{-18}$ uncertainties by 2019
Outline

- Atomic clocks based on caesium
- The role of optical clocks
- Single-ion optical frequency standards
- Minimising systematic frequency shifts
  - Motion
  - Electric fields
  - Magnetic fields
  - Gravity
- Optical frequency metrology
  - Stability, absolute frequencies and ratios
- Summary and future perspectives
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Atomic clock frequency standard basics

- Tune the frequency of the radiation to drive an atomic transition

\[ f_0 = 9.192\ 631\ 770\ \text{GHz} \]
Introduction of atomic time

1955
First caesium atomic frequency standard developed at NPL by Essen & Parry, accurate to 1 part in $10^{10}$

1967
The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.

2018
The best caesium fountain primary frequency standards now have accuracies of $1 - 2 \times 10^{-16}$
Improvements in caesium atomic clocks

- Essen's Cs clock
- Cs redefinition of the second
- Cs fountains
Cs fountain primary frequency standards

NPL-CsF2
INRIM ITCsF2
LNE-SYRTE FO2-Cs

NIST-F2
PTB-CSF2
NPL Commercial
Cs clocks are widely used

International time scale

Navigation

Network synchronisation

Patrizia Tavella
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Optical clock basics

- Caesium clock
  - Microwave radiation
    - $\sim 10^{10} \text{ Hz}$

- Optical clock
  - Optical radiation
    - $\sim 10^{15} \text{ Hz}$

Yann Le Coq
Performance of a frequency standard

Accuracy
Level of offset from correct frequency (systematic uncertainties)

Stability
Level of frequency fluctuations over time (statistical uncertainties)

\[ f' \]

\[ f_0 \]

Frequency

Time
Advantage of optical frequency standards

Stability:

Fractional instability

\[
\sigma(\tau) = \frac{\Delta \nu}{\nu} \frac{\eta}{(S/N)} \sqrt{\frac{T}{\tau}}
\]

<table>
<thead>
<tr>
<th></th>
<th>Microwave</th>
<th>Optical</th>
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<tbody>
<tr>
<td>(\Delta \nu)</td>
<td>(\sim 1 \text{ Hz})</td>
<td>(\sim 1 \text{ Hz})</td>
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<td>(\nu)</td>
<td>(\sim 10^{10} \text{ Hz})</td>
<td>(\sim 10^{15} \text{ Hz})</td>
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<tr>
<td>reach (10^{-15})</td>
<td>(\sim 1 \text{ day})</td>
<td>(\sim \text{ seconds})</td>
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Accuracy:

<table>
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<tr>
<th></th>
<th>Microwave</th>
<th>Optical</th>
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<tbody>
<tr>
<td></td>
<td>(\sim 10^{-15} - 10^{-16})</td>
<td>(\sim 10^{-17} - 10^{-18})</td>
</tr>
</tbody>
</table>
Improvements in optical clocks

- Microwave
- Optical (absolute frequency)
- Optical (estimated systematic uncertainty)

Graph showing improvements over time with markers for various technologies and years.
What difference does this make?
Impact of optical clocks

- Greater accuracy and stability opens up yet more opportunities for atomic clocks

Top-level SI

Fundamental physics  Geodesy  Astronomy, GNSS
Stabilising frequency to an atomic reference

Optical radiation

~ $10^{15}$ Hz

Laser cooling  State prepare  Probe clock transition  Detect

Steer frequency
What could possibly go wrong?

- Whether neutral atom or ion, need to think about:
  Fundamental: motion, E-fields, B-fields, gravitational effects

- Each setup may also have other issues to avoid
  Technical: shutter leakage / collisions / servo offsets / frequency chirps …

- Single ions are good because almost an isolated atom at rest

- Atoms good because large number gives better S/N

\[
\sigma(\tau) = \frac{\Delta \nu}{\nu} \frac{\eta}{(S/N)} \sqrt{\frac{T}{\tau}}
\]
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Ion clocks

- Nobel Prize 1989
  Dehmelt, Paul, Ramsey

- Dehmelt proposed using ions for optical frequency standards and ‘electron shelving’ for detection

![Paul trap diagram]

Narrow reference ("clock") transition

\[ \tau \approx 10 \text{ ns} \]

Cooling transition

\[ \tau \approx 1 \text{ s} \]

Detected photons (1000/s)

Ion being repeatedly driven

Ion in long lived state

Candidate systems (1)

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
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<tbody>
<tr>
<td>H</td>
<td>He</td>
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<tr>
<td>Li</td>
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<td>Na</td>
<td>Mg</td>
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<td>K</td>
<td>Ca</td>
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<td>Rb</td>
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<td>Cs</td>
<td>Ba</td>
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<td>Fr</td>
<td>Ra</td>
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</tbody>
</table>

Ions with alkali-like or quasi-alkali-like atomic structure
Alkali-like systems

199Hg⁺

- **2P₃/2**: F = 2 → F = 1
- **2P₁/2**: F = 1 → F = 0
- **2D₅/2**: F = 3 → F = 2

- 194 nm cooling
- 398 nm
- 282 nm (E2) clock transition
- $\Delta \nu_{nat} = 1.8$ Hz

88Sr⁺

- **2P₃/2**: F = 1 → F = 0
- **2P₁/2**: F = 2 → F = 1
- **2D₅/2**: F = 3 → F = 2

- 1033 nm
- 1092 nm
- 422 nm cooling
- 674 nm (E2) clock transition
- $\Delta \nu_{nat} = 0.4$ Hz

40Ca⁺

- **2P₃/2**: F = 1 → F = 0
- **2P₁/2**: F = 2 → F = 1
- **2D₅/2**: F = 3 → F = 2

- 397 nm cooling
- 854 nm
- 866 nm
- 729 nm (E2) clock transition
- $\Delta \nu_{nat} = 0.14$ Hz

171Yb⁺

- **2P₁/2**: F = 2 → F = 1
- **2P₃/2**: F = 3 → F = 2
- **2D₅/2**: F = 4 → F = 3
- **2D₃/2**: F = 0 → F = 1

- 935 nm
- 639 nm
- 370 nm cooling
- 436 nm (E2) clock transition
- $\Delta \nu_{nat} = 3.1$ Hz
- 467 nm (E3) clock transition
- $\Delta \nu_{nat} \sim 1$ nHz!
Candidate systems (2)

<table>
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<td>71</td>
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Ions with atomic structure similar to alkaline earth elements.
Alkaline-earth-like systems

**\(^{115}\text{In}^+\)**

- **1\(^{1}\)P\(_1\)**
- **3\(^{3}\)P\(_2\)**
- **3\(^{3}\)P\(_1\)**
- **3\(^{3}\)P\(_0\)**

**Transition: 231 nm**
- Cooling: 159 nm (360 kHz)
- Clock transition: \(\Delta n_{\text{nat}} = 0.8 \text{ Hz}\)

**\(^{27}\text{Al}^+\)**

- **1\(^{1}\)P\(_1\)**
- **3\(^{3}\)P\(_2\)**
- **3\(^{3}\)P\(_1\)**
- **3\(^{3}\)P\(_0\)**

**Transition: 167 nm**
- Cooling: 267.0 nm (0.5 kHz)
- Clock transition: 267.4 nm
- \(\Delta n_{\text{nat}} = 8 \text{ mHz}\)

**\(^{9}\text{Be}^+\)** auxiliary ion

- **2\(^{2}\)S\(_{1/2}\)**
- **2\(^{2}\)P\(_{3/2}\)**

**Transition: 313 nm**
- Raman: 313 nm
- Cooling: 313 nm
- Coulomb interaction

\(F=1\), \(F=2\)
Aluminium ion clock

- Quantum logic spectroscopy

[Initialisation to ground states]

[Clock spectroscopy on Al^+]

[Transfer Al^+ internal superposition state to motional superposition state]

[Transfer motional superposition state to internal superposition state in Be^+]

[P.O. Schmidt et al., Science 309, 749 (2005)]
Principles of ion trapping

Quadrupole potential:

\[ V(t) = A(t) \left( r^2 - 2z^2 \right) \]

\[ V(t) = \varepsilon(V_{dc} + V_{ac} \cos \Omega t) \frac{(r^2 - 2z^2)}{2r_o^2} \]

Radiofrequency voltage applied to top and bottom electrodes

→ ion trapped in time-averaged pseudo-potential minimum
Stable solutions

- Pseudo-potential gives stable trapping for parameters in the shaded region

\[
\alpha_z = -\frac{8e\varepsilon V_{dc}}{mr_o^2\Omega^2}
\]

and

\[
q_z = \frac{4e\varepsilon V_{ac}}{mr_o^2\Omega^2}
\]
Motion of the trapped ion

Stable ion motion can be separated into two parts:

**Secular motion**
Thermal motion associated with time-averaged confining potential (characteristic frequencies $\omega_r$ and $\omega_z$)

**Micromotion**
Driven oscillatory motion at frequency $\Omega$
(vanishes at trap centre)

Mathematical solution for ion’s position:

$$z(t) = \bar{z} + z_a \cos(\omega_z t) - z_a \frac{q_z}{2} \cos(\omega_z t) \cos(\Omega t) - \bar{z} \frac{q_z}{2} \cos(\Omega t)$$
Ion traps for optical frequency standards

### Endcap traps

- $V_1$
- $V_2$
- $V_{ac\cos\Omega t}$

### Linear traps

- rf electrode
- dc electrode
- $V_{ac\cos\Omega t}$

### Ring traps

- Ion
- Ring
- End-cap
- RF drive

[Images and diagrams of ion trap configurations from PTB and NPL]
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Ion motion causes Doppler shifts

- Motion leads to Doppler shift through

\[ \frac{\Delta f}{f_0} = \frac{v}{c} \]

- For **unconfined** ion at 1mK, Doppler broadening gives FWHM of optical probe ~ 1MHz

- **Confined** ion in harmonic potential

- Sidebands negligible if weak modulation, i.e. small enough amplitude of motion

\[ d < \frac{\lambda}{\pi} \]  
Lamb-Dicke criterion  
- eliminates 1\textsuperscript{st} order Doppler broadening
Ion motion causes Doppler shifts

- 2nd order Doppler shifts (relativistic time dilation) still present so need to measure residual motion

\[
\frac{\Delta f}{f_0} = -\frac{\langle v^2 \rangle}{2c^2}
\]

Secular motion
Micromotion

Sideband-to-carrier ratio

Micromotion

RF-photon correlation technique:

[J. Keller et al., J. Appl. Phys. 118, 04501 (2015)]
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Electric fields cause Stark shifts

- External electric fields induce a dipole moment in the ion, then interact with that induced dipole as a 2nd order effect (shifts proportional to $E^2$)

$$h\Delta f = -\frac{1}{2} \Delta \alpha_{sc} E^2$$

$$- \frac{1}{4} \alpha_{ten} \frac{[3M_F^2 - F(F+1)]}{F(2F-1)} (3 \cos^2 \theta - 1) E^2$$

$|e\rangle$  

$|g\rangle$

$\Delta \alpha_{sc} = \alpha_{sc}^e - \alpha_{sc}^g$

Electric fields cause Stark shifts

- Different frequency electric fields, so different polarisabilities
Stark shift – optical

- Strong extinction of cooling and repumper beams is vital
- Negligible shift due to probe laser at typical intensities (exception is 467 nm E3 transition in $^{171}\text{Yb}^+$)

Can interleave high/low power probes and extrapolate, or use a modified Ramsey scheme
Modified Ramsey schemes

- Problem: need to step frequency by exactly $\Delta$ to avoid shift
- Or else:
  - Insert $\pi$ pulse to reduce sensitivity of any error in the step
  - Invert phase of $\pi$ pulse to reduce sensitivity to pulse area
  - Interleave sequences with 90 deg phase shifts on $\pi/2$ pulses
  - Interleave sequences with different $T$ – auto balanced Ramsey

Ekkehard Peik

PRL 109, 213002 (2012); Phys. Rev. A 93, 010501 (2016); PRL 120, 053602 (2018)
Stark shift – thermal

- One of the hardest shifts to determine
- Blackbody radiation:
  \[ < E^2 > = 8.55 \times 10^{-5} T^4 \text{ (V}^2\text{/m}^2) \]
  \[ h\Delta f = -\frac{1}{2}\Delta\alpha_{sc}E^2 \]
- Difficult to characterise E-field when it’s not a blackbody distribution
- Ion mostly ‘sees’ trap structure. Need to characterise temperature rise and emissivities of all components, or use thermal imaging
- \( \Delta\alpha_{sc} \) (and hence BBR shift & uncertainty) generally smaller for ions than neutral atoms
Stark shift – rf trapping

- Motion in the trap leads not only to Doppler shifts, but also to the ion experiencing a time-averaged non-zero electric field.

- Measure the motion to deduce:
  
  Velocity $\rightarrow$ Amplitude of motion $\rightarrow$ $<E^2>$ $\rightarrow$ Stark shift
dc polarisabilities for different ions

<table>
<thead>
<tr>
<th></th>
<th>Ca(^+)</th>
<th>Sr(^+)</th>
<th>Yb(^+) E2</th>
<th>Yb(^+) E3</th>
<th>Hg(^+)</th>
<th>Al(^+)</th>
<th>In(^+)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta \alpha_{\text{SC}}) (10^{-41} \text{ Jm}^2/\text{V}^2)</td>
<td>-73.0</td>
<td>-47.938</td>
<td>+69</td>
<td>+8.88</td>
<td>+15</td>
<td>+0.82</td>
<td>+3.3</td>
</tr>
<tr>
<td>(\Delta \alpha_{\text{ten}}) (10^{-41} \text{ Jm}^2/\text{V}^2)</td>
<td>-24.51</td>
<td>-78.6</td>
<td>-136</td>
<td>+1</td>
<td>-3</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- Note that some polarisabilities are negative and some are positive

[A. Ludlow et al. Rev. Mod. Phys., 87, No. 2 (2015)]
Micromotion will cause both a Doppler and a Stark shift

\[ \frac{\Delta v_{\mu}}{v_0} = -\left( \frac{\Omega}{\omega_0} \right)^2 \left[ 1 + \frac{\Delta \alpha_0}{\hbar \omega_0} \left( \frac{m \Omega c}{e} \right)^2 \right] \sum_{x,y,z} R_i \]

Sr\(^+\) and Ca\(^+\) have negative \(\Delta \alpha\), so Doppler & Stark shifts cancel at the ‘magic trap drive frequency’

\[ \Omega_0 = \frac{e}{mc} \sqrt{-\frac{\hbar \omega_0}{\Delta \alpha_0}} \]

- Sr\(^+\): 14.4 MHz
- Ca\(^+\): 24.6 MHz

Designing traps to minimise shifts

- Motional effects → want low ion motional heating rate
  - Avoid noise on electrical signals and surface contaminants
  - Use large separation, $d$ between rf electrodes
    (ion heating = $1/d^\alpha$, where $2 < \alpha < 4$)
  - Operate with large $\Omega$ (and hence $V_{ac}$) to achieve large $\omega_i$

\[
\omega_i \approx \frac{q_i \Omega}{2\sqrt{2}}
\]

\[
q_z = \frac{4e\varepsilon V_{ac}}{mr_0^2 \Omega^2}
\]
Designing traps to minimise shifts

- **Thermal E-fields → want low temperature rise**
  - Operate with low $V_{ac}$ to minimise dielectric heating, and increase separation between $V_{ac}$ and ground
  - Careful choice of materials
  - Polish electrode surfaces to increase emissivity

- **rf E-fields → want no residual fields at trap centre**
  - Symmetric connections to rf electrodes
  - Compensation electrodes in pairs
Short break
Electric field gradients cause quadrupole shifts

- Electric field gradients interact with electric quadrupole moments

\[ \Delta f \propto Q_{dc} \cdot \Theta \cdot (3m_F^2 - F(F + 1)) \cdot (3 \cos^2 \beta - 1) \]

- States with ang. mom. \( J \geq 1 \) possess a quad. Mom., eg. \(^2\!D_{5/2}, \, ^3\!P_1\).
- Lattice clock transitions \(^1\!S_0 - ^3\!P_0\) are free from this effect
Quadrupole moments for various systems

<table>
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<th>Ca⁺</th>
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<th>Hg⁺</th>
<th>Al⁺</th>
<th>In⁺</th>
</tr>
</thead>
<tbody>
<tr>
<td>Θ (e a₀²)</td>
<td>1.83</td>
<td>2.6</td>
<td>2.08</td>
<td>-0.041</td>
<td>-0.510</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- No quadrupole moment in ¹¹⁵In⁺ or ¹⁷⁷Al⁺ because ¹S₀ – ³P₀
- For other systems, shift may be several Hz or more, so must be nulled.

[A. Ludlow et al. Rev. Mod. Phys., 87, No. 2 (2015)]
Nulling the quadrupole shift

$$\Delta f \propto (3 \cos^2 \beta - 1)$$

$\beta = \text{angle between B-field and E-field gradient}$

- Measure in three orthogonal B-field directions
- (Alternatively, measure 3 different pairs of Zeeman states)
- $(3 \cos^2 \beta - 1)$ averages to zero
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Magnetic fields – 1st order Zeeman shifts

Eliminated by probing two Zeeman components symmetrically placed about line centre, and taking the average frequency.

All components exhibit a linear Zeeman shift

$\Delta \nu_{\text{nat}} = 0.4 \text{ Hz}$

$\sim 10 \text{kHz in } 1 \mu\text{T for } ^{88}\text{Sr}^+$
Magnetic fields – 2\textsuperscript{nd} order Zeeman shifts

For odd isotope ions, with half-integer nuclear spin, can probe $m_F = 0 - m_F = 0$, so there is no 1st order Zeeman shift.

2\textsuperscript{nd} order Zeeman shift, $\Delta \nu = \kappa B^2$
### 2nd order Zeeman shift for different ions

<table>
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<th>Al⁺</th>
<th>In⁺</th>
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</thead>
<tbody>
<tr>
<td>2nd order coefficient Hz/mT²</td>
<td>+14.355</td>
<td>+3.1223</td>
<td>+52,096</td>
<td>-2,030</td>
<td>-18,900</td>
<td>-71.988</td>
<td>+4.09</td>
</tr>
<tr>
<td>2nd order Zeeman relative magnitudes</td>
<td>![Bar for Ca⁺]</td>
<td>![Bar for Sr⁺]</td>
<td>![Bar for Yb⁺ E2]</td>
<td>![Bar for Yb⁺ E3]</td>
<td>![Bar for Hg⁺]</td>
<td>![Bar for Al⁺]</td>
<td>![Bar for In⁺]</td>
</tr>
</tbody>
</table>

- The purple bars are 1,000 × larger

[A. Ludlow et al. Rev. Mod. Phys., 87, No. 2 (2015)]
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- Magnetic fields
  - **Gravity**

- Optical frequency metrology
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- Summary and future perspectives
Gravity potential affects clock frequency

- General relativity: time in different gravity potentials will run at different rates

- Frequency shift
  \[
  \frac{\Delta f}{f_0} = \frac{g \Delta h}{c^2}
  \]
  \( g = \) local acceleration due to gravity
  \( \Delta h = \) height above reference level
  \( c = \) speed of light

- Fractional change = \(10^{-16}\) for every 1 m change in height at Earth’s surface

- \(10^{-18}\) clocks sensitive to 1 cm height changes

Fact:
Your head ages faster than your feet by a couple of ns per year!
Potential above reference height

- Geoid - equipotential reference surface, local fluctuations
- GNSS gives height relative to ellipsoid
- Local modelling for difference between ellipsoid and geoid

Orthometric ht. \( (H) \) = Ht. Above Ellipsoid \( (HAE) \) – Geoid ht. \( (N) \)
Measuring gravity potentials at NMIs

- Surveys in 2013 – 2014 at INRIM, NPL, PTB, SYRTE
- 1 – 3 absolute gravity measurements at each site
- 35 – 99 relative gravity measurements in surrounding area
- Accuracy of potential above geoid is about 0.25 m²/s² (equivalent to 2.5 cm)
<table>
<thead>
<tr>
<th>Type of Shift</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Doppler shifts</strong></td>
<td>– due to motion of ion in trap</td>
</tr>
<tr>
<td><strong>Stark shifts</strong></td>
<td>– due to external electric fields</td>
</tr>
<tr>
<td></td>
<td>(optical, thermal, rf trap)</td>
</tr>
<tr>
<td><strong>Electric quadrupole shift</strong></td>
<td>– due to electric field gradients</td>
</tr>
<tr>
<td><strong>Zeeman shifts</strong></td>
<td>– due to external magnetic fields</td>
</tr>
<tr>
<td><strong>Gravitational redshift</strong></td>
<td>– due to location of ion</td>
</tr>
</tbody>
</table>
Uncertainty budget

- PTB Yb\(^+\) E3 – single-ion clock with best accuracy

<table>
<thead>
<tr>
<th>Effect</th>
<th>(\delta \nu/\nu_0) (10(^{-18}))</th>
<th>(u/\nu_0) (10(^{-18}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second-order Doppler shift</td>
<td>-3.7</td>
<td>2.1</td>
</tr>
<tr>
<td>Blackbody radiation shift</td>
<td>-70.5</td>
<td>1.8</td>
</tr>
<tr>
<td>Probe light related shift</td>
<td>0</td>
<td>1.1</td>
</tr>
<tr>
<td>Second-order Zeeman shift</td>
<td>-40.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Quadratic dc Stark shift</td>
<td>-1.2</td>
<td>0.6</td>
</tr>
<tr>
<td>Background gas collisions</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Servo error</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Quadrupole shift</td>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>Total</td>
<td>-115.8</td>
<td>3.2</td>
</tr>
</tbody>
</table>

[N. Huntemann et al., PRL 116, 063001 (2016)]
Which ions make the best clocks?

- Compact / simple systems – Ca$^+$, Sr$^+$
- Low systematic frequency shifts – In$^+$, Al$^+$
- High stability – Yb$^+$ (E3), Al$^+$

<table>
<thead>
<tr>
<th></th>
<th>Ca$^+$</th>
<th>Sr$^+$</th>
<th>Yb$^+$ E2</th>
<th>Yb$^+$ E3</th>
<th>Hg$^+$</th>
<th>Al$^+$</th>
<th>In$^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linewidth, $\Delta v$</td>
<td>✗</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
<td>✗</td>
</tr>
<tr>
<td>Stark shift, $\Delta \alpha_{SC}$</td>
<td>✗</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Quadrupole shift</td>
<td>✗</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>2nd order Zeeman</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Simple lasers</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✗</td>
</tr>
</tbody>
</table>

- $\Delta v$: Linewidth
- $\Delta \alpha_{SC}$: Stark shift
- Quadrupole shift
- 2nd order Zeeman
- Simple lasers
Outline

- Atomic clocks based on caesium
- The role of optical clocks
- Single-ion optical frequency standards
- Minimising systematic frequency shifts
  - Motion
  - Electric fields
  - Magnetic fields
  - Gravity

- Optical frequency metrology
  - Stability, absolute frequencies and ratios

- Summary and future perspectives
Measuring reproducibility and stability

\[ \sigma(\tau) = \frac{\Delta \nu}{\nu} \frac{\eta}{(S/N)} \sqrt{\frac{T}{\tau}} \]

- Compare two independent optical frequency standards
- Measure \((f_1 - f_2)\) for a period of time, repeatedly.
Stability depends on linewidths

Fractional instability

\[ \sigma(\tau) = \frac{\Delta \nu}{\nu} \frac{\eta}{(S/N)} \sqrt{\frac{T}{\tau}} \]

- Octupole stability limited only by laser linewidth as Yb\(^{+}\)(E3) natural linewidth \(~ 1\) nHz

Octupole:
\[ \sigma(\tau) = 5 \times 10^{-15} / \sqrt{\tau} \]
100ms probe pulses
10Hz linewidths

Quadrupole:
\[ \sigma(\tau) = 1 \times 10^{-14} / \sqrt{\tau} \]
30ms probe pulses
30Hz linewidths
Measuring the absolute frequency
Measuring the absolute frequency

Cs fountain

H maser

Femtosecond optical frequency comb

Ion trap

AOM

x2

Laser

Ion
Limited by uncertainty of Cs primary standards

Secondary representations of the second

- Optical frequency standards can be used to realise the SI second (although uncertainty cannot be better than Cs primary standard)
- List of secondary representations of the second now includes eight optical frequency standards

<table>
<thead>
<tr>
<th>Atom or ion</th>
<th>Transition</th>
<th>Wavelength</th>
<th>Recommended fractional uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{199}$Hg</td>
<td>$^1S_0 - ^3P_0$</td>
<td>266 nm</td>
<td>$5 \times 10^{-16}$</td>
</tr>
<tr>
<td>$^{27}$Al$^+$</td>
<td>$^1S_0 - ^3P_0$</td>
<td>267 nm</td>
<td>$19 \times 10^{-16}$</td>
</tr>
<tr>
<td>$^{199}$Hg$^+$</td>
<td>$^2S_{1/2} - ^2D_{5/2}$</td>
<td>282 nm</td>
<td>$19 \times 10^{-16}$</td>
</tr>
<tr>
<td>$^{171}$Yb$^+$</td>
<td>$^2S_{1/2} - ^2D_{3/2}$</td>
<td>436 nm</td>
<td>$6 \times 10^{-16}$</td>
</tr>
<tr>
<td>$^{171}$Yb$^+$</td>
<td>$^2S_{1/2} - ^2F_{7/2}$</td>
<td>467 nm</td>
<td>$6 \times 10^{-16}$</td>
</tr>
<tr>
<td>$^{171}$Yb</td>
<td>$^1S_0 - ^3P_0$</td>
<td>578 nm</td>
<td>$5 \times 10^{-16}$</td>
</tr>
<tr>
<td>$^{88}$Sr$^+$</td>
<td>$^2S_{1/2} - ^2D_{5/2}$</td>
<td>674 nm</td>
<td>$15 \times 10^{-16}$</td>
</tr>
<tr>
<td>$^{87}$Sr</td>
<td>$^1S_0 - ^3P_0$</td>
<td>698 nm</td>
<td>$4 \times 10^{-16}$</td>
</tr>
</tbody>
</table>
Measuring optical frequency ratios
Optical frequency ratios between labs

- Local comparisons
- Transportable clocks
- Satellite links (Two-way and GPS)
- Optical fibre links

Ca\(^+\): \(7.8 \times 10^{-17}\)

WUHAN

Ion optical ratios – variation of constants

- Sensitivity to variation of fine structure constant

<table>
<thead>
<tr>
<th>Ion</th>
<th>Relative sensitivity</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sr$^+$</td>
<td></td>
<td>0.43</td>
</tr>
<tr>
<td>Yb$^+$(E2)</td>
<td></td>
<td>1.00</td>
</tr>
<tr>
<td>Yb$^+$(E3)</td>
<td></td>
<td>-5.95</td>
</tr>
<tr>
<td>Hg$^+$</td>
<td></td>
<td>-2.94</td>
</tr>
<tr>
<td>Al$^+$</td>
<td></td>
<td>0.008</td>
</tr>
<tr>
<td>In$^+$</td>
<td></td>
<td>0.18</td>
</tr>
</tbody>
</table>

NIST - $^{199}$Hg$^+:^{27}$Al$^+$

NPL, PTB - $^{171}$Yb$^+$(E3):(E2)

[Rosenband et al., Science 319, 1808 (2008); Godun et al., PRL 113, 210801 (2014); Huntemann et al., PRL 113, 210802 (2014) ]
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**Summary and future perspectives**
Summary of ion optical clocks

- Systematic frequency shifts are generally low
- Laser systems generally simple
- Traps can be compact
- Sensitive tools for fundamental physics tests
- Single ions give poor frequency instability

\[
\sigma(\tau) = \frac{\Delta \nu}{\nu} \frac{\eta}{(S/N)} \sqrt{\frac{T}{\tau}}
\]

- Need to reduce instability:
  (a) Increase probe time to reduce \( \Delta \nu \sqrt{T} \) – no limit for Yb\(^+\)(E3)
  (b) Increase \( \nu \) by choosing an ion with UV transitions
  (c) Increase S/N by increasing number of ions
### Ions with UV transitions increase $v_0$

<table>
<thead>
<tr>
<th>Example</th>
<th>Highly charged ions</th>
<th>Thorium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>$\text{Ir}^{17+}$, sympathetically cooled</td>
<td>$\text{Th}^{2+}, \text{Th}^{3+}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>UV transition</th>
<th>Electrons more tightly bound</th>
<th>Nuclear transition</th>
</tr>
</thead>
</table>

| Low systematics | Transitions shift more towards the UV, hence smaller dc and rf electric polarisabilities | Nucleus is highly isolated from environment due to electron cloud |

| Highly relativistic | Larger relativistic shifts provide enhanced sensitivity to variation of constants | Sensitivity to variation of constants speculated to be enhanced by as much as 5–6 orders of magnitude |

*Ekkehard Peik*
Multi-ion clocks improve SNR by $1/\sqrt{N}$

- Challenge: to retain homogeneous frequency shifts across whole string of ions

- Linear traps - dc electric field gradients along axis, so need ions with low quadrupole moment (eg Al$^+$, In$^+$, Yb$^+(E3)$)

- Entanglement schemes give further $1/\sqrt{N}$ improvement; still many forms of decoherence present a challenging obstacle

<table>
<thead>
<tr>
<th>Stability</th>
<th>Time to reach $1 \times 10^{-18}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single ion</td>
<td>$1 \times 10^{-15} / \sqrt{\tau}$</td>
</tr>
<tr>
<td>10 uncorrelated ions ($1/\sqrt{N}$)</td>
<td>$3 \times 10^{-16} / \sqrt{\tau}$</td>
</tr>
</tbody>
</table>
Ions are still very promising candidates for optical clocks
Further reading

Text books
- F. Riehle, “Frequency Standards”, Wiley-VCH

Publications

- OC18 specifications document ... coming soon to www.oc18.eu